Circle C Is Centered At The Origin. If Q(10,0) Lies On Circle C, Which Of The Following Points Also Lies

Understanding the Geometric Foundations of Circle C Centered at the Origin

Circle C Is Centered At The Origin. If Q(10,0) Lies On Circle C, Which Of The Following Points Also Lies — this statement sets the stage for exploring fundamental concepts in coordinate geometry, specifically about circles centered at the origin. In this article, we will delve into how to determine whether a point lies on a given circle, analyze the properties of circles centered at the origin, and examine the implications of the point Q(10, 0) lying on circle C.

Understanding these concepts is crucial for students, educators, or anyone interested in geometry because it provides insight into how the position of points in the coordinate plane relates to the equations of circles. We will cover the equations of circles, the significance of the radius, and how to identify other points that satisfy the circle's equation.

Fundamentals of Circles in the Coordinate Plane

The Standard Equation of a Circle

In the coordinate plane, a circle with center (h, k) and radius r has the equation:

$$
(x - h)^2 + (y - k)^2 = r^2
$$

- When the circle is centered at the origin (0, 0), this simplifies to:

$$
x^2 + y^2 = r^2
$$

- The radius r is the distance from the center to any point on the circle.

Circle C Centered at the Origin

Given that circle C is centered at the origin, its equation is:

$$
x^2 + y^2 = r^2
$$

- To find the radius r, we need a point on the circle.

Given Point Q(10, 0):

- Since Q lies on circle C:

$$
(10)^2 + (0)^2 = r^2
$$
$$
100 + 0 = r^2
$$
$$
r^2 = 100
$$
$$
r = \sqrt{100} = 10
$$

Therefore, the equation of circle C is:

$$
x^2 + y^2 = 100
$$

This key step allows us to verify whether other points lie on the circle.

Determining if a Point Lies on Circle C

To check if a particular point P(x, y) lies on circle C, substitute the point's coordinates into the circle's equation:

- If the left side equals 100, the point lies on the circle.
- If the left side is less than 100, the point is inside the circle.
- If the left side is greater than 100, the point is outside the circle.

Example:

Suppose we want to verify whether point A(0, 10) lies on circle C:

$$
0^2 + 10^2 = 0 + 100 = 100
$$

Since it equals 100, point A(0, 10) lies on the circle.

Identifying Other Points on Circle C

Given the circle's equation $x^2 + y^2 = 100$, any point (x, y) satisfying this equation lies on the circle. Let's explore the types of points that satisfy this condition:

Points on the Circle at Specific Coordinates

- Along the x-axis:

- (10, 0): Already given as Q.
- (-10, 0):

$$
(-10)^2 + 0^2 = 100 + 0 = 100
$$

- So, (-10, 0) lies on the circle.

- Along the y-axis:

- (0, 10):

$$
0^2 + 10^2 = 0 + 100 = 100
$$

- Also on the circle.

- (0, -10):

$$
0^2 + (-10)^2 = 0 + 100 = 100
$$

- On the circle.

Thus, points symmetric about the axes are on the circle.

Points with Other Coordinates

- For any value of x, y can be found using:

\[

$$
y = \pm \sqrt{100 - x^2}
$$

Provided $x^2 \leq 100$.

Examples:

- $x = 6$:

$$
y = \pm \sqrt{100 - 36} = \pm \sqrt{64} = \pm 8
$$

- Points:

- (6, 8)
- (6, -8)

- $x = 8$:

$$
y = \pm \sqrt{100 - 64} = \pm \sqrt{36} = \pm 6
$$

- Points:

- (8, 6)
- (8, -6)

This process helps find multiple points on the circle.

Application: Which Points Also Lie on Circle C?

Suppose the question provides multiple options, such as:

1. (0, -10)
2. (5, 5)
3. (8, 6)
4. (12, 0)

Let's verify each:

Point 1: (0, -10)

$$
0^2 + (-10)^2 = 0 + 100 = 100
$$

- Lies on the circle.

Point 2: (5, 5)

$$
5^2 + 5^2 = 25 + 25 = 50
$$

- $50 \neq 100$, so it does not lie on the circle.

Point 3: (8, 6)

$$
8^2 + 6^2 = 64 + 36 = 100
$$

- Lies on the circle.

Point 4: (12, 0)

$$
12^2 + 0^2 = 144 + 0 = 144
$$

- $144 \neq 100$, so it does not lie on the circle.

Summary:

- Points (0, -10) and (8, 6) lie on circle C.
- Points (5, 5) and (12, 0) do not.

Implications for Geometry and Problem Solving

Understanding how to verify whether points lie on a circle centered at the origin extends beyond academic exercises. It has practical applications in fields like engineering, computer graphics, and navigation systems.

