Compute The Discriminant D(x, Y) Of The Function. F(x, Y) = X + Y4 - 6x-2y + 2 (Express Numbers In Exact

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Introduction to Discriminant in Algebraic Functions

Understanding the discriminant of a function is a fundamental aspect of algebra and calculus, as it provides critical information about the nature of the roots of the function, the shape of its graph, and the properties of its solutions. When dealing with polynomial functions, especially quadratic and higher-degree equations, the discriminant helps determine whether the roots are real or complex, distinct or repeated.

In this article, we will explore the process of computing the discriminant \(D(x, y) \) of a specific function:

\[
F(x, y) = x + y^4 - 6x - 2y + 2
\]

with a focus on expressing all numbers in their exact form. The goal is to analyze this function thoroughly, understand its structure, and learn how to compute its discriminant effectively.

Understanding the Given Function

Before diving into the discriminant calculation, it is essential to understand the structure of the function:

\[
F(x, y) = x + y^4 - 6x - 2y + 2
\]

Let's simplify this function by combining like terms:

$$
F(x, y) = (x - 6x) + y^4 - 2y + 2
$$
$$
F(x, y) = -5x + y^4 - 2y + 2
$$

This simplification helps us see the function as a combination of a linear term in x, a quartic term in y, and linear terms in y, along with a constant.

Now, the objective is to analyze this function's properties, specifically focusing on the discriminant, which traditionally involves quadratic functions. Since the function involves y^4, we might need to consider it as a quadratic in y^2, or perform substitutions to analyze roots.

What is the Discriminant?

The discriminant is a quantity that provides insight into the roots of a polynomial equation. For a quadratic equation of the form:

$$
ax^2 + bx + c = 0
$$

the discriminant D is given by:

$$
D = b^2 - 4ac
$$

- If $D > 0$, the quadratic has two distinct real roots.
- If $D = 0$, the quadratic has exactly one real root (a repeated root).
- If $D < 0$, the quadratic has two complex conjugate roots.

In the context of multivariable functions, the discriminant can be extended to analyze the behavior of the function with respect to a particular variable, often by fixing the other variables and considering the resulting polynomial.

Rearranging the Function for Discriminant Analysis

Given the form of $F(x, y)$, and considering $F(x, y) = 0$, we can treat the equation as a quadratic in y^2 or y.

Let's write the function as an equation:

$$
-5x + y^4 - 2y + 2 = 0
$$

or equivalently,

$$
y^4 - 2y + (2 - 5x) = 0
$$

Our goal is to analyze the roots of this quartic polynomial in y, which can be viewed as a quadratic in y^2 or y.

Expressing the Function as a Quadratic in y^2

Note that:

$$
y^4 = (y^2)^2
$$

So, if we substitute $z = y^2$, then:

$$
z^2 - 2 y + (2 - 5x) = 0
$$

However, since the term $-2 y$ is linear in y, it complicates expressing the entire quartic as a quadratic in z. Instead, it might be more straightforward to analyze the quartic directly, or consider the roots in terms of y.

Alternatively, consider the polynomial:

$$
P(y) = y^4 - 2 y + (2 - 5x)
$$

and analyze its roots for varying x.

Calculating the Discriminant of the Quartic Polynomial

The discriminant of a quartic polynomial:

$$
a y^4 + b y^3 + c y^2 + d y + e
$$

is a complex expression involving the coefficients a, b, c, d, e. It determines the nature of roots: whether roots are real or complex, repeated or distinct.

In our case:

$$
P(y) = y^4 + 0 y^3 + 0 y^2 - 2 y + (2 - 5x)
$$

so:

- $a = 1$
- $b = 0$
- $c = 0$
- $d = -2$
- $e = 2 - 5x$

The discriminant D of a quartic with these coefficients can be computed using the known formula or via software tools.

Discriminant Formula for a Quartic Polynomial

The discriminant D of a quartic polynomial:

$$
a y^4 + b y^3 + c y^2 + d y + e
$$

is given by a lengthy algebraic expression involving the coefficients, but for our specific polynomial, with $b = c = 0$, the formula simplifies significantly.

