Write The Equation Of The Line That Passes Through The Points (8, 1) And (2, 5) In Standard Form, Given

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Determining the equation of a line that passes through two specific points is a fundamental skill in algebra and coordinate geometry. Whether you're a student preparing for exams or someone working on a math-related project, understanding how to derive the line’s equation in standard form is essential. In this article, we will explore the step-by-step process to find the equation of the line passing through the points (8, 1) and (2, 5), with a focus on expressing the final answer in standard form.

Understanding the Problem

Before diving into calculations, let's clarify what the problem entails:

- Given points: (8, 1) and (2, 5)
- Goal: Find the equation of the line passing through these points
- Format: Standard form, which is typically written as Ax + By = C, where A, B, and C are integers, and A ≥ 0

By following structured steps, you'll be able to derive the line's equation accurately and efficiently.

Step 1: Find the Slope of the Line

The slope (m) indicates the steepness and direction of the line. It is calculated using the coordinates of the two points:

Formula for Slope

$$
m = \frac{y_2 - y_1}{x_2 - x_1}
$$

Given points:
- $(x_1, y_1) = (8, 1)$
- $(x_2, y_2) = (2, 5)$

Calculating the Slope

$$
m = \frac{5 - 1}{2 - 8} = \frac{4}{-6} = -\frac{2}{3}
$$

So, the slope of the line is $-\frac{2}{3}$.

Step 2: Write the Equation in Point-Slope Form

The point-slope form of a line's equation is:

$$
y - y_1 = m(x - x_1)
$$

Choose either of the two points to substitute into the formula. Let's use (8, 1):

$$
y - 1 = -\frac{2}{3}(x - 8)
$$

Expanding the Equation

Distribute the slope:

$$
y - 1 = -\frac{2}{3}x + \frac{16}{3}
$$

Now, solve for y:

$$
y = -\frac{2}{3}x + \frac{16}{3} + 1
$$

Express 1 as a fraction with denominator 3:

$$
y = -\frac{2}{3}x + \frac{16}{3} + \frac{3}{3} = -\frac{2}{3}x + \frac{19}{3}
$$

The equation in slope-intercept form is:

$$
y = -\frac{2}{3}x + \frac{19}{3}
$$

Note: While slope-intercept form is useful, our goal is the standard form.

Step 3: Convert to Standard Form

To write the equation in standard form (Ax + By = C), eliminate fractions and rearrange terms.

Eliminate the Denominators

Multiply both sides of the equation by 3 to clear fractions:

$$
3y = -2x + 19
$$

Rearranged:

$$
2x + 3y = 19
$$

This is the equation in standard form.

Final Standard Form Equation

$$
\boxed{2x + 3y = 19}
$$

In this form, A = 2, B = 3, and C = 19.

Additional Tips for Writing Standard Form Equations

- Ensure that A ≥ 0: If A is negative, multiply the entire equation by -1 to make A positive.
- Simplify coefficients: Make sure all coefficients are integers and simplified as much as possible.
- Check your work: Substitute the original points into the final equation to verify correctness.

Verification: Confirm the Points Lie on the Line

Verify that both points satisfy the equation:

- For (8, 1):

$$
2(8) + 3(1) = 16 + 3 = 19 \quad \checkmark
$$

- For (2, 5):

$$
2(2) + 3(5) = 4 + 15 = 19 \quad \checkmark
$$

Both points satisfy the equation, confirming its correctness.

Summary

To summarize, the process to find the equation of the line passing through (8, 1) and (2, 5) in standard form involves:

1. Calculating the slope:

$$
m = -\frac{2}{3}
$$

2. Using the point-slope form with one of the points:

$$
y - 1 = -\frac{2}{3}(x - 8)
$$

3. Expanding and simplifying to slope-intercept form:

$$
y = -\frac{2}{3}x + \frac{19}{3}
$$

4. Clearing fractions and rewriting in standard form:

$$
2x + 3y = 19
$$

This final equation accurately represents the line passing through the given points and is expressed in standard form, suitable for various applications in algebra and coordinate geometry.

