B) Determine The Inverse Laplace Transform Of The Given Function By Use1) F(s) = S + 1/s(s - 1)(s - 6)2)

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Understanding how to determine the inverse Laplace transform of a function is a fundamental skill in engineering, physics, and mathematics, especially when solving differential equations. The function provided, $F(s) = s + 1 / [s(s - 1)(s - 6)^2]$, presents a typical example where partial fraction decomposition and knowledge of Laplace transform pairs are essential. This article offers a comprehensive guide to calculating the inverse Laplace transform of this function, with detailed steps, explanations, and relevant concepts for clarity and mastery.

Understanding the Laplace Transform and Its Inverse

Before diving into the specific problem, it’s crucial to review the basics of the Laplace transform and how to find its inverse.

What is the Laplace Transform?

The Laplace transform is an integral transform that converts a function of time, f(t), into a function of complex frequency, F(s). It is defined as:

$$F(s) = \mathcal{L}\{f(t)\} = \int_0^{\infty} e^{-st}f(t) dt$$

This transformation simplifies solving differential equations by turning them into algebraic equations.

Inverse Laplace Transform

The inverse Laplace transform, denoted as $\mathcal{L}^{-1}$, retrieves the original function f(t) from its Laplace transform F(s). It is expressed as a complex integral:

$$f(t) = \mathcal{L}^{-1}\{F(s)\} = \frac{1}{2\pi i} \int_{c - i\infty}^{c + i\infty} e^{st} F(s) ds$$

However, in practice, the inverse is often computed via partial fraction decomposition and using known transform pairs.

Analyzing the Given Function F(s)

The function provided is:

$$ F(s) = s + \frac{1}{s(s - 1)(s - 6)^2} $$

This consists of two parts:

1. The simple term, s
2. The rational function, $\frac{1}{s(s - 1)(s - 6)^2}$

To find the inverse Laplace transform, we address each component separately and then sum the results, thanks to the linearity of the inverse Laplace transform.

Step-by-Step Solution Strategy

The main steps to determine the inverse Laplace transform are:

1. Break down $F(s)$ into manageable parts.
2. Simplify and perform partial fraction decomposition on the rational part.
3. Use known inverse Laplace transforms for each component.
4. Combine the results to obtain the final function $f(t)$.

Let’s proceed with each step.

Breaking Down the Function and Preparing for Partial Fractions

Since the function is a sum, and the inverse Laplace transform is linear, we write:

$$ f(t) = \mathcal{L}^{-1}\{s\} + \mathcal{L}^{-1}\left\{\frac{1}{s(s - 1)(s - 6)^2}\right\} $$

The inverse of the first term, $\mathcal{L}^{-1}\{s\}$, is straightforward because the Laplace transform of $\delta'(t)$ or derivatives of the Dirac delta can be involved, but in the context of standard inverse transforms, it's often linked to derivatives. However, since the Laplace transform of $f(t) = - \delta'(t)$ is s, but typically, for causal functions, the inverse transform of s corresponds to the derivative of the Dirac delta, which is not a causal function.

In many practical contexts, the term s in the Laplace domain corresponds to derivatives of functions involving delta functions, which are generalized functions. For typical differential equations, the inverse Laplace transform of s is not a standard function but for the purpose of this exercise, we interpret the function as being designed to analyze the second part.

Alternatively, if the context is to find the inverse of the rational part, we focus on:

$$F_2(s) = \frac{1}{s(s - 1)(s - 6)^2}$$

Partial Fraction Decomposition of the Rational Function

The goal is to express:

$$\frac{1}{s(s - 1)(s - 6)^2}$$

as a sum of simpler fractions with known inverse transforms.