The discriminant for this quartic simplifies to:

$$
D = 256 a^3 e^3 - 192 a^2 b d e^2 - 128 a^2 c^2 e^2 + 144 a^2 c d^2 e - 27 a^2 d^4 + 144 a b^2 c e^2 - 6 a b^2 d^2 e - 80 a c^3 e + 18 a c^2 d^2 - 4 b^3 d^3 - 27 b^4
$$

$$
e^2 + 16 b^4 c^3 - 4 b^3 c^2 d + 18 b^2 c^3 d - 4 b^3 c d^2
$$

But since $b = c = 0$, many terms vanish, leaving:

$$
D = 256 a^3 e^3 - 27 a^2 d^4
$$

Plugging in $a=1$, $d=-2$, and $e=2 - 5x$:

$$
D = 256 (1)^3 (2 - 5x)^3 - 27 (1)^2 (-2)^4
$$
$$
D = 256 (2 - 5x)^3 - 27 (16)
$$
$$
D = 256 (2 - 5x)^3 - 432
$$

Final Expression for the Discriminant $D(x)$

Thus, the discriminant of the quartic polynomial in y is:

$$
\boxed{
D(x) = 256 (2 - 5x)^3 - 432
}
$$

This expression allows us to analyze the roots of the quartic polynomial for different values of x, and consequently, the behavior of the original function $F(x, y)$.

Interpreting the Discriminant $D(x)$

The sign of $D(x)$ determines the nature of roots in y:

- If $D(x) > 0$: The quartic polynomial has four real roots, implying certain properties for the original function.
- If $D(x) = 0$: The polynomial has repeated roots, indicating potential points of tangency or degenerate solutions.

- If $D(x) < 0$: The polynomial has two real roots and two complex conjugate roots, indicating different geometric properties.

Let's analyze the critical points where the discriminant equals zero:

$$
256 (2 - 5x)^3 - 432 = 0
$$
$$
256 (2 - 5x)^3 = 432
$$
$$
(2 - 5x)^3 = \frac{432}{256} = \frac{27}{16}
$$

Taking cube roots:

$$
2 - 5x = \sqrt[3]{\frac{27}{16}} = \frac{\sqrt[3]{27}}{\sqrt[3]{16}} = \frac{3}{\sqrt[3]{16}}
$$

Therefore,

$$
5x = 2 - \frac{3}{\sqrt[3]{16}}
$$
$$
x = \frac{2}{5} - \frac{3}{5 \sqrt[3]{16}}
$$

This critical value of x marks the transition point where the nature of roots in the polynomial changes.

Graphical Interpretation and Applications

Understanding the discriminant

Frequently Asked Questions

What is the general method to compute the discriminant D(x, y) of the function F(x, y) = x + y^4 -

6x - 2y + 2?

To compute the discriminant D(x, y) of the function, first identify the quadratic form within F(x, y), then construct its associated matrix, and finally calculate the determinant of that matrix, ensuring all numbers are expressed in exact form.

How is the quadratic form extracted from the function F(x, y) = x + y^4 - 6x - 2y + 2 for discriminant calculation?

Since F(x, y) includes terms in y^4 and linear terms in x and y, the quadratic form relevant for the discriminant is based on the quadratic terms, which in this case is the y^4 term. However, as y^4 is degree 4, the quadratic form relates to the second-degree terms, and the discriminant focuses on the quadratic part involving y^2 or similar terms.

Is the discriminant D(x, y) of the function F(x, y) equal to the determinant of the Hessian matrix?

Yes, for functions of multiple variables, the discriminant D(x, y) can be computed as the determinant of the Hessian matrix, which contains second derivatives of the function. However, since F(x, y) includes a y^4 term, the Hessian must be carefully computed considering the highest degree terms.

How do you compute the second derivatives needed for the Hessian matrix of F(x, y) = x + y^4 - 6x - 2y + 2?

Calculate the partial derivatives: ∂^2F/∂x^2, ∂^2F/∂x∂y, ∂^2F/∂y∂x, and ∂^2F/∂y^2. For this function, ∂^2F/∂x^2 = 0, ∂^2F/∂x∂y = 0, and ∂^2F/∂y^2 = 12y^2, since the only y-dependent quadratic term is y^4.

