Which Side Lengths Form A Right Triangle? Choose All Answers That Apply: Choose All Answers That Apply:

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Understanding the properties of right triangles is fundamental in geometry, trigonometry, and various real-world applications such as engineering, architecture, and navigation. A key question often asked is: Which side lengths can form a right triangle? This article explores the conditions under which side lengths create a right triangle, the mathematical principles involved, and how to identify valid sets of side lengths. Whether you're a student preparing for exams or a professional applying geometric concepts, mastering this topic will enhance your comprehension of right triangles.

What Is a Right Triangle?

A right triangle is a type of triangle that contains exactly one right angle—an angle measuring 90 degrees. The sides of a right triangle are classified as:
- Legs: The two sides that form the right angle.
- Hypotenuse: The side opposite the right angle, which is always the longest side.

Understanding the relationship between these sides is crucial in determining whether a given set of side lengths can form a right triangle.

The Pythagorean Theorem

The defining feature of a right triangle is the Pythagorean theorem, which states:

In a right triangle, the square of the hypotenuse (c) equals the sum of the squares of the other two sides (a and b):

\[a^2 + b^2 = c^2 \]

This relationship is both necessary and sufficient for the sides to form a right triangle. If the sides satisfy this equation, they form a right triangle; if not, they do not.

Determining Valid Side Lengths for a Right Triangle

Given three side lengths, a, b, and c, how can we determine whether they form a right triangle? The key is to identify the hypotenuse and check whether the Pythagorean theorem holds.

Steps to verify:
1. Arrange the side lengths in order from smallest to largest: $a \leq b \leq c$.
2. Check the Pythagorean condition: whether $a^2 + b^2 = c^2$.

If the above holds true, then the sides form a right triangle.

Which Side Lengths Form A Right Triangle? – The Criteria

The question "which side lengths form a right triangle?" can be answered by considering the following criteria:

1. Pythagorean Condition

- Exact equality: $a^2 + b^2 = c^2$
- Valid when the sides satisfy this condition precisely, indicating a perfect right triangle.

2. Inequality Conditions for Non-Right Triangles

- If $a^2 + b^2 > c^2$, the triangle is acute (all angles less than 90°).
- If $a^2 + b^2 < c^2$, the triangle is obtuse (one angle greater than 90°).

Therefore, only when $a^2 + b^2 = c^2$ do the side lengths form a right triangle.

Choosing All Answers That Apply: Valid Sets of

Side Lengths

When presented with multiple options, identifying all that can form a right triangle involves verifying each set against the Pythagorean theorem.

Example options:
- (3, 4, 5)
- (5, 12, 13)
- (8, 15, 17)
- (7, 24, 25)
- (6, 8, 10)
- (2, 3, 4)
- (9, 40, 41)

Step-by-step verification:

For each set:
1. Arrange sides in ascending order.
2. Calculate the squares of the two smaller sides and compare their sum to the square of the largest side.

Sample calculations:

- (3, 4, 5):
- $3^2 + 4^2 = 9 + 16 = 25$
- $5^2 = 25$
- Equal; forms a right triangle.

- (5, 12, 13):
- $5^2 + 12^2 = 25 + 144 = 169$
- $13^2 = 169$
- Equal; forms a right triangle.

- (8, 15, 17):
- $8^2 + 15^2 = 64 + 225 = 289$
- $17^2 = 289$
- Equal; forms a right triangle.

- (7, 24, 25):
- $7^2 + 24^2 = 49 + 576 = 625$
- $25^2 = 625$
- Equal; forms a right triangle.

- (6, 8, 10):
- $6^2 + 8^2 = 36 + 64 = 100$
- $10^2 = 100$
- Equal; forms a right triangle.

- (2, 3, 4):
- $2^2 + 3^2 = 4 + 9 = 13$
- $4^2 = 16$

- Not equal; does not form a right triangle.

- (9, 40, 41):
- $9^2 + 40^2 = 81 + 1600 = 1681$
- $41^2 = 1681$
- Equal; forms a right triangle.

Conclusion: The sets (3, 4, 5), (5, 12, 13), (8, 15, 17), (7, 24, 25), (6, 8, 10), and (9, 40, 41) can all form right triangles.

Special Cases and Pythagorean Triples

Sets of three positive integers satisfying the Pythagorean theorem are known as Pythagorean triples. Some common primitive triples include:
- (3, 4, 5)
- (5, 12, 13)
- (8, 15, 17)
- (7, 24, 25)
- (9, 40, 41)

Non-primitive triples are multiples of these primitive triples, such as (6, 8, 10), which is just 2 times (3, 4, 5).

Note: When selecting side lengths, any multiple of a Pythagorean triple also forms a right triangle, provided all sides are scaled proportionally.

