Which Equation Represents A Circle With A Center Point Of (-2, 3) That Passes Through The Point (4, -3)

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Understanding the equation of a circle is fundamental in coordinate geometry. In this article, we will explore how to determine the specific equation of a circle given its center and a point through which it passes. Specifically, we will analyze the problem of finding the equation of a circle with a center at (-2, 3) that passes through the point (4, -3). By examining the principles involved and applying the appropriate formulas, readers will gain a comprehensive understanding of how to derive the circle's equation.

Fundamentals of the Equation of a Circle

Standard Form of a Circle Equation

The equation of a circle in the coordinate plane is typically expressed in its standard form:

$$ (x - h)^2 + (y - k)^2 = r^2 $$

where:

- (h, k) is the center of the circle.
- r is the radius of the circle.

This form explicitly reveals the center and radius, making it straightforward to analyze and graph the circle.

Determining the Center and Radius

Given the center (h, k), the key task is to find the radius r. If the circle passes through a point (x, y), then the radius is simply the distance between the center and this point.

Mathematically, this is expressed as:

$$ r = \sqrt{(x - h)^2 + (y - k)^2} $$

or, in the standard form, once r^2 is known, the equation can be written directly.

Applying the Concepts to Our Problem

Given Data

- Center point: $(-2, 3)$
- Point on the circle: $(4, -3)$

Our goal is to:

1. Calculate the radius r using the distance formula.
2. Write the equation of the circle in standard form.

Calculating the Radius r

Using the distance formula:

$$r = \sqrt{(x - h)^2 + (y - k)^2}$$

Substituting the given points:

$$r = \sqrt{(4 - (-2))^2 + (-3 - 3)^2}$$

Simplify the expressions:

$$r = \sqrt{(4 + 2)^2 + (-6)^2}$$

$$r = \sqrt{6^2 + (-6)^2}$$

$$r = \sqrt{36 + 36}$$

$$r = \sqrt{72}$$

$$r = \sqrt{36 \times 2}$$

$$r = 6 \sqrt{2}$$

Therefore, the radius r is $6 \sqrt{2}$.

Formulating the Equation

Knowing r, the equation of the circle is:

$$(x - h)^2 + (y - k)^2 = r^2$$

Substitute $h = -2$, $k = 3$, and $r^2 = (6 \sqrt{2})^2$:

$$(x - (-2))^2 + (y - 3)^2 = (6 \sqrt{2})^2$$

Simplify:

$$(x + 2)^2 + (y - 3)^2 = 36 \times 2$$

$$(x + 2)^2 + (y - 3)^2 = 72$$

This is the standard form of the circle's equation.

Final Equation and Its Significance

The precise equation representing the circle with center at (-2, 3) passing through (4, -3) is:

$$(x + 2)^2 + (y - 3)^2 = 72$$

This equation fully describes the circle's geometry, including its location and size.

Understanding the Components of the Equation

- Center: The coordinates $(-2, 3)$ are evident from the form $(x - h)^2 + (y - k)^2$, where $h = -2$ and $k = 3$.
- Radius: The radius is $r = 6 \sqrt{2}$. Its length indicates the distance from the center to any point on the circle, which in this case, includes the point $(4, -3)$.

Additional Insights and Applications

Visualizing the Circle

Plotting the circle involves marking the center at (-2, 3) and drawing a circle with radius $6 \sqrt{2}$. Since $\sqrt{2} \approx 1.414$, the radius approximately equals:

$$6 \times 1.414 \approx 8.485$$

This provides a tangible sense of the circle's size in the coordinate plane.

Why Is This Important?

Understanding how to derive the equation of a circle from given points and centers is essential in various fields, including:

- Geometry and algebraic problem-solving
- Computer graphics, where circles are used in rendering and design
- Engineering, for designing circular components
- Physics, in modeling circular motion

Common Mistakes to Avoid

- Mixing up the signs in the standard form, especially when dealing with the center's coordinates.
- Forgetting to square the radius when writing the equation.
- Confusing the point passing through with the center point, leading to incorrect radius calculation.
- Not simplifying the radius properly, which may lead to errors in the final equation.

Additional Examples and Practice Problems

To reinforce understanding, consider practicing with different centers and points. For example:

- Find the equation of a circle with center (h, k) passing through a given point (x, y).
- Determine the radius and write the equation for the circle given various points.

