Which Graph Shows The Solution To The System Of Linear Inequalities?2x 3y <12y < 3On A Coordinate

Which Graph Shows The Solution To The System Of Linear Inequalities? 2x + 3y < 12, y < 3 On A Coordinate

Understanding how to interpret and graph systems of linear inequalities is fundamental in algebra and coordinate geometry. When presented with a system such as "2x + 3y < 12" and "y < 3," the goal is to determine which graph accurately depicts all the solutions satisfying both inequalities simultaneously. This comprehensive guide will walk you through the concepts, steps, and tips needed to identify the correct graph for this system, ensuring you have a clear understanding of linear inequalities, their graphical representations, and how to analyze multiple inequalities together.

Understanding Systems of Linear Inequalities

Before diving into the specific inequalities, it’s crucial to understand what systems of inequalities represent and how they are visualized.

What Are Linear Inequalities?

A linear inequality is an inequality that involves a linear expression in two variables, such as x and y. Examples include:
- 2x + 3y < 12
- y < 3

These inequalities define regions in the coordinate plane where the solutions satisfy the inequalities.

What Is a System of Linear Inequalities?

A system of inequalities consists of two or more inequalities considered simultaneously. The solution to the system is the set of all points (x, y) that satisfy all inequalities at once.

For example:
- 2x + 3y < 12
- y < 3

The solution set is the intersection of the regions defined by each inequality.

Graphing Linear Inequalities: The Basics

Graphing inequalities involves shading regions of the coordinate plane that satisfy the inequality. Here's the general process:

Step 1: Graph the Boundary Line

- Convert the inequality to an equation to find the boundary line.
- For "<" or ">", draw a dashed line since points on the line are not included.
- For "≤" or "≥", draw a solid line.

Step 2: Shade the Correct Region

- Pick a test point not on the boundary line (commonly (0,0) if it's not on the line).
- Substitute the test point into the inequality.
- If the inequality holds true, shade the side of the boundary line containing the test point.
- If not, shade the opposite side.

Step 3: Repeat for All Inequalities

- The solution to the system is the overlapping shaded region where all inequalities are satisfied.

Analyzing the Given System of Inequalities

Let's analyze the specific inequalities:

1. $2x + 3y < 12$
2. $y < 3$

Understanding each individually helps in determining the combined solution.

Graphing $2x + 3y < 12$

- First, convert to equality: $2x + 3y = 12$
- Find intercepts:

- When x = 0: 3y = 12 → y = 4
- When y = 0: 2x = 12 → x = 6
- Draw the boundary line through points (0,4) and (6,0):
- Since the inequality is "<", draw the line as dashed.
- To find which side to shade:
- Pick a test point, like (0,0):
- 2(0) + 3(0) = 0 < 12 → true
- So, shade the side containing (0,0), which is below or to one side of the line.

Graphing y < 3

- Boundary line: y = 3
- Since the inequality is "<", draw the line as dashed.
- Test point (0,0):
- 0 < 3 → true
- Shade below the line y=3.

Identifying the Solution Region

The overall solution to the system is the intersection of:

- The region below the dashed line 2x + 3y = 12
- The region below the dashed line y = 3

The overlapping shaded region is where both conditions are satisfied simultaneously.

How To Recognize the Correct Graph

When choosing the correct graph among multiple options, consider the following:

1. Boundary Lines

- Confirm that the boundary lines are dashed (since inequalities are "<" or "<=").
- Ensure the boundary lines are positioned correctly based on intercepts.

2. Shaded Regions

- Check that the shading corresponds to the inequalities:
- For $2x + 3y < 12$, shaded below the line connecting (0,4) and (6,0).
- For $y < 3$, shaded below $y=3$ line.
- The solution is the intersection of these shaded regions.

3. Test Points

- Verify that the test point (0,0) lies within the shaded region for both inequalities.

Common Mistakes to Avoid

- Using solid lines for "<" or ">" inequalities.
- Shading on the wrong side of the boundary.
- Overlooking the direction of inequalities, especially when they involve "<" or ">".
- Forgetting to test a point to confirm shading.

Visual Representation: Step-by-Step Graphing Guide

Here's a step-by-step approach to graph and analyze the system:

1. Draw the boundary line for $2x + 3y = 12$:
- Plot points (0,4) and (6,0).
- Draw a dashed line through these points.

2. Shade the region for $2x + 3y < 12$:
- Test (0,0): $0 < 12$? Yes → shade below the line.

3. Draw the boundary line for $y = 3$:
- Draw a dashed horizontal line at $y=3$.

4. Shade the region for $y < 3$:
- Test (0,0): $0 < 3$? Yes → shade below $y=3$.

5. Identify the intersection:
- The solution region is where both shadings overlap, which is:
- Below the line $2x + 3y = 12$
- Below $y=3$

6. Verify with test points:
- Pick (1,2): check both inequalities:
- $2(1) + 3(2) = 2 + 6 = 8 < 12$? Yes.
- $y=2 < 3$? Yes.
- Since (1,2) satisfies both, it's within the solution region.