Practical Applications of Identifying Right Triangles

Knowing which side lengths form a right triangle is essential in various fields:

- **Construction and Architecture:** Ensuring structures are perpendicular and stable.
- **Navigation:** Using the Pythagorean theorem to calculate distances.
- **Computer Graphics:** Rendering right angles and calculating distances between points.
- **Trigonometry:** Calculating angles and side lengths in real-world problems.

Common Mistakes to Avoid

While verifying side lengths, keep in mind:
- Always order the sides to identify the hypotenuse correctly.
- Check for measurement errors; small inaccuracies can lead to incorrect conclusions.
- Remember that not all triangles with sides satisfying the Pythagorean theorem are right triangles if the sides are approximate or rounded, so precise calculations are essential.

Summary

To determine which side lengths form a right triangle, consider the following:
- The sides must satisfy the Pythagorean theorem: $a^2 + b^2 = c^2$.
- The sides can be primitive Pythagorean triples or multiples thereof.
- Any set of side lengths failing this condition does not form a right triangle.
- Remember that the hypotenuse is the longest side; always verify that the largest side is indeed the hypotenuse before applying the theorem.

In conclusion, when faced with multiple options, apply the Pythagorean theorem to each set of side lengths to identify all that can form right triangles.

Final Note: Mastering how to identify right triangles through side lengths is a foundational skill in geometry. Practice with different sets of numbers and familiarize yourself with common Pythagorean triples to strengthen your understanding and problem-solving abilities.

Frequently Asked Questions

Which side lengths can form a right triangle? Choose all that apply.

3, 4, 5

Given side lengths 5, 12, and 13, do they form a right triangle?

Yes, because $5^2 + 12^2 = 13^2$ ($25 + 144 = 169$).

Can side lengths 6, 8, and 10 make a right triangle?

Yes, since $6^2 + 8^2 = 10^2$ ($36 + 64 = 100$).

Are the side lengths 7, 24, and 25 suitable for a right triangle?

Yes, because $7^2 + 24^2 = 25^2$ ($49 + 576 = 625$).

Do side lengths 9, 40, and 41 form a right triangle?

Yes, since $9^2 + 40^2 = 41^2$ ($81 + 1600 = 1681$).

Are side lengths 1, 1, and √2 possible for a right triangle?

Yes, they satisfy the Pythagorean theorem ($1^2 + 1^2 = (\sqrt{2})^2$).

Can side lengths 5, 5, and 7 form a right triangle?

No, because $5^2 + 5^2 \neq 7^2$ ($25 + 25 \neq 49$).

Does the set of side lengths 8, 15, and 17 form a right triangle?

Yes, since $8^2 + 15^2 = 17^2$ ($64 + 225 = 289$).

Are side lengths 10, 24, and 26 suitable for a right triangle?

Yes, because $10^2 + 24^2 = 26^2$ ($100 + 576 = 676$).

Additional Resources

Which Side Lengths Form A Right Triangle? Choose All Answers That Apply

Understanding the conditions under which three side lengths form a right triangle is fundamental in geometry, with applications spanning from basic mathematics education to advanced engineering and architectural design. This article delves into the mathematical principles that determine whether a set of side lengths can constitute a right triangle, analyzing the key criteria, the Pythagorean theorem, and the various ways to identify valid right triangles.

Introduction to Right Triangles and Their Significance

A right triangle is a type of triangle characterized by having one angle measuring exactly 90 degrees. This right angle is crucial because it introduces specific relationships between the lengths of the sides—namely, the hypotenuse (the side opposite the right angle) and the legs (the other two sides).

Understanding which side lengths form a right triangle is not only an academic exercise but also a practical concern in real-world applications such as construction, navigation, and computer graphics. The ability to determine whether a set of lengths can form a right triangle enables engineers and designers to ensure structural integrity and precision.

The Pythagorean Theorem: The Cornerstone of Right Triangle Identification

At the core of identifying right triangles lies the Pythagorean theorem. Formulated by the ancient Greek mathematician Pythagoras, the theorem states:

> In a right triangle, the square of the hypotenuse (the longest side) is equal to the sum of the squares of the other two sides.

Mathematically, if the sides are labeled as a, b, and c, where c is the hypotenuse, then:

$$
a^2 + b^2 = c^2
$$

This equation provides a straightforward test: given three side lengths, if the sum of the squares of the two shorter sides equals the square of the longest side, then the lengths form a right triangle.

Determining Valid Side Lengths: The Criteria

To establish whether three side lengths can form a right triangle, several criteria and considerations are necessary:

1. Identification of the Longest Side

Since the hypotenuse is always the longest side in a right triangle, any three lengths must be ordered to identify the potential hypotenuse:

- Arrange the lengths such that c is the maximum of the three.

- The other two sides are a and b.

2. Application of the Pythagorean Condition

Once ordered:

$$
a^2 + b^2 \stackrel{?}{=} c^2
$$

- If equality holds, the lengths form a right triangle.