Key takeaways include:

- The importance of the circle's radius and how it defines the set of all points satisfying the circle's equation.
- The symmetry of points about axes for circles centered at the origin.
- The method of substituting point coordinates into the circle's equation for verification.

Additional Topics for Exploration

To deepen understanding, students and enthusiasts can explore:

- Deriving the equation of a circle given two points: Find the circle passing through points A and B with known positions.
- Finding tangent points: Determine where a line intersects the circle.
- Transformations and translations: How moving the circle's center affects its equation and points on it.
- Graphing circles: Visualizing the circle and key points to develop spatial intuition.

Conclusion

In summary, when a circle C is centered at the origin and passes through a point Q(10, 0), the radius of the circle is 10. The equation of the circle is $x^2 + y^2 = 100$. To determine whether a point lies on this circle, substitute its coordinates into the equation and check for equality. Points with coordinates satisfying this equation, such as (0, -10) and (8, 6), also lie on the circle. This process underscores fundamental principles of coordinate geometry, including the relationship between a circle's radius, center, and points on its circumference.

Mastering these concepts enables learners to solve a wide array of geometric problems, develop spatial reasoning, and appreciate the elegance of geometric relationships in the coordinate plane.

Frequently Asked Questions

What is the equation of a circle centered at the origin passing through the point Q(10,0)?

The equation is $x^2 + y^2 = 100$, since the radius is 10 units.

If circle C is centered at the origin and passes through Q(10,0), which other points with the same distance from the origin lie on the circle?

Any point that is 10 units from the origin, such as (-10, 0), (0, 10), or (0, -10).

Does the point (7, 7) lie on the circle centered at the origin passing through Q(10,0)?

No, because the distance from the origin is $\sqrt{(7^2 + 7^2)} = \sqrt{98} \approx 9.9$, which is not 10.

Which of these points lie on the circle: (0, 10), (10, 0), (10, 10), or (0, -10)?

The points (0, 10), (10, 0), and (0, -10) lie on the circle, since they are 10 units from the origin.

How can you determine if a point lies on the circle centered at the origin with radius 10?

Calculate the distance from the origin; if it equals 10, the point lies on the circle.

Is the point (-10, 0) on the circle centered at the origin passing through Q(10,0)?

Yes, because its distance from the origin is $\sqrt{(-10)^2 + 0^2} = 10$.

What is the general form of the equation for the circle centered at the origin passing through (10,0)?

The equation is $x^2 + y^2 = 100$.

If a point P has coordinates (x, y) and lies on the circle centered at the origin with radius 10, what is the relationship between x and y?

They satisfy the equation $x^2 + y^2 = 100$.

Can the point (6, 8) lie on the circle centered at the origin passing through Q(10,0)?

Yes, because its distance from the origin is $\sqrt{6^2 + 8^2} = \sqrt{100} = 10$, so it lies on the circle.

Which points are definitely on the circle centered at the origin passing through (10, 0)?

Any point where $x^2 + y^2 = 100$, such as (10, 0), (0, 10), (-10, 0), and (0, -10).

Additional Resources

Circle Geometry Analysis: Determining Points on a Circle Centered at the Origin

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Introduction

In the realm of coordinate geometry, understanding the properties of circles and their points is fundamental. Whether you're a student preparing for exams or an educator designing problem sets, grasping how to identify whether a point lies on a given circle is a crucial skill. Today, we explore an intriguing question: "Circle C is centered at the origin. If Q(10,0) lies on Circle C, which of the following points also lies on the same circle?" This problem is more than an exercise in calculation; it exemplifies the elegance of geometric reasoning combined with algebraic application.

In this comprehensive review, we will dissect the problem, analyze the principles involved, and provide step-by-step guidance to identify points lying on the circle. The goal is to develop an intuitive and systematic approach to solving similar problems, emphasizing clarity and depth of understanding.

Understanding the Fundamentals: Circle Equation and Center

Before diving into the specific problem, it's essential to review the basic properties of circles in the coordinate plane.

The Standard Equation of a Circle

A circle with center (h, k) and radius r is described by:

$$
(x - h)^2 + (y - k)^2 = r^2
$$

- Center: (h, k)
- Radius: r

When the circle is centered at the origin $(0, 0)$, the equation simplifies to:

$$
x^2 + y^2 = r^2
$$

This simplification is particularly convenient because the center's coordinates are zero, eliminating the linear terms in x and y.

Analyzing the Given Data: Center at the Origin and Point Q(10,0)

Given that Circle C is centered at the origin, and Q(10,0) lies on the circle, we can determine the radius of the circle directly.

Step 1: Determine the Radius r

Since Q(10,0) lies on the circle:

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$$
(10)^2 + (0)^2 = r^2
$$
$$
100 + 0 = r^2
$$
$$
r^2 = 100
$$
$$
r = \sqrt{100} = 10
$$

Key insight: The radius of circle C is 10 units.

