Consider The Line Represented By: Y + 4 = 2/5(x - 9)Write An Equation Representing A Different Line With

Consider The Line Represented By: Y + 4 = 2/5(x - 9)Write An Equation Representing A Different Line With

Understanding how to interpret and manipulate linear equations is fundamental in algebra and coordinate geometry. The given line, expressed in point-slope form, offers a foundation for exploring how different equations can represent various lines with unique properties, slopes, and intercepts. In this comprehensive guide, we will analyze the original equation, convert it into various forms, and then explore how to formulate different lines with specific features. Whether you're a student seeking to deepen your understanding or an educator preparing lessons, this article aims to be an authoritative resource.

Analyzing the Given Line Equation

To begin, let's carefully examine the provided equation:

$$ Y + 4 = \frac{2}{5}(x - 9) $$

This equation is written in the point-slope form, which is generally expressed as:

$$ y - y_1 = m(x - x_1) $$

where:

- (x_1, y_1) is a point on the line
- m is the slope of the line

Identifying Components:

- Slope $m = \frac{2}{5}$
- Point (x_1, y_1): To find this point, compare with the form $y - y_1 = m(x - x_1)$:

$$ y + 4 = \frac{2}{5}(x - 9) \Rightarrow y - (-4) = \frac{2}{5}(x - 9) $$

Thus, $y_1 = -4$, $x_1 = 9$.

Key features of the original line:

- Slope: $\frac{2}{5}$
- Passes through $(9, -4)$

Converting the Equation into Different Forms

Understanding various forms of the linear equation allows for versatile applications and easier interpretation.

1. Slope-Intercept Form (y = mx + b)

Starting with the point-slope form:

$$ y + 4 = \frac{2}{5}(x - 9) $$

Distribute:

$$ y + 4 = \frac{2}{5}x - \frac{2}{5} \times 9 $$
$$ y + 4 = \frac{2}{5}x - \frac{18}{5} $$

Subtract 4 from both sides:

$$ y = \frac{2}{5}x - \frac{18}{5} - 4 $$

Express 4 as $\frac{20}{5}$:

$$ y = \frac{2}{5}x - \frac{18}{5} - \frac{20}{5} $$

Combine:

$$ y = \frac{2}{5}x - \frac{38}{5} $$

Result:

$$ y = \frac{2}{5}x - \frac{38}{5} $$

Interpretation:

- Slope $m = \frac{2}{5}$ indicates a moderate upward tilt.
- Y-intercept $b = -\frac{38}{5} \approx -7.6$.

2. Standard Form (Ax + By = C)

Starting from the slope-intercept form:

$$ y = \frac{2}{5}x - \frac{38}{5} $$

Multiply through by 5 to clear fractions:

$$ 5y = 2x - 38 $$

Rearranged:

$$ 2x - 5y = 38 $$

This is the standard form, with integer coefficients.

Formulating a Different Line: Variations and Examples

Having understood the original line's equations, we now explore how to create different lines with specific characteristics.

1. Same Slope, Different Y-Intercept

- The slope remains $\frac{2}{5}$, but the line passes through a different point.

Example:

Choose point $(4, 2)$.

Using point-slope form:

$$ y - 2 = \frac{2}{5}(x - 4) $$

Convert to slope-intercept form:

$$ y - 2 = \frac{2}{5}x - \frac{2}{5} \times 4 $$
$$ y - 2 = \frac{2}{5}x - \frac{8}{5} $$
$$ y = \frac{2}{5}x - \frac{8}{5} + 2 $$

Express 2 as $\frac{10}{5}$:

$$ y = \frac{2}{5}x + \frac{2}{5} $$

Equation:

$$ y = \frac{2}{5}x + \frac{2}{5} $$

This line has the same slope but a different y-intercept, illustrating parallelism.

2. Same Point, Different Slope

- Keep the point $(9, -4)$, but choose a different slope.

Example:

Set $m = -\frac{1}{2}$:

$$y + 4 = -\frac{1}{2}(x - 9)$$

Convert to slope-intercept:

$$y + 4 = -\frac{1}{2}x + \frac{1}{2} \times 9$$
$$y + 4 = -\frac{1}{2}x + \frac{9}{2}$$

Subtract 4 (which is $\frac{8}{2}$):

$$y = -\frac{1}{2}x + \frac{9}{2} - \frac{8}{2}$$
$$y = -\frac{1}{2}x + \frac{1}{2}$$

Equation:

$$y = -\frac{1}{2}x + \frac{1}{2}$$

This line intersects the original at some point, but with a different slope, it is neither parallel nor coincident.

3. Perpendicular and Parallel Lines

- Parallel Line: Same slope, different intercept.

- Perpendicular Line: Negative reciprocal slope.