Choosing the Correct Graph

Based on the above analysis, the correct graph should show:

- A dashed line for $2x + 3y = 12$, passing through (0,4) and (6,0).
- A dashed horizontal line at $y=3$.
- Shaded regions:
- Below the $2x + 3y = 12$ line.
- Below the $y=3$ line.
- The overlapping shaded region representing the solution set.

In multiple-choice options, look for:
- Accurate placement of boundary lines.
- Correct shading consistent with the inequalities.
- Proper dashed lines for "<" inequalities.

Conclusion: Understanding and Interpreting Graphs of Linear Inequalities

Mastering graphing systems of inequalities involves understanding boundary lines, shading correctly, and analyzing the intersection of regions. When confronted with a system like "$2x + 3y < 12$" and "$y < 3$," carefully plot the boundary lines, verify the shading, and test points to confirm your solution region. Recognizing these features in the provided options will lead you to the correct graph that accurately depicts the solution set.

Practicing with various systems enhances your ability to quickly and accurately interpret graphs, a vital skill in algebra, calculus, and real-world problem solving involving inequalities.

Final tip: Always double-check your boundary lines, shading, and test points to confirm the graph matches the inequalities precisely. With practice, identifying the correct graph becomes intuitive, allowing you to solve similar problems efficiently and confidently.

Frequently Asked Questions

What does the graph of a system of linear inequalities represent?

It shows the region where all the inequalities in the system are satisfied simultaneously, typically shaded areas on the coordinate plane.

How can I identify the solution area for the system $2x + 3y < 12$ and $y < 3$ on a graph?

You should graph both inequalities, shade the regions satisfying each, and find the overlapping shaded region, which represents the solution set.

What is the significance of the boundary lines when graphing inequalities like $2x + 3y < 12$?

The boundary line, $2x + 3y = 12$, is drawn as a dashed line if the inequality is strict ($<$ or $>$), indicating points just touching the line are not included in the solution.

How do I determine which side of the boundary line to shade for $y < 3$?

Test a point not on the line, like $(0,0)$. If it satisfies $y < 3$, shade that side; otherwise, shade the opposite side.

What does the double inequality $12y < 3$ mean in the context of graphing?

It appears to be a typo or formatting error; likely, it was meant to be two separate inequalities: $2x + 3y < 12$ and $y < 3$. Each is graphed separately, and their solution is the intersection.

How can I tell if a point is part of the solution to the system $2x + 3y < 12$ and $y < 3$?

Substitute the point's coordinates into both inequalities; if both are true, the point lies in the solution region.

What are common mistakes when graphing systems of linear inequalities?

Common mistakes include using solid lines for strict inequalities, shading the wrong side, and forgetting to test points to determine shading areas.

Is the solution to the system always a bounded region?

Not necessarily; the solution can be bounded or unbounded depending on the inequalities. For example, inequalities like $y < 3$ produce unbounded regions.

Can the graph of the system 2x + 3y < 12 and y < 3 be represented with a single shaded region?

Yes, the solution is the intersection of the regions satisfying each inequality, resulting in a specific shaded area on the coordinate plane.

What tools can help me accurately graph the solution to this system?

Graphing calculators, online graphing tools like Desmos, or graph paper and a ruler can help accurately depict the boundary lines and shading regions.

Additional Resources

Which Graph Shows The Solution To The System Of Linear Inequalities? 2x + 3y < 12 and y < 3 On A Coordinate Plane

Understanding how to determine which graph represents the solution to a system of linear inequalities is an essential skill in algebra and coordinate geometry. When faced with the system $2x + 3y < 12$ and $y < 3$, the challenge is to interpret these inequalities correctly and visualize their combined solution region on a coordinate plane. This article provides a comprehensive guide to analyzing such systems, explaining how to graph inequalities accurately, and identifying the graph that shows the solution set.

Understanding the System of Inequalities

Before diving into graphing, it's critical to interpret each inequality individually, understanding their boundary lines and the regions they encompass.

What Are Linear Inequalities?

Linear inequalities are inequalities involving linear expressions, such as $ax + by < c$, $ax + by > c$, $ax + by \leq c$, or $ax + by \geq c$. These define a half-plane on the coordinate plane, which is either above, below, to the left, or to the right of the boundary line.

The Given System

The system provided is:

1. $2x + 3y < 12$
2. $y < 3$

The goal is to find the region in the coordinate plane that satisfies both inequalities simultaneously.

Step-by-Step Approach to Graphing the System

Step 1: Graph the Boundary Lines

Each inequality corresponds to a boundary line, which is obtained by replacing the inequality sign with an equal sign.

