Consider The Lines L1 : 2 4t, 1 + 3t, 2t And L2 : S + 5, S 3, 2 4s.(a) Show That The Lines Intersect.(b)

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Understanding the Problem: Analyzing the Given Lines

When studying lines in three-dimensional space, a common task is to determine whether two lines intersect, are parallel, or are skewed (neither intersecting nor parallel). The problem at hand involves two parametric lines, labeled L1 and L2, with given equations that include parameters t and s.

Specifically, the lines are defined as follows:

- Line L1:
$x = 2 + 4t$
$y = 1 + 3t$
$z = 2t$

- Line L2:
$x = s + 5$
$y = s + 3$
$z = 2s$

The task is divided into two parts:

1. Part (a): Show that the lines intersect.
2. Part (b): (Not explicitly detailed here but likely related to further properties like finding the point of intersection or verifying if the lines are skewed or parallel).

In this article, we will explore how to analyze these lines, determine whether they intersect, and understand the underlying concepts involved in such problems.

Understanding Parametric Equations of Lines in 3D Space

Parametric equations describe a line in space using a parameter (t or s). For a line in three dimensions, the general form is:

$$
\begin{cases}
x = x_0 + a t \\
y = y_0 + b t \\
z = z_0 + c t
\end{cases}
$$

where (x_0, y_0, z_0) is a point on the line, and (a, b, c) is the direction vector indicating the line's orientation.

Line L1:
- Point on L1 when $t=0$: $(2, 1, 0)$
- Direction vector: $\vec{d_1} = (4, 3, 2)$

Line L2:
- Point on L2 when $s=0$: $(5, 3, 0)$
- Direction vector: $\vec{d_2} = (1, 1, 2)$

Understanding these components helps in analyzing whether the lines intersect, are parallel, or skew.

Part (a): Showing That the Lines Intersect

To determine whether two lines in three-dimensional space intersect, we need to find a common point (x, y, z) that satisfies both lines' parametric equations for some values of parameters t and s.

Step 1: Set Up the Equations

From the parametric equations:

$$
\begin{cases}
x = 2 + 4t \quad \text{(from L1)} \\
x = s + 5 \quad \text{(from L2)}
\end{cases}
$$

$$
\begin{cases}
y = 1 + 3t \quad \text{(from L1)} \\

y = s + 3 \quad \text{(from L2)}
\end{cases}
\]

\[
\begin{cases}
z = 2t \quad \text{(from L1)} \\
z = 2s \quad \text{(from L2)}
\end{cases}
\]

To find the intersection point, these equations must be simultaneously satisfied for some \(t\) and \(s\).

Step 2: Equate the Corresponding Coordinates

From the \(x\)-coordinate equations:

\[
2 + 4t = s + 5
\]
which simplifies to:

\[
s = 2 + 4t - 5 = -3 + 4t
\]

From the \(y\)-coordinate equations:

\[
1 + 3t = s + 3
\]
which simplifies to:

\[
s = 1 + 3t - 3 = -2 + 3t
\]

From the \(z\)-coordinate equations:

\[
2t = 2s
\]

which simplifies to:

$$
t = s
$$

Now, we have three expressions for s:

- $s = -3 + 4t$
- $s = -2 + 3t$
- $t = s$

Substituting $s = t$ into the first two equations:

$$
t = -3 + 4t \implies -3 + 4t = t \implies -3 = -3t \implies t = 1
$$

Similarly, check the second:

$$
t = -2 + 3t \implies -2 + 3t = t \implies -2 = -2t \implies t = 1
$$

Both yield $t=1$, and since $s = t$, then $s=1$.

Step 3: Find the Point of Intersection

Using $t=1$:

$$
x = 2 + 4(1) = 2 + 4 = 6
$$

$$
y = 1 + 3(1) = 1 + 3 = 4
$$

$$
z = 2(1) = 2
$$

Check with $s=1$ in L2:

$$
x = 1 + 5 = 6
$$

$$
y = 1 + 3 = 4
$$

$$
z = 2(1) = 2
$$

The point $(6, 4, 2)$ satisfies both lines' parametric equations, confirming that the lines indeed intersect at this point.

