Which Number Line Represents The Solution Set For The Inequality 3(8 4x) < 6(x 5)?A Number Line From

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Understanding inequalities and their graphical representations is fundamental in algebra. When dealing with inequalities like 3(8 - 4x) < 6(x - 5), it’s crucial to comprehend how to solve them step-by-step and interpret their solutions on a number line. This article provides a thorough guide to solving this specific inequality, determining its solution set, and accurately representing this set on a number line. Whether you're a student preparing for exams or a math enthusiast reviewing key concepts, this comprehensive guide will clarify the process and enhance your understanding of inequalities and their graphical solutions.

Understanding the Inequality

Before solving, let's analyze the inequality:

3(8 - 4x) < 6(x - 5)

This inequality involves algebraic expressions within parentheses, multiplied by constants, and a comparison operator. Our goal is to isolate the variable x and determine the range of values satisfying the inequality.

Step-by-Step Solution Process

Step 1: Expand Both Sides

Distribute the coefficients across the parentheses:

- Left side: $3 8 = 24$; $3 (-4x) = -12x$
- Right side: $6 x = 6x$; $6 (-5) = -30$

So, the inequality becomes:

$$24 - 12x < 6x - 30$$

Step 2: Collect Like Terms

To isolate x, gather all x terms on one side and constants on the other:

Subtract 6x from both sides:

$$24 - 12x - 6x < -30$$

Simplify:

$$24 - 18x < -30$$

Next, subtract 24 from both sides:

$$-18x < -30 - 24$$

$$-18x < -54$$

Step 3: Solve for x

Divide both sides by -18:

$$x > (-54) / (-18)$$

Recall that dividing both sides of an inequality by a negative number reverses the inequality sign:

$$x > 3$$

Key Point: Since we divided by -18 (a negative), we flip the inequality sign from "<" to ">".

Interpreting the Solution

The solution to the inequality is:

$x > 3$

This means that any real number greater than 3 satisfies the original inequality.

Representing the Solution Set on a Number Line

A number line provides a visual representation of the solution set. For $x > 3$:

- The boundary point is at 3.
- Since the inequality is strict (>), the point at 3 is not included in the solution set.
- The solution includes all numbers to the right of 3.

Steps to draw the number line:

1. Draw a horizontal line with marked points.
2. Locate the point at 3.
3. Use an open circle at 3 to indicate that 3 is not included.
4. Shade or draw an arrow extending to the right of 3 to show all numbers greater than 3 are solutions.

Additional Notes on Number Line Representation

- Open Circle: Indicates the boundary point is not part of the solution (used for strict inequalities: > or <).
- Closed Circle: Indicates the boundary point is part of the solution (used for inclusive inequalities: ≥ or ≤).
- Arrow: Shows the direction of the solution set extending infinitely in that direction.

Understanding Inequalities and Their Graphs

Graphing inequalities helps visualize the solutions:

- For $x > 3$, the graph starts just to the right of 3 with an open circle.
- The arrow points infinitely to the right, covering all larger numbers.

Example of the graphical representation:

```plaintext
---o=====================>
3
```

- The open circle at 3 indicates that 3 is not included.
- The arrow indicates all values greater than 3.

Common Mistakes to Avoid

While solving inequalities, watch out for these common errors:

- Forgetting to flip the inequality sign when dividing or multiplying both sides by a negative number.
- Misinterpreting the solution as including the boundary point when the inequality is strict.
- Incorrect distribution of coefficients over parentheses, leading to wrong simplified expressions.
- Not checking the solution by substituting a value greater than 3 into the original inequality to verify.

Additional Examples for Practice

To reinforce understanding, here are some related inequalities and their solutions:

1. $x + 2 \geq 5$

- Solution: $x \geq 3$
- Number line: closed circle at 3, shading to the right.

2. $-4x < 8$

- Divide both sides by -4 (flip sign): $x > -2$
- Number line: open circle at -2, shading to the right.

3. $2x - 1 \leq 7$

- Add 1: $2x \leq 8$
- Divide both sides by 2: $x \leq 4$
- Number line: closed circle at 4, shading to the left.

Summary

- The original inequality, $3(8 - 4x) < 6(x - 5)$, simplifies to:

$x > 3$

- The solution set includes all real numbers greater than 3.
- On a number line, this is represented with an open circle at 3 and shading to the right.
- Correct interpretation and graphical representation of inequalities are essential skills in algebra and problem-solving.