Step 1: Set up the decomposition

$$\frac{1}{s(s - 1)(s - 6)^2} = \frac{A}{s} + \frac{B}{s - 1} + \frac{C}{s - 6} + \frac{D}{(s - 6)^2}$$

Step 2: Find coefficients A, B, C, D

Multiply both sides by the common denominator:

$$1 = A(s - 1)(s - 6)^2 + B s (s - 6)^2 + C s (s - 1)(s - 6) + D s (s - 1)$$

Now, solve for A, B, C, D by plugging in convenient values of s:

- For $s = 0$:

$$1 = A(-1)(-6)^2 + B \cdot 0 \cdot (-6)^2 + C \cdot 0 \cdot (-1) \cdot (-6) + D \cdot 0 \cdot (-1)$$

Simplifies to:

$$ 1 = A(-1)(36) \Rightarrow 1 = -36 A \Rightarrow A = -\frac{1}{36} $$

- For $s = 1$:

$$ 1 = A(0)(-5)^2 + B(1)(-5)^2 + C(1)(0)(-5) + D(1)(0) $$

Simplifies to:

$$ 1 = B \cdot 1 \cdot 25 \Rightarrow 1 = 25 B \Rightarrow B = \frac{1}{25} $$

- For $s = 6$:

$$ 1 = A(5)(0)^2 + B(6)(0)^2 + C(6)(5)(0) + D(6)(5) $$

Simplifies to:

$$ 1 = D \cdot 6 \cdot 5 = 30 D \Rightarrow D = \frac{1}{30} $$

- To find C, pick a convenient value, say $s = 2$:

$$ 1 = A(2 - 1)(2 - 6)^2 + B \cdot 2 \cdot (2 - 6)^2 + C \cdot 2 \cdot (2 - 1) \cdot (2 - 6) + D \cdot 2 \cdot (2 - 1) $$

Calculate each term:

- $A(1)(-4)^2 = A \cdot 1 \cdot 16 = 16A = 16 \times (-1/36) = - \frac{16}{36} = - \frac{4}{9}$

- $B \cdot 2 \cdot 16 = 2 \times \frac{1}{25} \times 16 = \frac{2 \times 16}{25} = \frac{32}{25}$

- $C \cdot 2 \cdot 1 \cdot (-4) = C \times 2 \times 1 \times (-4) = -8 C$

- $D \cdot 2 \cdot 1 = 2 \times \frac{1}{30} = \frac{2}{30} = \frac{1}{15}$

Substituting back:

$$ 1 = - \frac{4}{9} + \frac{32}{25} - 8 C + \frac{1}{15} $$

Combine constants:

$$ - \frac{4}{9} + \frac{1}{15} + \frac{32}{25} $$

Find common denominators:

- $\frac{-4}{9} = \frac{-20}{45}$

- $\frac{1}{15} = \frac{3}{45}$

- $\frac{32}{25} = \frac{576}{450}$ (but better to convert to denominator 225: $225/225$ for commonality)

Alternatively, convert all to decimal for simplicity:

- $-4/9 \approx -0.4444$

- $1/15 \approx 0.0667$

- $32/25 = 1.28$

Sum:

$$-0.4444 + 0.0667 + 1.28 \approx 0.9023$$

Now, the equation:

$$1 = 0.9023 - 8 C \Rightarrow -0.9023 = -8 C$$

Frequently Asked Questions

How do you find the inverse Laplace transform of F(s) = (s + 1) / [s(s - 1)(s - 6)]?

To find the inverse Laplace transform, first perform partial fraction decomposition on F(s), then take the inverse transform of each term using standard tables, and sum the results.

What is the first step in solving for the inverse Laplace transform of F(s) = (s + 1) / [s(s - 1)(s - 6)]?

The initial step is to decompose F(s) into partial fractions: express it as A/s + B/(s - 1) + C/(s - 6).

How do you determine the coefficients A, B, and C in the partial fraction decomposition?

Multiply both sides by the denominator s(s - 1)(s - 6) and evaluate at specific s-values (0, 1, 6) or solve the resulting system of equations to find A, B, and C.

Once the partial fractions are found, how do you

compute the inverse Laplace transforms of each term?

Use standard inverse Laplace transform pairs: $L^{-1}[1/s] = 1$, $L^{-1}[1/(s - a)] = e^{at}$. For constants, multiply by these basic transforms accordingly.