Conclusion: Lines Intersect at Point (6, 4, 2)

Since we have found consistent parameters $t=1$ and $s=1$ that lead to the same point $(6, 4, 2)$, the lines L1 and L2 intersect at this point in three-dimensional space.

Additional Insights and Geometric Interpretation

Understanding whether lines intersect in 3D space involves examining their parametric equations, direction vectors, and points. The approach used here—setting equations equal and solving for parameters—is standard in vector geometry.

Key concepts involved include:

- Parametric equations: Describing lines using parameters.
- Equating coordinates: To find common points.
- Solving systems of equations: To determine the existence of solutions.

Geometric implications:

- If the lines intersect at exactly one point, they are called intersecting lines.
- If they are parallel but do not intersect, their direction vectors are scalar multiples, and their points do not coincide.
- If they are neither parallel nor intersecting, they are called skew lines.

In this case, the calculations confirm an intersection point, indicating that the lines are intersecting.

Summary of the Process

Here's a step-by-step summary of how to determine if two lines in 3D intersect:

1. Write the parametric equations of both lines.
2. Equate the corresponding coordinate equations.
3. Solve for the parameters t and s.
4. Check for consistency among solutions.
5. Substitute back to find the intersection point.

This systematic approach applies broadly to similar problems in vector geometry and three-dimensional analytical geometry.

Additional Considerations and Practice

To reinforce understanding, consider practicing with other pairs of lines, including:

- Lines that are parallel: $\text{direction vectors proportional}$, but points are different.
- Skew lines: Neither parallel nor intersecting.
- Coincident lines: Same line, overlapping completely.

Understanding these distinctions is fundamental for advanced studies in vector calculus, computer graphics, and geometry.

Final Thoughts

The analysis of lines in three-dimensional space is a foundational skill in vector geometry. By mastering parametric equations, solving systems of equations, and interpreting geometric conditions, you can confidently determine relationships between lines—whether they intersect, are parallel, or are skewed. The example discussed here illustrates how to methodically approach the problem and arrive at a definitive conclusion regarding the intersection of lines L1 and L2.

Keywords: 3D lines, parametric equations, line intersection, vector geometry, analytical geometry, solving

systems, intersection point

Frequently Asked Questions

What are the parametric equations of lines L1 and L2?

Line L1 is given by the parametric equations: $x = 2 + 4t$, $y = 1 + 3t$, $z = 2t$. Line L2 is given by: $x = s + 5$, $y = s + 3$, $z = 2s$.

How do you determine if two lines in 3D intersect?

To determine if two lines intersect, equate their parametric equations and check if there exists values of parameters t and s that satisfy all three equations simultaneously.

What is the first step to show that lines L1 and L2 intersect?

Set the parametric equations of L1 and L2 equal to each other: $2 + 4t = s + 5$, $1 + 3t = s + 3$, $2t = 2s$, and solve for t and s.

How do you solve for the parameters t and s to find the intersection point?

From the third equation, $2t = 2s$, which simplifies to $t = s$. Substitute $t = s$ into the first and second equations to solve for t and s.

What is the solution for t and s that shows the lines intersect?

From the equations: $2 + 4t = t + 5$ and $1 + 3t = t + 3$. Solving these gives $t = 0.5$ and $s = 0.5$, indicating the lines intersect at those parameter values.

What is the intersection point of lines L1 and L2?

Substituting $t = 0.5$ into L1: $x = 2 + 4(0.5) = 4$, $y = 1 + 3(0.5) = 2.5$, $z = 2(0.5) = 1$. The intersection point is (4, 2.5, 1).

How do you verify that the found point lies on both lines?

Substitute the point (4, 2.5, 1) back into the parametric equations of both lines to confirm it satisfies both sets.

What does it mean if the parametric equations yield consistent solutions **for t and s?**

It means the lines intersect at a common point, confirming their intersection.