Sample Practice Problem:

Find the equation of a circle with center at $(1, -2)$ passing through $(4, 2)$.

Solution Steps:

1. Calculate the radius using the distance formula.
2. Write the standard form equation.

This practice helps solidify the method of deriving circle equations from given data.

Conclusion

In summary, finding the equation of a circle based on its center and a point on the circle involves calculating the radius via the distance formula and then substituting into the standard form of the circle's equation. For the given problem, the equation is:

$$ (x + 2)^2 + (y - 3)^2 = 72 $$

This equation uniquely characterizes the circle with center at (-2, 3) passing through (4, -3). Mastery of these concepts enables students and professionals to analyze, graph, and apply circles effectively across various mathematical and real-world contexts.

Frequently Asked Questions

What is the general form of the equation of a circle with center (-2, 3)?

The general form is $(x + 2)^2 + (y - 3)^2 = r^2$, where r is the radius.

How do you find the radius of the circle passing through the point (4, -3) with center (-2, 3)?

Calculate the distance between the center and the point: $r = \sqrt{(4 + 2)^2 + (-3 - 3)^2} = \sqrt{6^2 + (-6)^2} = \sqrt{36 + 36} = \sqrt{72} = 6\sqrt{2}$.

What is the specific equation of the circle with center (-2, 3) passing through (4, -3)?

Using the radius $r = 6\sqrt{2}$, the equation is $(x + 2)^2 + (y - 3)^2 = (6\sqrt{2})^2 = 72$.

How do you derive the equation of a circle given its center and a point on the circle?

Find the radius by calculating the distance between the center and the point, then substitute into the standard circle equation $(x - h)^2 + (y - k)^2 = r^2$.

What is the importance of the radius in the equation of a circle?

The radius determines the size of the circle and appears as r^2 in the standard form equation.

Can you verify that the point (4, -3) lies on the circle with center (-2, 3) and radius $6\sqrt{2}$?

Yes, by plugging in (4, -3) into the equation: $(4 + 2)^2 + (-3 - 3)^2 = 6\sqrt{2})^2$, which simplifies to $36 + 36 = 72$, confirming it's on the circle.

What is the final equation of the circle centered at (-2, 3) passing through (4, -3)?

The equation is $(x + 2)^2 + (y - 3)^2 = 72$.

Additional Resources

Understanding the Equation of a Circle: Center and Radius Fundamentals

When delving into the geometry of circles, one of the foundational concepts is understanding how to derive their equations based on given points and properties. The question at hand—"Which equation represents a circle with a center point of (-2, 3) that passes through the point (4, -3)?"—serves as an excellent example to explore the core principles and formulas involved in circle equations. To thoroughly address this, we will examine the general form of a circle's equation, how to determine its radius using known points, and subsequently, how to formulate its specific equation.

Fundamentals of a Circle's Equation

Before moving into calculations, it’s essential to comprehend the standard forms of a circle's equation and the components involved.

Standard Form of a Circle's Equation

The most common and recognizable form of a circle's equation in a coordinate plane is:

$$
(x - h)^2 + (y - k)^2 = r^2
$$

where:

- (h, k) is the center of the circle.
- r is the radius of the circle.

This form clearly indicates the geometric relationship between the circle's center and a point on its circumference. When given the center coordinates, the primary task becomes calculating the radius, which can then be substituted into this formula.

General Equation of a Circle

Another form is the expanded general form:

$$
x^2 + y^2 + Dx + Ey + F = 0
$$

which can be transformed into the standard form by completing the square. However, given the information in our problem, the standard form is more straightforward.

Determining the Radius of the Circle

The key step in formulating the circle's equation is calculating its radius. Given:

- Center point (h, k) = (-2, 3)
- A point on the circle (x, y) = (4, -3)

The radius r is the distance between the center and this point, which can be computed using the distance formula:

$$
r = \sqrt{(x - h)^2 + (y - k)^2}
$$

Step-by-step calculation:

1. Identify the known points:

- Center: $(-2, 3)$

- Point on the circle: $(4, -3)$

2. Calculate differences in x and y:

- $\Delta x = 4 - (-2) = 4 + 2 = 6$

- $\Delta y = -3 - 3 = -6$

3. Compute the squared differences:

- $(\Delta x)^2 = 6^2 = 36$

- $(\Delta y)^2 = (-6)^2 = 36$

4. Sum the squared differences:

$$
36 + 36 = 72
$$

5. Calculate the radius:

$$
r = \sqrt{72} = \sqrt{36 \times 2} = 6 \sqrt{2}
$$

Therefore, the radius of the circle is $6 \sqrt{2}$.