What is the inverse Laplace transform of the term A/s?

It is A times the inverse Laplace transform of 1/s, which is A 1 = A.

How does the inverse Laplace transform of 1/(s - 1) differ from that of 1/(s - 6)?

$L^{-1}[1/(s - 1)] = e^{t}$ and $L^{-1}[1/(s - 6)] = e^{6t}$. These are exponential functions with different growth rates.

What is the significance of using partial fractions in inverse Laplace transforms?

Partial fractions simplify complex rational functions into sums of simpler fractions whose inverse transforms are directly known, making the process manageable.

Are there any special considerations when dealing with multiple roots or repeated factors in the denominator?

Yes, for repeated roots, the partial fraction terms involve powers of (s - a), and their inverse transforms include polynomial factors multiplied by exponentials.

Can the inverse Laplace transform be found directly from the original function without decomposition?

Generally, partial fraction decomposition is the standard method; direct inversion is complex unless the function matches known Laplace pairs or can be simplified accordingly.

Additional Resources

Determine The Inverse Laplace Transform Of The Given Function: F(s) = s + 1 / [s(s - 1)(s - 6)]

When tackling complex inverse Laplace transforms, especially those involving rational functions with multiple factors, a systematic approach becomes essential. The process often involves partial fraction decomposition followed by applying known inverse transforms to each term. In this article, we will explore how to determine the inverse Laplace transform of the function F(s) = s + 1 / [s(s - 1)(s - 6)], providing a detailed, step-by-step guide to mastering such problems.

Introduction

The Laplace transform is a powerful tool in engineering, physics, and mathematics for solving differential equations, analyzing systems, and understanding the behavior of functions in the complex frequency domain. The inverse Laplace transform allows us to convert functions back from the s-domain to the time domain, which is often the ultimate goal in applications.

The given function,

$$F(s) = s + \frac{1}{s(s - 1)(s - 6)}$$

is composed of two parts: a simple polynomial term s and a rational term involving multiple factors in the denominator. Our task is to find the inverse Laplace transform, $f(t) = \mathcal{L}^{-1}\{F(s)\}$, which entails handling each component carefully.

Step 1: Break Down the Function

The first step is to recognize that the inverse Laplace transform of a sum is the sum of the inverse transforms:

$$f(t) = \mathcal{L}^{-1}\{s\} + \mathcal{L}^{-1}\left\{\frac{1}{s(s - 1)(s - 6)}\right\}$$

- The inverse of s is straightforward.
- The inverse of the rational part requires partial fraction decomposition.

Step 2: Inverse Laplace of the Polynomial Part s

Recall that:

$$\mathcal{L}^{-1}\{s\} = \delta'(t)$$

where $\delta'(t)$ is the derivative of the Dirac delta function. However, in practical applications, especially when dealing with causal systems, the more relevant part is often the rational function. For simplicity and clarity, if the context involves functions of $t \geq 0$, the inverse of s is often considered as a function that corresponds to the derivative of a step function, which is a delta function.

In many engineering contexts, the inverse Laplace transform of s is taken as:

$$
f(t) = \delta'(t)
$$

But for most applications involving causal signals, focus is on the second term. We will proceed with the main focus on the second part, which involves the rational fraction.

Step 3: Focus on the Rational Part

The core challenge lies in:

$$
F_2(s) = \frac{1}{s(s - 1)(s - 6)}
$$

Why Partial Fraction Decomposition?

Partial fractions allow us to decompose a complex rational function into simpler terms, each with known inverse transforms. Once decomposed, each term corresponds to a standard Laplace pair.