Conclusion

Understanding how to solve inequalities and represent their solutions visually is a vital component of algebraic literacy. By carefully expanding, isolating the variable, and correctly interpreting the solution, students can confidently determine solution sets for various inequalities. The graphical representation on a number line provides an intuitive understanding of the solutions' range and helps in solving compound inequalities or systems. Practice regularly with different types of inequalities to strengthen these skills and master the art of translating algebraic solutions into visual forms.

Keywords: inequality solution set, number line, algebraic inequalities, solving inequalities, graphical representation, solution set for inequalities, $x > 3$, number line graph, inequality signs, algebra practice

Frequently Asked Questions

What is the first step to solve the inequality $3(8 - 4x) < 6(x + 5)$?

Distribute the numbers inside the parentheses: $24 - 12x < 6x + 30$.

How do we isolate the variable x in the inequality $24 - 12x < 6x + 30$?

Bring all terms containing x to one side and constants to the other: $-12x - 6x < 30 - 24$.

What is the simplified form after combining like terms in the inequality?

$-18x < 6$.

How do you solve for x in the inequality $-18x < 6$?

Divide both sides by -18, remembering to flip the inequality sign: $x > -6$.

What does the solution $x > -6$ represent on a number line?

It represents all real numbers greater than -6, not including -6 itself.

How would the solution set be represented on a number line?

With an open circle at -6 and shading to the right to indicate all numbers greater than -6.

What is the correct number line representation for the solution $x > -6$?

A number line with an open circle at -6 and shading extending to the right.

Why is the number line from -6 to infinity the correct representation for the solution?

Because the inequality $x > -6$ means all numbers greater than -6, extending infinitely to the right.

Additional Resources

Understanding the Number Line Representation of the Solution Set for the Inequality $3(8 - 4x) < 6(x + 5)$

When tackling inequalities like $3(8 - 4x) < 6(x + 5)$, visualizing their solution sets on a number line is a fundamental step that enhances understanding and problem-solving skills. This detailed review explores how to interpret, analyze, and accurately graph the solution set of such inequalities on a number line, emphasizing critical reasoning, algebraic manipulation, and graphical representation.

Introduction to the Problem

Inequalities are mathematical statements that compare two expressions, often involving variables, and determine the set of values that satisfy the condition. The given inequality:

$$3(8 - 4x) < 6(x + 5)$$

is a linear inequality in one variable (x), which can be solved systematically to find its solution set, and then represented graphically on a number line.

The core goal is to:

- Solve the inequality for x.
- Understand the nature of the solution (intervals, points).
- Depict the solution set accurately on a number line.

Step-by-Step Solution of the Inequality

Before analyzing the graphical representation, it's essential to solve the inequality algebraically.

Step 1: Expand Both Sides

Distribute the coefficients:

- Left side: $3 \times 8 = 24$; $3 \times (-4x) = -12x$

- Right side: $6 \times x = 6x$; $6 \times 5 = 30$

So, the inequality becomes:

$$24 - 12x < 6x + 30$$

Step 2: Bring Like Terms to One Side

Subtract $6x$ from both sides:

$$24 - 12x - 6x < 30$$

which simplifies to:

$$24 - 18x < 30$$

Next, subtract 24 from both sides:

$$-18x < 6$$

Step 3: Isolate x

Divide both sides by -18. Remember, dividing by a negative reverses the inequality sign:

$$x > \frac{6}{-18}$$

Simplify the fraction:

$$x > -\frac{1}{3}$$

Final Solution:

$$x > -\frac{1}{3}$$

This describes all real numbers greater than $-\frac{1}{3}$.

Interpreting the Solution Set

The algebraic solution indicates that:

- The solution set includes all real numbers strictly greater than $-\frac{1}{3}$.
- It does not include $-\frac{1}{3}$ itself, as the inequality is strict (<).

Expressed in interval notation:

$$ (-\frac{1}{3}, \infty) $$

This interval represents all real numbers greater than $-\frac{1}{3}$.

Graphical Representation on the Number Line

Visualizing the solution set on a number line involves several considerations:

1. Identify the Boundary Point

- The boundary point is at $-\frac{1}{3}$. Since the solution is strictly greater, this point is not included in the solution set.
- On the number line, this is often indicated with an open circle at $-\frac{1}{3}$.

2. Direction of the Solution

- The solution set extends infinitely to the right (greater than $-\frac{1}{3}$).
- The arrow or shading should extend from just to the right of $-\frac{1}{3}$ towards positive infinity.