What is the explicit form of the Hessian matrix for F(x, y) = x + y^4 - 6x - 2y + 2?

The Hessian matrix H is:

H =
| ∂^2F/∂x^2 ∂^2F/∂x∂y |
| ∂^2F/∂y∂x ∂^2F/∂y^2 |

which simplifies to:

| 0 0 |
| 0 12y^2 |.

How is the discriminant D(x, y) calculated from the

Hessian matrix of this function?

The discriminant $D(x, y)$ is the determinant of the Hessian matrix. For this function, $D(x, y) = (0)(12y^2) - (0)(0) = 0$, indicating the Hessian is degenerate, which affects the classification of critical points.

Given the Hessian determinant is zero, what does that imply about the critical points of F(x, y)?

A zero Hessian determinant indicates the critical point is degenerate or inconclusive for classifying as a local minimum, maximum, or saddle point. Further analysis is needed to determine the nature of the critical points.

Can the discriminant D(x, y) help identify the nature of the function's critical points in this case?

Since the Hessian determinant is zero everywhere, the discriminant alone cannot determine the nature of the critical points. Additional methods, such as analyzing higher derivatives or the behavior of the function, are required.

How do you express the final discriminant D(x, y) in exact numbers for the given function?

The discriminant $D(x, y)$ is expressed as the determinant of the Hessian matrix, which is 0, so in exact form, $D(x, y) = 0$. If considering specific points, substitute y values into $12y^2$ to evaluate the Hessian's entries accordingly.

Additional Resources

Compute The Discriminant D(x, Y) Of The Function F(x, Y) = X + Y^4 - 6x - 2y + 2

Introduction

In the realm of algebraic geometry and multivariable calculus, understanding the behavior of functions often hinges on analyzing their critical points, singularities, and the nature of their level sets. A crucial component in this analysis is the discriminant, which provides insights into the nature of solutions, bifurcations, and degeneracies present within a function.

This article delves into the detailed computation of the discriminant $D(x, y)$ of the function:

$$
F(x, y) = x + y^4 - 6x - 2y + 2
$$

Our goal is to systematically examine the function's structure, compute its discriminant, interpret its geometric significance, and understand the implications of the results obtained.

Understanding the Function $F(x,y)$

Before proceeding with the discriminant calculation, it is essential to understand the function's form and potential characteristics.

$$
F(x, y) = x + y^4 - 6x - 2y + 2
$$

Simplifying:

$$
F(x, y) = (x - 6x) + y^4 - 2y + 2 = -5x + y^4 - 2y + 2
$$

This polynomial in two variables combines a linear term in x with a quartic in y, along with linear and constant terms. Its structure suggests that for fixed x, the level sets are quartic curves in y, and vice versa.

The Role of the Discriminant in Multivariable Functions

The discriminant, in the context of multivariable functions, often refers to a polynomial expression derived from the function's partial derivatives that signals degeneracies such as multiple roots or singular points. For a function $F(x, y)$, the discriminant is generally associated with the condition where the gradient vanishes or where the solutions to the equation $F(x, y) = 0$ have singularities.

More precisely, for a polynomial F, the discriminant can be viewed as the resultant of F and its derivative with respect to one of its variables, often used to eliminate a variable and find critical parameter values.

Computing the Discriminant $D(x, y)$

Given the function $F(x, y)$, the typical approach involves:

1. Finding the critical points: Solving the system of equations where both partial derivatives vanish:

$$
\frac{\partial F}{\partial x} = 0, \quad \frac{\partial F}{\partial y} = 0
$$

2. Analyzing the resultant: Computing the resultant of these derivatives to eliminate the variable x or y, which yields the discriminant polynomial $D(x, y)$.

Step 1: Compute Partial Derivatives

$$
\frac{\partial F}{\partial x} = 1 - 6 = -5
$$
$$
\frac{\partial F}{\partial y} = 4 y^3 - 2
$$

Notice that:

- The partial derivative with respect to x is a constant -5.

- The partial derivative with respect to y is a cubic polynomial $4 y^3 - 2$.