Step 4: Perform Partial Fraction Decomposition

Set:

$$
\frac{1}{s(s - 1)(s - 6)} = \frac{A}{s} + \frac{B}{s - 1} + \frac{C}{s - 6}
$$

Solving for A, B, and C:

Multiply both sides by $s(s - 1)(s - 6)$:

$$
1 = A(s - 1)(s - 6) + B s (s - 6) + C s (s - 1)
$$

Now, expand and simplify:

$$
1 = A(s^2 - 7s + 6) + B(s^2 - 6s) + C(s^2 - s)
$$

Expressed as:

$$
1 = A s^2 - 7A s + 6A + B s^2 - 6B s + C s^2 - C s
$$

Group like terms:

$$
1 = (A + B + C) s^2 + (-7A - 6B - C) s + 6A
$$

This equality holds for all s, so equate coefficients:

- Coefficient of s^2:

$$
A + B + C = 0
$$

- Coefficient of s:

$$
-7A - 6B - C = 0
$$

- Constant term:

$$
6A = 1 \Rightarrow A = \frac{1}{6}
$$

Now substitute $A = \frac{1}{6}$ into the other equations:

1. $A + B + C = 0$:

$$
\frac{1}{6} + B + C = 0 \Rightarrow B + C = -\frac{1}{6}
$$

2. $-7A - 6B - C = 0$:

$$
-7 \times \frac{1}{6} - 6B - C = 0 \Rightarrow -\frac{7}{6} - 6B - C = 0
$$

Express C from the first:

$$
C = -\frac{1}{6} - B
$$

Plug into the second:

$$
-\frac{7}{6} - 6B - \left(-\frac{1}{6} - B\right) = 0
$$

Simplify:

$$
-\frac{7}{6} - 6B + \frac{1}{6} + B = 0
$$

Combine like terms:

$$
\left(-\frac{7}{6} + \frac{1}{6}\right) + (-6B + B) = 0
$$

$$
-\frac{6}{6} - 5B = 0 \Rightarrow -1 - 5B = 0
$$

Solve for B:

$$
-5B = 1 \Rightarrow B = -\frac{1}{5}
$$

Now find C:

$$
C = -\frac{1}{6} - B = -\frac{1}{6} - \left(-\frac{1}{5}\right) = -\frac{1}{6} + \frac{1}{5}
$$

Find common denominator (30):

$$
-\frac{5}{30} + \frac{6}{30} = \frac{1}{30}
$$

Final coefficients:

$$
A = \frac{1}{6}, \quad B = -\frac{1}{5}, \quad C = \frac{1}{30}
$$

Step 5: Rewrite the Rational Function

The partial fraction expansion is:

\[
\frac{1}{s(s - 1)(s - 6)} = \frac{1/6}{s} - \frac{1/5}{s - 1} + \frac{1/30}{s - 6}
\]

Step 6: Find the Inverse Laplace Transform of Each Term

Recall standard Laplace inverse pairs:

- \(\mathcal{L}^{-1}\left\{\frac{1}{s}\right\} = 1\) (unit step function)
- \(\mathcal{L}^{-1}\left\{\frac{1}{s - a}\right\} = e^{a t}\)

Applying these:

\[
\mathcal{L}^{-1}\left\{\frac{1/6}{s}\right\} = \frac{1}{6} \times 1 = \frac{1}{6}
\]

\[
\mathcal{L}^{-1}\left\{-\frac{1/5}{s - 1}\right\} = -\frac{1}{5} e^{t}
\]

\[
\mathcal{L}^{-1}\left\{\frac{1/30}{s - 6}\right\} = \frac{1}{30} e^{6t}
\]

Combined inverse:

\[
f_2(t) = \frac{1}{6} - \frac{1}{5} e^{t} + \frac{1}{30} e^{6t}
\]

Step 7: Combine All Parts

Recall that the original function's inverse is:

\[
f(t) = \mathcal{L}^{-1}\{s\} + \mathcal{L}^{-1}\left\{\frac{1}{s(s - 1)(s - 6)}\right\}
\]

```
- As previously discussed, the inverse of \(s\) corresponds to a delta
function or, in some contexts, can be interpreted as the derivative of the
step function. For practical purposes, in causal systems, the main
contribution is from the second part.

Final expression:

\[
\boxed{
f(t) = \delta'(t) + \frac{1}{6} - \frac{1}{5} e^{t} + \frac{1}{30} e^{6t}
}
\]
```

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