Formulating the Equation of the Circle

With the center $(-2, 3)$ and radius $6 \sqrt{2}$, the standard form of the circle's equation becomes:

$$
(x - (-2))^2 + (y - 3)^2 = (6 \sqrt{2})^2
$$

which simplifies to:

$$
(x + 2)^2 + (y - 3)^2 = (6 \sqrt{2})^2
$$

Calculating the right-hand side:

$$
(6 \sqrt{2})^2 = 6^2 \times (\sqrt{2})^2 = 36 \times 2 = 72
$$

Final equation:

$$
(x + 2)^2 + (y - 3)^2 = 72
$$

This is the explicit equation of the circle with the given center passing through the specified point.

Deep Dive: Analyzing the Components of the Equation

Each part of the circle's equation has geometric significance:

- $(x + 2)$: The horizontal shift from the origin, indicating the center is 2 units left of the y-axis.
- $(y - 3)$: The vertical shift from the origin, indicating the center is 3 units above the x-axis.
- 72: The square of the radius, revealing the distance from the center to any point on the circle.

Understanding these components allows for better visualization and manipulation of the circle's properties.

Verification: Confirming the Point Lies on the Circle

It's important to verify that the point (4, -3) truly satisfies the equation:

$$
(x + 2)^2 + (y - 3)^2 = 72
$$

Substitute the point:

$$
(4 + 2)^2 + (-3 - 3)^2 = 6^2 + (-6)^2 = 36 + 36 = 72
$$

Since the equation holds true, the point indeed lies on the circle, confirming the correctness of our derived equation.

Additional Considerations and Variations

While the problem specifies a particular circle, it’s useful to explore related concepts and variations:

1. Equation in Expanded Form

Expanding the standard form:

\[
(x + 2)^2 + (y - 3)^2 = 72
\]

Gives:

\[
x^2 + 4x + 4 + y^2 - 6y + 9 = 72
\]

Combine like terms:

\[
x^2 + y^2 + 4x - 6y + (4 + 9) = 72
\]

Simplify:

\[
x^2 + y^2 + 4x - 6y - 59 = 0
\]

This form is less intuitive but useful for certain algebraic manipulations.

2. Graphical Interpretation

Plotting the circle involves:

- Marking the center at \((-2, 3)\).
- Drawing a circle with radius \(6 \sqrt{2} \approx 8.485\).
- Ensuring the circle passes through (4, -3).

Understanding the geometric layout aids in visual comprehension and problem-solving.

3. Parametric Equations

Alternatively, the circle can be expressed parametrically:

$$
x = h + r \cos \theta
$$
$$
y = k + r \sin \theta
$$

where θ ranges from 0 to 2π. For our circle:

$$
x = -2 + 6 \sqrt{2} \cos \theta
$$
$$
y = 3 + 6 \sqrt{2} \sin \theta
$$

This form is particularly useful in calculus and physics applications.

Summary and Final Remarks

In conclusion, the process of determining the accurate equation of a circle based on its center and a point on its circumference involves:

- Recognizing the standard form of the circle's equation.
- Computing the radius using the distance formula.
- Substituting the center coordinates and radius into the standard form to derive the explicit equation.

For the given problem, the precise equation is:

$$
\boxed{(x + 2)^2 + (y - 3)^2 = 72}
$$

This equation embodies the geometric conditions specified—center at $(-2, 3)$ and passing through $(4, -3)$. Understanding each step in this process enhances conceptual clarity and equips learners with a robust approach to tackling similar problems involving circle equations. Whether for graphing, algebraic manipulation, or geometric reasoning, mastering these principles is essential for a comprehensive understanding of circle properties in coordinate geometry.

In essence, a clear grasp of the relationship between a circle’s center, radius, and passing points allows for accurate formulation of its equation, serving as a fundamental skill in mathematics and related fields.

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