Step 2: Critical Point Conditions

Set derivatives to zero:

$$
-5 = 0 \quad \Rightarrow \quad \text{No solution}
$$

$$
4 y^3 - 2 = 0 \quad \Rightarrow \quad y^3 = \frac{1}{2}
$$

But since $\frac{\partial F}{\partial x} = -5 \neq 0$, the gradient never vanishes, and therefore, the function F has no critical points where both derivatives vanish simultaneously.

Interpretation:

The key insight here is that the function $F(x, y)$ is strictly monotonic in x because its partial derivative with respect to x is a non-zero constant -5. This implies that the level sets are non-degenerate in x, and the function does not have internal critical points associated with x.

However, the behavior in y is governed by a cubic polynomial, which can have up to three real roots depending on the value of the parameter.

Discriminant of the Univariate Polynomial in y

Given the earlier observations, the relevant discriminant for understanding the nature of the solutions is that of the polynomial in y:

$$
Q(y) = 4 y^3 - 2
$$

The discriminant of a cubic polynomial $a y^3 + b y^2 + c y + d$ is given by:

$$
\Delta = 18 a b c d - 4 b^3 d + b^2 c^2 - 4 a c^3 - 27 a^2 d^2
$$

In our case, the polynomial:

$$
Q(y) = 4 y^3 + 0 y^2 + 0 y - 2
$$

with coefficients:

$$
a = 4, \quad b = 0, \quad c = 0, \quad d = -2
$$

Plugging into the discriminant formula:

$$
\Delta = 18 \times 4 \times 0 \times 0 \times (-2) - 4 \times 0^3 \times (-2) + 0^2 \times 0^2 - 4 \times 4 \times 0^3 - 27 \times 4^2 \times (-2)^2
$$

Simplify step-by-step:

- First term: $18 \times 4 \times 0 \times 0 \times (-2) = 0$

- Second term: $- 4 \times 0^3 \times (-2) = 0$

- Third term: $0^2 \times 0^2 = 0$

- Fourth term: $- 4 \times 4 \times 0^3 = 0$

- Fifth term: $- 27 \times 16 \times 4 = - 27 \times 16 \times 4$

Calculate:

$$
27 \times 16 = 432
$$

$$
432 \times 4 = 1728
$$
$$
\Rightarrow -1728
$$

Thus, the discriminant:

$$
\boxed{D_y = -1728}
$$

Since the discriminant is non-zero, the cubic polynomial $Q(y)$ has three distinct roots over the complex numbers (by the nature of cubic discriminant), and in particular, no multiple roots.

Geometric and Analytical Significance

The key takeaway from the discriminant calculation is:

- The cubic polynomial $Q(y) = 4 y^3 - 2$ has three distinct roots because the discriminant is non-zero.

- These roots are:

$$
y^3 = \frac{1}{2} \quad \Rightarrow \quad y = \sqrt[3]{\frac{1}{2}}
$$

which has a single real root:

$$
y_{1} = \sqrt[3]{\frac{1}{2}}
$$

and two complex conjugates:

$$
y_{2,3} = \sqrt[3]{\frac{1}{2}} \times \omega, \quad \omega = e^{2\pi i/3}, e^{4\pi i/3}
$$

The real root:

$$
y_{r} = \sqrt[3]{\frac{1}{2}}
$$

corresponds to the critical point in y, but since $\partial F / \partial x$ is constant and non-zero, the function \(F\) does not have internal critical points where both derivatives vanish.

The Overall Discriminant \(D(x, y)\)

Given that the partial derivatives reveal no simultaneous zeros, the classical discriminant \(D(x, y)\) associated with the vanishing gradient is trivial in this context.

However, if we interpret the discriminant of the relation \(F(x, y) = 0\) with respect to \(x\) or \(y\), we can consider the following:

Express \(x\) in terms of \(y\):

\[
F(x, y) = 0 \quad \Rightarrow \quad -5x + y^4 - 2 y + 2 = 0
\]
\[
-5x = - y^4 + 2 y - 2 \quad \Rightarrow \quad x = \frac{1}{5} (y^4 - 2 y + 2)
\]

This defines \(x\) as a quartic polynomial in \(y\). The critical points of the level set correspond to points where the derivative of \(x\) with respect to \(y\) is zero:

\[
\frac{d x}{d y} = \frac{1}{5} (4 y^3 - 2) = 0
\]
\[

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