Conclusion

Knowing how to derive the equation of a line from two points, especially in standard form, is a crucial skill. The key steps involve finding the slope, writing the equation in point-slope form, converting to slope-intercept form, and finally rearranging into standard form. With practice, these steps become intuitive, enabling quick and accurate solutions to similar problems. Remember to always verify your final equation by substituting the original points to ensure correctness. Whether used in academic settings or real-world applications, mastering this process enhances your understanding of linear equations and their representations.

Frequently Asked Questions

How do you find the equation of a line passing through points (8, 1) and (2, 5) in standard form?

First, find the slope using (y2 - y1) / (x2 - x1), then find the point-slope form, and finally convert it to standard form Ax + By = C.

What is the slope of the line passing through (8, 1) and (2, 5)?

The slope is (5 - 1) / (2 - 8) = 4 / -6 = -2/3.

How do I write the equation of the line in point-slope form using the points (8, 1) and (2, 5)?

Using point (8, 1) and slope -2/3, the point-slope form is y - 1 = (-2/3)(x - 8).

How do I convert the point-slope form to standard form for this line?

Expand and rearrange: y - 1 = (-2/3)(x - 8) → multiply both sides by 3 to clear denominator, then simplify to get Ax + By = C.

What is the standard form of the line passing through (8, 1) and (2, 5)?

The standard form is 2x + 3y = 22.

Why is the standard form of a line important?

Standard form facilitates easy comparison of lines, identification of intercepts, and is useful for solving systems of equations.

Can you show the step-by-step process to derive the standard form from the points?

Yes. First, find the slope (-2/3), then use point-slope form with point (8, 1), expand to get 3(y - 1) = -2(x - 8), simplify to 3y - 3 = -2x + 16, then rearranged to 2x + 3y = 19, which is the standard form.

Are the points (8, 1) and (2, 5) collinear, and how does that affect the equation?

Yes, both points lie on the same line, so the derived equation accurately represents the line passing through both points.

What should I check after deriving the standard form to ensure it's correct?

Substitute the original points into the equation to verify they satisfy it; both should satisfy the standard form equation.

What is the significance of the coefficients in the standard form equation 2x + 3y = 22?

The coefficients 2 and 3 are related to the slope and intercepts of the line, and the constant 22 confirms the specific line passing through the given points.

Additional Resources

Line Equation Derivation: Mastering the Standard Form Through Point-Slope Analysis

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When it comes to understanding the foundational concepts of algebra and coordinate geometry, one of the most essential skills is being able to derive the equation of a straight line passing through two points. Whether for academic purposes, problem-solving, or practical applications like engineering and computer graphics, knowing how to express that line in standard form is invaluable. Today, we will delve into the process of finding the equation of a line passing through the points (8, 1) and (2, 5), culminating in an explicit, detailed expression of that line in standard form.

Understanding the Fundamentals: Points and Line Equations

Before jumping into calculations, it’s crucial to understand what the problem entails.

The Points:
- Point A: (8, 1)
- Point B: (2, 5)

These points lie on the coordinate plane, with each point defined by an x-coordinate and a y-coordinate.

Goal:
Find the equation of the line passing through these two points, expressed in standard form, which is generally written as:

$$
Ax + By = C
$$

where A, B, and C are real numbers, and A and B are not both zero.

Why Standard Form?

The standard form is widely used because it provides a concise way to characterize a line, particularly useful for solving systems of equations, analyzing intersections, and graphing. It also makes it easier to determine intercepts immediately.

Step 1: Find the Slope of the Line

The first and most critical step in deriving the line's equation is calculating its slope, often denoted as m.