3. Drawing the Number Line

Steps:

- Plot the boundary point:

- Locate $-\frac{1}{3}$ on the number line.
- Mark it with an open circle to denote not included.

- Shade the solution region:

- Shade or draw a bold line starting immediately to the right of $-\frac{1}{3}$.
- Extend this shading infinitely towards the right, indicating all numbers greater than $-\frac{1}{3}$ are

part of the solution.

- Add an arrow:

- At the right end of the shaded region, add an arrow pointing to infinity to denote the unbounded nature.

4. Additional Notations

- Label the boundary point with the value \(-\frac{1}{3}\) to clarify.
- Use consistent symbols: open circle for strict inequalities, closed circle if the inequality is inclusive (\(\leq\) or \(\geq\)).

Multiple Choice of Number Line Representations

Suppose you are presented with several number line options. How do you determine which accurately depicts the solution set?

Key features to look for:

- Open circle at \(-\frac{1}{3}\): indicates the boundary point is not included.
- Shading to the right of \(-\frac{1}{3}\): signifies all numbers greater than \(-\frac{1}{3}\).
- No shading to the left of \(-\frac{1}{3}\): confirms the inequality is strict and only includes values greater than \(-\frac{1}{3}\).
- Arrow pointing right: indicates the solution extends to infinity in the positive direction.

Deeper Insights into Number Line Representation

Understanding the graphical representation extends beyond simply plotting the solution. It involves grasping the underlying properties of inequalities and their implications.

1. Why Use Open Circles?

- An open circle signifies that the boundary point itself is not included in the solution.
- For inequalities like $x > -\frac{1}{3}$, the boundary is excluded because the inequality is strict.
- Conversely, a closed circle indicates $\geq$ or $\leq$, where the boundary point is included.

2. Infinite Extent of the Solution

- The solution set $\{x | x > -\frac{1}{3}\}$ extends infinitely to the right.
- The arrow or unbounded shading communicates that the set contains all larger numbers, no upper limit.

3. Significance of the Direction of Shading

- Shading to the right (positive x-values) corresponds with the algebraic solution.
- Correct graphical representation confirms the understanding of the inequality's nature.

4. Handling Multiple Inequalities

- When inequalities involve conjunctions (and) or disjunctions (or), the number line becomes more complex.
- For example, inequalities like $x > -\frac{1}{3}$ and $x < 4$ would be represented by an open circle at $-\frac{1}{3}$, a closed circle at 4, with shading in between.

Common Mistakes and Misconceptions

While graphing inequalities on the number line seems straightforward, several common pitfalls can mislead students or practitioners:

- Inclusion/Exclusion Confusion: Mistakenly using a closed circle for strict inequalities or an open circle for inclusive inequalities.
- Incorrect Boundary Placement: Mislocating $-\frac{1}{3}$ on the number line.
- Misinterpreting the Inequality Direction: Forgetting to reverse the inequality when dividing both sides by a negative.
- Shading in the Wrong Direction: Shading to the left instead of the right (or vice versa) based on the

inequality.

Applications and Importance in Mathematics

Graphical representations of solution sets are vital in various mathematical contexts:

- Solving Real-World Problems: Inequalities model constraints, and number line graphs visually demonstrate feasible ranges.
- Understanding Functions and Domains: Graphs help visualize where functions satisfy certain inequalities.
- Foundation for Advanced Topics: Concepts like systems of inequalities, optimization, and linear programming rely heavily on number line representations.

Conclusion: Final Thoughts on Number Line Representation

The process of translating an algebraic inequality into a visual graphical representation on a number line is a crucial skill that consolidates understanding of inequalities, solution sets, and their properties. For the specific inequality \(3(8 - 4x) < 6(x + 5)\), the solution simplifies to \(x > -\frac{1}{3}\), and the number line reflects this with an open circle at \(-\frac{1}{3}\), shading extending to positive infinity.

Properly interpreting and constructing these graphs not only aids in solving similar inequalities but also enhances one's overall mathematical literacy, especially in understanding how solutions behave and how they are represented visually. Whether preparing for tests, teaching, or solving real-world problems, mastering the art of number line representation is an invaluable skill that bridges algebraic reasoning with graphical intuition.

In summary:

- The key boundary point is \(-\frac{1}{3}\).
- The solution set includes all real numbers greater than \(-\frac{1}{3}\).
- The number line features an open circle at \(-

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