(x^(2))/(49)+(y^(2))/(36)=1 Orientation Of Majoran's Center Coordinates Of Foci: Coordinates Of Vertices

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The equation $\frac{x^2}{49} + \frac{y^2}{36} = 1$ represents an ellipse centered at the origin. Understanding the orientation, the position of the foci, the vertices, and other key features of this ellipse is fundamental in conic section analysis. This article provides an in-depth exploration of these aspects, including how to identify the center, determine the coordinates of the foci and vertices, and understand the orientation of the ellipse.

Understanding the Standard Form of an Ellipse

The general form of an ellipse centered at the origin is given by:

$$
\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1
$$

where:

- a and b are the semi-major and semi-minor axes, respectively.
- The larger denominator under the squared term indicates the major axis.

In our specific case:

$$
\frac{x^2}{49} + \frac{y^2}{36} = 1

\]

- $a^2 = 49 \Rightarrow a = 7$
- $b^2 = 36 \Rightarrow b = 6$

Since $a > b$, the major axis is along the x-axis, which influences the orientation of the ellipse.

Orientation of the Ellipse

Horizontal vs. Vertical Ellipses

- When the larger denominator is under the $\ x^2$ term (a^2), the ellipse is oriented horizontally, with the major axis along the x-axis.
- When the larger denominator is under the $\ y^2$ term (b^2), the ellipse is oriented vertically, with the major axis along the y-axis.

In our equation, since $a^2 = 49$ is greater than $b^2 = 36$, the major axis is along the x-axis. Therefore, the ellipse extends more along the horizontal direction.

Center of the Ellipse

The standard form indicates the ellipse is centered at the origin $(0,0)$. This point is called the center of the ellipse and is the midpoint between its vertices and foci.

- Coordinates of the center: $\boxed{(0, 0)}$

Vertices of the Ellipse

Vertices are the points on the ellipse that lie along the major axis and represent its maximum and minimum extent.

Vertices Along the Major Axis

Since the major axis is along the x-axis:

- The vertices are at $(\pm a, 0)$.

Calculating these:

$$
\boxed{
\begin{aligned}
V_1 &= (-a, 0) = (-7, 0) \\
V_2 &= (a, 0) = (7, 0)
\end{aligned}
}
$$

These points are the furthest points along the major axis.

Vertices Along the Minor Axis

- The co-vertices are at $(0, \pm b)$:

$$
\boxed{
\begin{aligned}
CV_1 &= (0, -6) \\
CV_2 &= (0, 6)
\end{aligned}
}
$$

However, these are not the vertices but co-vertices, which are minor axis extremities.

Foci of the Ellipse

The foci are special points located along the major axis, closer to the center, and are essential for defining the shape of the ellipse.

Calculating the Distance to the Foci (c)

The foci are located at $(\pm c, 0)$ along the x-axis, where:

$$
c^2 = a^2 - b^2
$$

Substituting values:

$$
c^2 = 49 - 36 = 13
$$

$$
c = \sqrt{13} \approx 3.6056
$$

Coordinates of the Foci

- The foci are at $(\pm c, 0)$:

$$
\boxed{
\begin{aligned}
F_1 &= (-\sqrt{13}, 0) \approx (-3.6056, 0) \\
F_2 &= (\sqrt{13}, 0) \approx (3.6056, 0)
\end{aligned}
}
$$

These points are vital in the geometric definition of the ellipse, as the sum of the distances from any point on the ellipse to the two foci remains constant and equals $2a$.

Summary of Key Coordinates

Feature	Coordinates	Description
Center	$(0, 0)$	Midpoint of the ellipse
Vertices	$(-7, 0)$, $(7, 0)$	Along the major axis
Co-vertices	$(0, -6)$, $(0, 6)$	Along the minor axis
Foci	$(-3.6056, 0)$, $(3.6056, 0)$	Focus points along the major axis

Applications and Significance of the Ellipse Properties

Understanding the orientation and key points of an ellipse has various applications across fields such as astronomy, engineering, and physics.

- Astronomy: The orbits of planets are elliptical, with the Sun at one focus.
- Engineering: Elliptical reflectors use the property that rays emanating from one focus reflect to the other focus.
- Physics: Elliptical trajectories occur in systems with central forces.

Visual Representation and Graphing

Visualizing the ellipse helps in comprehending the geometric relationships:

1. Plot the center at $(0,0)$.
2. Plot the vertices at $(-7, 0)$ and $(7, 0)$.
3. Plot the foci at $(-3.6056, 0)$ and $(3.6056, 0)$.
4. Draw the ellipse passing through the vertices, ensuring the foci are correctly positioned.