Parallel to original:

$$y = \frac{2}{5}x + c$$

Choose $c = 5$:

$$y = \frac{2}{5}x + 5$$

Perpendicular to original:

Original slope is $\frac{2}{5}$, so the perpendicular slope is:

$$m_{perpendicular} = -\frac{5}{2}$$

Equation passing through $(9, -4)$:

$$y + 4 = -\frac{5}{2}(x - 9)$$

Convert to slope-intercept:

$$y + 4 = -\frac{5}{2}x + \frac{5}{2} \times 9$$
$$y + 4 = -\frac{5}{2}x + \frac{45}{2}$$

Subtract 4 ($\frac{8}{2}$):

$$y = -\frac{5}{2}x + \frac{45}{2} - \frac{8}{2}$$
$$y = -\frac{5}{2}x + \frac{37}{2}$$

Equation:

$$y = -\frac{5}{2}x + \frac{37}{2}$$

This line is perpendicular to the original.

Creating a Line with Specific Features

Suppose you want to construct a line with particular attributes. Here are common scenarios:

1. Line with a Given Slope and Y-Intercept

- For example, slope $m = 3$, y-intercept $b = -2$:

$$y = 3x - 2$$

2. Line Passing Through Two Points

- For points (x_1, y_1) and (x_2, y_2), the slope is:

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

- Then use point-slope form with either point.

Example:

Points: $(2, 3)$ and $(4, 7)$:

$$m = \frac{7 - 3}{4 - 2} = \frac{4}{2} = 2$$

Equation using point $(2, 3)$:

$$y - 3 = 2(x - 2)$$
$$y = 2x - 4 + 3$$
$$y = 2x - 1$$

Applications and Significance of Different Line Equations

Understanding how to manipulate and generate various linear equations is crucial across multiple fields:

- Physics: Describing motion with constant velocity (slope as speed).
- Economics: Cost functions, revenue models.
- Engineering: Signal processing, system modeling.
- Mathematics Education: Teaching concepts of slope,

Frequently Asked Questions

What is the slope of the line represented by the equation y + 4 = (2/5)(x - 9)?

The slope of the line is 2/5.

How do you convert the equation y + 4 = (2/5)(x - 9) into slope-intercept form?

Distribute the 2/5 to get y + 4 = (2/5)x - (18/5), then subtract 4 from both sides to get y = (2/5)x - (18/5) - 4, which simplifies to y = (2/5)x - (18/5) - (20/5) = (2/5)x - (38/5).

What is the y-intercept of the line y = (2/5)x - (38/5)?

The y-intercept is -38/5, which is approximately -7.6.

Write an equation of a line parallel to the given line and passing through the point (0, 0).

Since parallel lines have the same slope, the equation is y = (2/5)x.

Write an equation of a line perpendicular to the given line and passing through the point (0, 0).

Perpendicular lines have slopes that are negative reciprocals. The slope of the given line is 2/5, so the perpendicular slope is -5/2. The equation passing through (0,0) is y = (-5/2)x.

How can you write a different line with the same slope but a different y-intercept?

Choose a different y-intercept, for example, 3, and write y = (2/5)x + 3.

If the original line passes through the point (9, -4), how can you verify this point lies on the line?

Substitute x = 9 into the slope-intercept form y = (2/5)x - (38/5): y = (2/5)9 - (38/5) = 18/5 - 38/5 = -20/5 = -4, confirming the point (9, -4) lies on the line.

What is the significance of the point (9, -4) in the original line's equation?

The point (9, -4) is a point on the line, which can be used to derive or verify the line's equation.

Additional Resources

Understanding the Equation of a Line: A Comprehensive Exploration of Y + 4 = (2/5)(x - 9)

Introduction to the Equation and Its Significance

Mathematics, especially algebra, offers a toolkit for describing and analyzing lines on a coordinate plane. One of the fundamental forms used is the point-slope form, which allows us to write the equation of a line when a point on the line and its slope are known. The equation provided—Y + 4 = (2/5)(x - 9)—is a classic example of this form, and understanding it thoroughly is essential for grasping more complex concepts in analytic geometry.

This piece aims to dissect this specific equation, interpret its components, and explore how to generate different lines with similar or related properties. We will delve into the structure of the equation, identify key characteristics such as slope and intercepts, and then discuss methods to derive different lines with various equations based on the original.

Breaking Down the Given Equation

Point-Slope Form Explained

The equation:

\[Y + 4 = \frac{2}{5}(x - 9) \]

is written in the point-slope form, which generally takes the shape:

$$ y - y_1 = m(x - x_1) $$

where:

- (x_1, y_1) is a point on the line.
- m is the slope of the line.

In the given equation, it is rearranged slightly:

$$ Y + 4 = \frac{2}{5}(x - 9) $$

which can be rewritten as:

$$ y - (-4) = \frac{2}{5}(x - 9) $$

indicating that the point $(9, -4)$ lies on the line, and the slope of the line is $\frac{2}{5}$.

Key components:

- Point: $(9, -4)$
- Slope: $m = \frac{2}{5}$

Understanding these components allows us to analyze the behavior of the line graphically, such as its steepness and where it passes through the coordinate plane.