- If $a^2 + b^2 > c^2$, the triangle is acute (all angles less than 90°).

- If $a^2 + b^2 < c^2$, the triangle is obtuse (one angle greater than 90°).

3. Triangle Inequality Theorem

Before considering rightness, the side lengths must satisfy the triangle inequality:

- Each pair of sides must sum to more than the third:

$$
a + b > c,\quad a + c > b,\quad b + c > a
$$

- If any of these inequalities fail, the lengths cannot form a triangle at all.

Applying the Criteria: Examples and Analysis

Let us examine specific examples to illustrate which side lengths form right triangles.

Example A: Sides 3, 4, 5

- Sorted: 3, 4, 5

- Check triangle inequality:

- $3 + 4 = 7 > 5$ ✓

- Check Pythagorean theorem:

- $3^2 + 4^2 = 9 + 16 = 25$

- $5^2 = 25$

- Since both are equal, these lengths form a right triangle.

Example B: Sides 5, 12, 13

- Sorted: 5, 12, 13

- Triangle inequality:

- $5 + 12 = 17 > 13$ ✓

- Pythagorean check:

- $5^2 + 12^2 = 25 + 144 = 169$

- $13^2 = 169$

- Equal, so these sides form a right triangle.

Example C: Sides 7, 24, 25

- Sorted: 7, 24, 25

- Triangle inequality:

- $7 + 24 = 31 > 25$ ✓

- Pythagorean check:

- $7^2 + 24^2 = 49 + 576 = 625$

- $25^2 = 625$

- Equal, forming a right triangle.

Example D: Sides 2, 3, 4

- Sorted: 2, 3, 4

- Triangle inequality:

- $2 + 3 = 5 > 4$ ✓

- Pythagorean check:

- $2^2 + 3^2 = 4 + 9 = 13$

- $4^2 = 16$

- Since $13 \neq 16$, the triangle is not right-angled.

- The sums indicate the triangle is acute or obtuse, but not right-angled.

Choosing All Answers That Apply: Which Side Lengths Form A Right Triangle?

Based on the principles outlined, the question often appears in multiple-choice formats where multiple options could be correct. The general rule is:

- If the three side lengths satisfy the Pythagorean theorem with the largest side as the hypotenuse, then they form a right triangle.

- All options where $a^2 + b^2 = c^2$ (with c being the longest side) are correct.

- Options where the equality does not hold are not correct unless they satisfy the Pythagorean condition.

Common Sets of Side Lengths and Their Right Triangle Status

Side Lengths	Right Triangle?	Explanation
3, 4, 5	Yes	Classic Pythagorean triple
5, 12, 13	Yes	Pythagorean triple
8, 15, 17	Yes	Pythagorean triple
7, 24, 25	Yes	Pythagorean triple
9, 40, 41	Yes	Pythagorean triple
2, 3, 4	No	Not Pythagorean; the sum of squares doesn't match hypotenuse

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Special Cases and Variations

1. Non-Integer Side Lengths

- The Pythagorean theorem applies regardless of whether side lengths are integers or real numbers.

- For example, sides 1.5, 2.0, and 2.5 could form a right triangle if:

$$
1.5^2 + 2.0^2 = 2.25 + 4 = 6.25
$$

$$
\text{Hypotenuse}^2 = 2.5^2 = 6.25
$$

- Since the equality holds, these lengths form a right triangle.

2. Scaling of Pythagorean Triples

- Any Pythagorean triple can be scaled by a common factor to generate new right triangles.

- For example, scaling 3-4-5 by 2 yields 6-8-10, which also satisfy the Pythagorean theorem.

3. Degenerate Cases

- When the sum of two sides equals the third (e.g., 2, 2, 4), no triangle is formed; these are considered degenerate.

Conclusion: Synthesizing the Criteria

To accurately answer which side lengths form a right triangle, one must:

- Order the sides to identify the potential hypotenuse.

- Verify the triangle inequality to confirm a valid triangle.

- Apply the Pythagorean theorem to check for equality.

- Select all options where the Pythagorean condition holds true.

In multiple-choice scenarios, the correct answers are all side length sets satisfying \(a^2 + b^2 = c^2\), with the largest side as the hypotenuse. Recognizing these criteria allows for quick and accurate identification, whether dealing with classic integer triples or arbitrary real lengths.

Final Remarks

Understanding which side lengths form a right triangle is an essential aspect of geometric problem-solving. The Pythagorean theorem remains the definitive tool for this purpose. By systematically applying the criteria discussed, one can confidently determine the validity of a set of lengths for forming right triangles in both theoretical and practical contexts.

Keywords: right triangle, Pythagorean theorem, side lengths, hypotenuse, geometric criteria, triangle inequality, P

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