This visualization confirms the horizontal orientation and the relative positions of key features.

Conclusion

The ellipse represented by $\frac{x^2}{49} + \frac{y^2}{36} = 1$ is oriented along the x-axis, with the center at the origin. Its vertices are at $(-7, 0)$ and $(7, 0)$, and the foci are approximately at $(-3.6056, 0)$ and $(3.6056, 0)$. Understanding these properties enables a comprehensive grasp of the ellipse's shape, orientation, and geometric significance. Whether for academic purposes or practical applications, mastering the coordinates of the ellipse's critical points enhances spatial reasoning and analytical skills.

Keywords: ellipse, standard form, major axis, vertices, foci, center, orientation, coordinates, conic sections, geometry

Frequently Asked Questions

What is the standard form of the ellipse given by $(x^2)/49 + (y^2)/36 = 1$?

The standard form is $(x^2)/a^2 + (y^2)/b^2 = 1$, where $a^2=49$ and $b^2=36$, representing an ellipse centered at the origin with horizontal major axis.

What is the orientation of the major axis in the ellipse $(x^2)/49 + (y^2)/36 = 1$?

The major axis is horizontal because $a^2=49 > 36$, so the major axis runs along the x-axis.

How do you find the coordinates of the foci for the ellipse $(x^2)/49 + (y^2)/36 = 1$?

Calculate $c = \sqrt{(a^2 - b^2)} = \sqrt{(49 - 36)} = \sqrt{13} \approx 3.605$. The foci are at $(\pm c, 0)$, i.e., approximately $(\pm 3.605, 0)$.

What are the coordinates of the vertices of the ellipse $(x^2)/49 +$

$(y^2)/36 = 1$?

Vertices are at the endpoints of the major axis: $(\pm a, 0)$, which are at $(\pm 7, 0)$.

How are the center coordinates of the ellipse $(x^2)/49 + (y^2)/36 = 1$ determined?

Since the equation is centered at the origin, the center coordinates are (0, 0).

What is the significance of the major and minor axes in the ellipse $(x^2)/49 + (y^2)/36 = 1$?

The major axis is the longest diameter passing through the foci and vertices along the x-axis, while the minor axis is perpendicular to it along the y-axis, defining the ellipse's shape.

How does the orientation of the ellipse change if the equation is $(x^2)/36 + (y^2)/49 = 1$?

In this case, since $b^2=49 > 36$, the major axis would be vertical along the y-axis.

How can the equation $(x^2)/49 + (y^2)/36 = 1$ be rewritten to identify the lengths of the axes?

It can be written as $x^2/7^2 + y^2/6^2 = 1$, indicating the semi-major axis length $a=7$ and semi-minor axis length $b=6$.

What is the relationship between the axes lengths and the foci in the ellipse $(x^2)/49 + (y^2)/36 = 1$?

The foci are located at $(\pm c, 0)$, where $c = \sqrt{(a^2 - b^2)}$. Here, $c \approx 3.605$, which is less than the semi-major axis length $a=7$, ensuring the ellipse's shape.

Why is the center of the ellipse (0, 0) in the equation (x^2)/49 + (y^2)/36 = 1?

Because the equation is in standard form with no terms shifting the center, indicating the ellipse is centered at the origin.

Additional Resources

(x^(2))/(49)+(y^(2))/(36)=1 Orientation Of Majoran's Center Coordinates Of Foci: Coordinates Of Vertices

In the realm of conic sections, the ellipse stands out as a fundamental curve with applications spanning astronomy, engineering, and mathematics education. Understanding its geometric properties—particularly the orientation, center, foci, and vertices—is essential for both theoretical insights and practical implementations. Today, we delve into the specific ellipse defined by the equation:

$(x^2)/49 + (y^2)/36 = 1$

This article aims to unravel the intricate details of this ellipse, focusing on its orientation, the position of its center, and the coordinates of its foci and vertices. We'll explore the mathematical principles underpinning these features, providing a comprehensive yet accessible guide for students, educators, and enthusiasts alike.

Understanding the Standard Equation of an Ellipse

Before analyzing the specific ellipse, it is crucial to grasp the general form of the equation:

$(x^2)/a^2 + (y^2)/b^2 = 1$

where:

- a and b are the semi-major and semi-minor axes, respectively.
- The larger denominator indicates the major axis's orientation.