Definition of Slope:
$$
m = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1}
$$

Calculation:
Given the points:
- $(x_1, y_1) = (8, 1)$
- $(x_2, y_2) = (2, 5)$

Applying the formula:
$$

$$
m = \frac{5 - 1}{2 - 8} = \frac{4}{-6} = -\frac{2}{3}
$$

Interpretation: The slope of the line is $-\frac{2}{3}$. This negative slope indicates the line descends as it moves from left to right.

Step 2: Find the Equation in Point-Slope Form

The point-slope form is an intermediary step that leverages the slope and a known point on the line. The formula is:

$$
y - y_1 = m(x - x_1)
$$

Choosing Point A (8, 1):

$$
y - 1 = -\frac{2}{3}(x - 8)
$$

This form is convenient because it explicitly combines the slope with a point, making the transition to other forms straightforward.

Step 3: Convert to Slope-Intercept Form (Optional but Helpful)

While our goal is the standard form, many find it helpful to first express the line in slope-intercept form ($y = mx + b$) to verify correctness and facilitate conversion.

Applying the point-slope formula:

$$
y - 1 = -\frac{2}{3}(x - 8)
$$

Distribute:

$$
y - 1 = -\frac{2}{3}x + \frac{16}{3}
$$

Add 1 (which is \(\frac{3}{3}\)) to both sides:

\[
y = -\frac{2}{3}x + \frac{16}{3} + \frac{3}{3} = -\frac{2}{3}x + \frac{19}{3}
\]

Thus, the slope-intercept form is:

\[
y = -\frac{2}{3}x + \frac{19}{3}
\]

Step 4: Convert to Standard Form

The key task is converting the slope-intercept form into the standard form \(Ax + By = C\).

Step-by-step process:

1. Eliminate fractions for clarity and to keep coefficients integral:

\[
y = -\frac{2}{3}x + \frac{19}{3}
\]

Multiply both sides by 3:

\[
3y = -2x + 19
\]

2. Rearrange terms to bring all variables to one side:

\[
2x + 3y = 19
\]

3. Identify coefficients:

\[
A = 2,\quad B = 3,\quad C = 19
\]

This is already in standard form.

Final equation in standard form:

\[

\boxed{
2x + 3y = 19
}
\]

Additional Considerations and Variations

While the above derivation yields the equation in a clean, standard form, it's worth exploring some nuances and alternative representations.

1. Sign conventions:

The standard form often prefers \(A\) to be positive. If the coefficients were negative, multiplying through by \(-1\) would be appropriate to maintain consistency.

2. Ensuring integer coefficients:

Our process naturally led to integer coefficients. Sometimes, equations involve fractions, and multiplying through by the least common denominator (LCD) helps clear fractions.

3. Verification:

Always verify the line passes through the given points:

- For (8, 1):

\[
2(8) + 3(1) = 16 + 3 = 19 \quad \checkmark
\]

- For (2, 5):

\[
2(2) + 3(5) = 4 + 15 = 19 \quad \checkmark
\]

Both points satisfy the equation, confirming correctness.

Conclusion: The Complete Equation

Having systematically calculated and verified, the equation of the line passing through points (8, 1) and (2, 5) in standard form is:

$$
\boxed{
2x + 3y = 19
}
$$

This equation succinctly encapsulates the line’s geometry and serves as a critical tool for further analysis, including finding intersections, graphing, and solving related algebraic problems.

Summary of Key Steps:

- Calculate the slope using the two points.
- Use the point-slope form to find the initial line equation.
- Convert to slope-intercept form for clarity.
- Eliminate fractions to achieve a clean standard form.
- Verify with original points.

Final Thoughts

Mastering the derivation of a line’s equation from two points is a fundamental skill in algebra. It combines understanding of slopes, algebraic manipulation, and verification. This process underscores the importance of systematic problem-solving and clarity in mathematical expression. Whether for academic pursuits or practical applications, the ability to express a line in standard form is a testament to one's grasp of coordinate geometry principles.

By practicing these steps regularly, students and professionals alike can develop confidence in their analytical abilities and deepen their understanding of the geometric relationships that underpin much of mathematics and its applications.

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