Converting to Slope-Intercept Form

To deepen our understanding, converting the point-slope form into the slope-intercept form:

$$ y = mx + b $$

is helpful. Starting from:

$$ y + 4 = \frac{2}{5}(x - 9) $$

we expand and simplify:

$$ y + 4 = \frac{2}{5}x - \frac{2}{5} \times 9 $$

$$ y + 4 = \frac{2}{5}x - \frac{18}{5} $$

Subtract 4 from both sides:

$$ y = \frac{2}{5}x - \frac{18}{5} - 4 $$

Express 4 as a fraction with denominator 5:

$$ y = \frac{2}{5}x - \frac{18}{5} - \frac{20}{5} $$

Combine like terms:

$$ y = \frac{2}{5}x - \frac{38}{5} $$

This form makes it straightforward to identify the y-intercept:

- Y-intercept: $(0, -\frac{38}{5})$

and the slope:

- Slope: $\frac{2}{5}$

Geometric Interpretation and Graphical Features

Slope and Its Implications

The slope $\frac{2}{5}$ indicates that for every increase of 5 units in x, y increases by 2 units. This positive slope signifies an upward trend from left to right on the graph.

The slope's fraction form also reveals the line's steepness:

- A slope of $\frac{2}{5}$ is gentle, neither very steep nor flat.
- The rise over run is 2 over 5, which can be visualized as a moderate incline.

Y-Intercept and X-Intercept

- The Y-intercept occurs at $y = -\frac{38}{5} \approx -7.6$, indicating where the line crosses the Y-axis.
- To find the X-intercept, set $y = 0$:

$$
0 = \frac{2}{5}x - \frac{38}{5}
$$

Multiply through by 5 to clear fractions:

$$
0 = 2x - 38
$$

$$
2x = 38
$$

$$
x = 19
$$

Therefore, the line crosses the X-axis at $(19, 0)$.

Deriving Equations for Different Lines

Having thoroughly examined the original line, we now turn to the core question: "Write an equation representing a different line with..."

This invites exploration into how to generate equations of other lines that are related or share certain properties with the original.

1. Lines with the Same Slope but Different Intercepts

Since the slope $\frac{2}{5}$ determines the angle of inclination, lines with the same slope are called parallel lines.

How to write such lines:

- Keep the slope $m = \frac{2}{5}$.
- Choose different points (x_1, y_1) to pass through.
- Use point-slope form:

$$
y - y_1 = \frac{2}{5}(x - x_1)
$$

Example:

Suppose we pick $(0, 2)$, then the equation:

$$
y - 2 = \frac{2}{5}(x - 0) \Rightarrow y = \frac{2}{5}x + 2
$$

This line is parallel to the original but crosses the Y-axis at 2.

---

2. Lines with Different Slopes

To create a different line, we may choose a different slope.

For example:

- Slope: $m = -\frac{1}{2}$

- Using the point $(9, -4)$:

$$
y + 4 = - \frac{1}{2}(x - 9)
$$

Expanding:

$$
y + 4 = - \frac{1}{2}x + \frac{9}{2}
$$

Subtract 4:

$$
y = - \frac{1}{2}x + \frac{9}{2} - 4
$$

Express 4 as $\frac{8}{2}$:

$$
y = - \frac{1}{2}x + \frac{9}{2} - \frac{8}{2} = - \frac{1}{2}x + \frac{1}{2}
$$

Equation:

$$
y = - \frac{1}{2}x + \frac{1}{2}
$$

This line has a negative slope, crossing the Y-axis at 0.5.

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3. Lines with Specific Intercepts

Suppose we want the new line to pass through a specific point or have a particular intercept:

- Y-intercept at $(0, c)$:

Using the slope $\frac{2}{5}$:

\[

y = \frac{2}{5}x + c
\]

- If \(c = 3 \), then:

\[
y = \frac{2}{5}x + 3
\]

- The line passes through \((0, 3) \), and is parallel to the original.

- X-intercept:

Set \(y = 0 \):

\[
0 = \frac{2}{5}x + c
\]

\[
x = - \frac{5}{2} c
\]

Thus, changing \(c \) shifts the line vertically, affecting intercepts but not slope.

Transformations and Variations of the Original Line

1. Vertical Shifts

- Moving the line up or down involves changing \(y \)-intercept.

- For example, shifting the original line upward by 2 units:

\[
y = \frac{2}{5}x - \frac{38}{5} + 2
\]

Express 2 as \(\frac{10}{5} \):

\[
y = \frac{2}{5}x - \frac{38}{5} + \frac{10}{5} = \frac{2}{5}x - \frac{28}{5}
\]

- This is a parallel line, shifted upward.

Similarly, shifting downward by 3 units:

$$
y = \frac{2}{5}x - \frac{38}{5} - 3
$$

Express 3 as $\frac{15}{5}$:

$$
y = \frac{2}{5}x - \frac{38}{5} - \frac{15}{5} = \frac{2}{5}x - \frac{53}{5}
$$

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