In the given equation:

$(x^2)/49 + (y^2)/36 = 1$

- $a^2 = 49$, hence $a = 7$
- $b^2 = 36$, hence $b = 6$

Since 49 > 36, the major axis is along the x-axis, meaning the ellipse is elongated horizontally.

Determining the Orientation of the Ellipse

The orientation of an ellipse refers to the direction of its major axis relative to the coordinate axes. For the given equation:

$(x^2)/49 + (y^2)/36 = 1$

- The larger denominator (49) appears under x^2, indicating the major axis aligns with the x-axis.
- The ellipse extends further along the x-direction than along y, confirming a horizontal orientation.

Key Point:
When the larger denominator is under x^2, the major axis is horizontal; if under y^2, it is vertical.

Locating the Center of the Ellipse

The standard form:

$$(x - h)^2/a^2 + (y - k)^2/b^2 = 1$$

describes an ellipse centered at (h, k). In our given equation, there are no additional shifts; the equation is centered at the origin:

Center Coordinates: $(0, 0)$

This simplifies the analysis, as all key features—vertices, foci—are symmetric about the origin.

Vertices of the Ellipse

Vertices are the points on the ellipse that lie along its major axis, marking the maximum extents.

Calculation of Vertices

Given:
- Major axis along x-axis
- $a = 7$

Vertices are located at:

- $(h \pm a, k)$

Since the center is at $(0, 0)$:

- Vertices at $(\pm 7, 0)$

Interpretation

- The left vertex: $(-7, 0)$
- The right vertex: $(7, 0)$

These points represent the endpoints of the ellipse along its major axis, marking its widest horizontal stretch.

The Foci: Focus of the Ellipse

Foci are special points situated along the major axis, closer to the center, such that the sum of the distances from any point on the ellipse to the foci is constant.

Calculating the Foci Coordinates

The distance from the center to each focus along the major axis is given by:

$c = \sqrt{(a^2 - b^2)}$

Plugging in the values:

$c = \sqrt{(49 - 36)} = \sqrt{13} \approx 3.605$

Since the major axis is along x:

- Foci at $(h \pm c, k)$

Given the center at (0, 0):

- Foci at $(\pm\sqrt{13}, 0)$

Approximate Coordinates:

- (3.605, 0)
- (-3.605, 0)

Significance of the Foci

These points are crucial in defining the shape of the ellipse. Any point on the ellipse maintains the property that the sum of its distances to the two foci remains constant, equal to $2a = 14$.

Summary of Key Features

Feature	Coordinates	Description
Center	(0, 0)	The midpoint of the ellipse
Vertices	(-7, 0) and (7, 0)	Ends of the major axis along the x-axis
Foci	$(-\sqrt{13}, 0)$ and $(\sqrt{13}, 0)$	Foci points along the major axis
Orientation	Horizontal	Major axis aligned with the x-axis

Understanding these features provides a comprehensive picture of the ellipse’s shape and position in the coordinate plane.

Visualizing the Ellipse

Visual representation is paramount to fully grasp the geometric properties:

- Draw the coordinate axes.
- Mark the center at (0, 0).
- Plot the vertices at (-7, 0) and (7, 0).
- Locate the foci approximately at (-3.605, 0) and (3.605, 0).
- Sketch the ellipse passing through the vertices, symmetric about both axes, with the foci lying along the major axis.

Such visualization aids in understanding the relative positions of all key points, reinforcing the mathematical calculations.

Practical Applications and Significance

Understanding the properties of this ellipse extends beyond theoretical interest:

- Astronomy: Planetary orbits often approximate elliptical paths with the sun at a focus.
- Engineering: Elliptical reflective surfaces utilize the focus properties for directing sound or light.
- Optics: Elliptical mirrors are used in telescopes and optical devices.

The precise knowledge of the center, vertices, and foci enables scientists and engineers to design systems that leverage elliptical properties effectively.

Conclusion

The ellipse defined by the equation $(x^2)/49 + (y^2)/36 = 1$ exemplifies the classic characteristics of a horizontally oriented ellipse centered at the origin. Its vertices at (-7, 0) and (7, 0) mark its extremities along the major axis, while the foci at approximately (-3.605, 0) and (3.605, 0) are pivotal in understanding its geometric and optical properties.

Through analyzing the orientation, center, vertices, and foci, we gain a comprehensive understanding of this conic section, bridging algebraic equations with geometric intuition. Whether in academic settings, scientific research, or engineering designs, such insights into ellipses foster a deeper appreciation of the elegant interplay between algebra and geometry, underscoring the enduring importance of conic sections in both theory and application.

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