- For $2x + 3y = 12$
- For $y = 3$

The boundary lines are critical because the solution regions are the half-planes determined by these lines.

Step 2: Plot the Boundary Lines

Line 1: $2x + 3y = 12$

- Find intercepts:
- x-intercept: Set $y=0$, then $2x = 12 \rightarrow x=6$
- y-intercept: Set $x=0$, then $3y=12 \rightarrow y=4$

- Plot points:
- (6, 0)
- (0, 4)

- Draw a straight line through these points. Since the inequality is < (less than), the boundary line will be dashed to indicate that points on the line are not included in the solution.

Line 2: $y = 3$

- This is a horizontal line crossing the y-axis at 3.
- Draw a dashed horizontal line through $y=3$, since the inequality is < (less than).

Step 3: Determine Which Side of the Boundary Lines Represent the Solution

This involves testing a point not on the boundary line (commonly the origin, (0,0)).

For $2x + 3y < 12$:

- Test point: (0,0)
- Plug into inequality: $2(0) + 3(0) = 0 < 12$ → True
- So, the half-plane below or above the line, depending on the inequality, contains the solution.

Since the inequality is less than, the solution is the half-plane below the line $2x + 3y = 12$.

For $y < 3$:

- Test point: (0,0)
- Since $0 < 3$, the point satisfies the inequality.
- The solution region is below the line $y=3$ (since $y<3$), which indicates the half-plane below or toward the $y=3$ line.

Step 4: Shade the Correct Regions

- For $2x + 3y < 12$, shade the region below the dashed line connecting (6,0) and (0,4).
- For $y < 3$, shade the region below the horizontal dashed line $y=3$.

Note: The solution to the system is the intersection of these two shaded regions.

Visualizing the Solution Region

Understanding the combined solution involves recognizing the overlap between the two shaded half-planes:

- The region below the line $2x + 3y = 12$ (a sloped line).
- The region below the line $y=3$ (a horizontal line).

The intersection, therefore, is the part of the coordinate plane below both lines.

Identifying the Correct Graph

When presented with multiple graphs, look for the one that:

- Has a dashed line for $2x + 3y = 12$.
- Has a dashed horizontal line at $y=3$.
- Shades the region below both lines.
- The overlapping shaded region should be bounded above by $y=3$ and below by the line $2x + 3y = 12$, extending infinitely to the sides unless explicitly limited.

Additional Tips for Analyzing and Graphing Such Systems

1. Always Convert Inequalities to Equalities First

Plot the boundary lines by setting the inequality to an equal sign. This helps visualize the boundary.

2. Use Test Points

Test points (like (0,0)) to determine which side of the boundary lines to shade.

3. Consider the Nature of the Inequality Sign

- < or > indicates the boundary line is dashed.
- ≤ or ≥ indicates the boundary line is solid.

4. Find the Intersection Points

Identify where the boundary lines intersect to understand the limits of the solution region.

5. Verify the Solution Region

Make sure the shaded region satisfies both inequalities simultaneously.

Common Pitfalls to Avoid

- Confusing the direction of shading.
- Assuming the boundary line is part of the solution when the inequality is strict (< or >).
- Overlooking the need for dashed or solid boundary lines.

Summary: Which Graph Shows The Solution?

The correct graph will depict:

- The boundary line $2x + 3y = 12$ as a dashed line.
- The boundary line $y=3$ as a dashed horizontal line.
- The shaded region below both lines, representing the intersection of the half-planes $2x + 3y < 12$ and $y < 3$.
- The intersection area will be an infinite region bounded above by $y=3$ and below by $2x + 3y = 12$, extending to the left and right.

Final Thoughts

Mastering the process of graphing systems of inequalities involves a combination of algebraic manipulation, visual interpretation, and logical reasoning. Recognizing the correct boundary lines, shading the appropriate regions, and identifying the intersection are crucial skills for solving such problems. When presented with multiple graphs, focus on the dashed or solid lines and the shading pattern—these visual cues are key to selecting the graph that accurately represents the solution to the system.

By following this detailed guide, you'll be better equipped to analyze, graph, and interpret systems of linear inequalities confidently.

Which Graph Shows The Solution To The System Of Linear Inequalities 2x 3y Lt 12y Lt 3on A Coordinate

Which Graph Shows The Solution To The System Of Linear Inequalities 2x 3y Lt 12y Lt 3on A Coordinate

Related Articles

- What Reasons Does Warren Provide In This Passage To Support The Claim? Select Three Options. The Policy
- A Borrower Sells A Mortgaged Property For Less Than What Is Owed On The Loan Balance. This Is An Example
- A Planet In Another Solar System Orbits A Star With A Mass Of 4, 00 X 108 Kg. At One Point In Its Orbit,

Back to Home