Equation of Circle C

With the center at the origin and the radius determined, the equation of Circle C becomes:

$$
x^2 + y^2 = 100
$$

This equation is the foundation for testing whether other points lie on the circle.

Identifying Points That Lie on the Circle

Suppose we are presented with multiple candidate points, such as:

- $A(0,10)$
- $B(10,10)$
- $C(7,7\sqrt{3})$
- $D(0,-10)$

Our task is to verify which of these points satisfy the circle's equation.

Step-by-Step Verification Process

The verification process involves substituting the coordinates of each point into the circle's equation:

$$
x^2 + y^2 = 100
$$

- If the equation holds true, the point lies on the circle.
- If it does not, the point is outside or inside the circle, but not on it.

Let's apply this to each candidate point.

Practical Examples: Testing Candidate Points

Point A(0,10)

$$
0^2 + 10^2 = 0 + 100 = 100
$$

Since $100 = r^2$, point A lies on the circle.

Point B(10,10)

$$
10^2 + 10^2 = 100 + 100 = 200
$$

Because $200 \neq 100$, point B does not lie on the circle.

Point C(7,7\sqrt{3})

Calculate y:

$$
7\sqrt{3} \approx 7 \times 1.732 = 12.124
$$

Now,

$$
7^2 + (7\sqrt{3})^2 = 49 + (7^2 \times 3) = 49 + (49 \times 3) = 49 + 147 = 196
$$

Since $196 \neq 100$, point C does not lie on the circle.

Point D(0,-10)

$$
0^2 + (-10)^2 = 0 + 100 = 100
$$

The point D lies on the circle.

Summary of Results

Point	Calculation	On Circle?

$A(0,10)$	$0^2 + 10^2 = 100$	Yes
$B(10,10)$	$100 + 100 = 200$	No
$C(7,7\sqrt{3})$	$49 + 147 = 196$	No
$D(0,-10)$	$0 + 100 = 100$	Yes

Thus, points A and D lie on the circle, while points B and C do not.

Broader Implications: Recognizing Patterns and Applying Principles

This problem emphasizes several key principles in coordinate geometry:

1. Center and Radius Relationship: The circle's equation directly derives from the center and radius, simplifying analysis when the center is at the origin.
2. Distance Formula as a Tool: Verifying whether a point lies on a circle involves applying the distance formula, which, in the case of a circle centered at the origin, reduces to checking if $x^2 + y^2 = r^2$.
3. Symmetry in Coordinates: Points symmetric about axes often reveal whether they are on the circle, especially for circles centered at the origin.

Extending the Example: Additional Points and Variations

Suppose we are given other points, such as:

- $E(-6,8)$
- $F(8,6)$
- $G(0,10)$

Let's verify:

Point E(-6,8):

$$
(-6)^2 + 8^2 = 36 + 64 = 100
$$
On circle? Yes.

Point F(8,6):

$$
8^2 + 6^2 = 64 + 36 = 100
$$
On circle? Yes.

Point G(0,10):

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$$0 + 100 = 100$$

On circle? Yes.

This demonstrates the symmetry and how multiple points can lie on the circle, especially when they satisfy the equation.

Practical Applications and Common Mistakes

Applications:

- Navigation and GPS: Determining whether a point lies within a certain radius from a fixed point.
- Design and Engineering: Ensuring points or objects are positioned on specific geometric loci.
- Computer Graphics: Rendering circles and verifying pixel positions.

Common Mistakes to Avoid:

- Confusing the radius squared with the radius during calculations.
- Neglecting the signs of coordinates; negative values are squared, so signs do not affect the calculation.
- Assuming points lie on the circle without verifying the equation.

Final Thoughts: Mastering Circle Point Verification

The core takeaway from this analysis is the elegant simplicity of the circle equation centered at the origin: $x^2 + y^2 = r^2$. By knowing the radius, which we derive from a known point on the circle, we can efficiently test whether other points belong to the circle.

This problem exemplifies how algebra and geometry intertwine, offering powerful tools for spatial reasoning. Whether applied in academic settings, engineering, or computer graphics, mastering the technique of point verification on circles enhances spatial awareness and problem-solving efficiency.

Summary

- The circle centered at the origin with point Q(10,0) on it has a radius of 10 units.
- The equation of the circle: $x^2 + y^2 = 100$.
- Points satisfying this equation lie on the circle.
- Verification involves substituting the point's coordinates into the equation.
- Symmetry and understanding of the distance formula streamline the process.
- Recognizing these principles aids in solving more complex geometric problems efficiently.

By internalizing these concepts, you're equipped to approach circle-related questions with confidence, precision, and a deeper appreciation for the beauty of geometric relationships.

End of Article

Circle C Is Centered At The Origin If Q 10 0 Lies On Circle C Which Of The Following Points Also Lies

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