Why is it important to check all three coordinates when testing intersection?

Because lines are in three-dimensional space, all three coordinates (x, y, z) must satisfy the equations simultaneously for the lines to intersect.

How is the concept of lines intersecting relevant in 3D geometry problems?

Understanding line intersections helps in solving spatial problems, determining points of connection, and analyzing geometric configurations in 3D space.

Additional Resources

Consider The Lines L1 : 2 4t, 1 + 3t, 2t And L2 : S + 5, S 3, 2 4s

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Analyzing the Intersection of Two Parametric Lines: A Comprehensive Investigation

In the realm of analytical geometry, understanding the relationships between lines—whether they intersect, are parallel, or skew—is fundamental. The exploration of lines defined parametrically offers a methodical approach to decipher these relationships. This article delves into the problem of examining two lines, designated as L1 and L2, given by their parametric equations, with the goal of determining whether they intersect. The process involves detailed algebraic manipulations, geometric interpretations, and verification techniques, providing a thorough understanding suitable for academic review and practical applications.

Introduction to the Problem Statement

Given two lines defined parametrically:

- Line L1:
\[

\begin{cases}
x = 2 + 4t \\
y = 1 + 3t \\
z = 2t
\end{cases}
\]

- Line L2:
\[
\begin{cases}
x = s + 5 \\
y = s \\
z = 2 + 4s
\end{cases}
\]

The core objectives are:

1. Part (a): Show that the two lines intersect.
2. Part (b): Determine the point of intersection if it exists.

This investigation embodies the practical application of parametric equations in spatial geometry, illustrating the process of intersection verification, which is essential in disciplines such as computer graphics, robotics, and physics.

Understanding Parametric Equations of Lines

Before delving into the analysis, it is vital to comprehend the parametric form of lines:

- A line in three-dimensional space can be represented by a point and a direction vector.
- The parametric equations express the coordinates of any point on the line in terms of a parameter (t for L1, s for L2).

For L1:
- Starting point: \((2, 1, 0)\)
- Direction vector: \(\vec{d}_1 = (4, 3, 2)\)

For L2:
- Starting point: \((5, 0, 2)\)
- Direction vector: \(\vec{d}_2 = (1, 1, 4)\)

The intersection of these lines hinges on whether there exist parameters t and s satisfying all three coordinate equations simultaneously.

Part (a): Demonstrating That The Lines Intersect

Step 1: Set Up the System of Equations

To determine if L1 and L2 intersect, we need to find values t and s such that:

$$
\begin{cases}
2 + 4t = s + 5 \quad \text{(x-coordinate)} \\
1 + 3t = s \quad \text{(y-coordinate)} \\
2t = 2 + 4s \quad \text{(z-coordinate)}
\end{cases}
$$

This system equates the parametric expressions for x, y, and z to find common points.

Step 2: Simplify and Solve the System

Let's analyze each equation separately.

Equation 1 (x-coordinate):

$$
2 + 4t = s + 5 \Rightarrow s = 2 + 4t - 5 = 4t - 3
$$

Equation 2 (y-coordinate):

$$
1 + 3t = s
$$

Substituting s from Equation 1 into Equation 2:

$$
1 + 3t = 4t - 3
$$

$$
1 + 3t = 4t - 3
$$

Solve for t:

$$
1 + 3t = 4t - 3 \Rightarrow 1 + 3t - 4t = -3 \Rightarrow 1 - t = -3
$$

$$
-t = -4 \Rightarrow t = 4
$$

Now, find s using $t = 4$:

$$
s = 4(4) - 3 = 16 - 3 = 13
$$

Equation 3 (z-coordinate):

$$
2t = 2 + 4s
$$

Substitute $t = 4$ and $s = 13$:

$$
2 \times 4 = 2 + 4 \times 13
$$

$$
8 = 2 + 52
$$

$$
8 = 54
$$

This is not true, indicating inconsistency.

Step 3: Interpretation of the Result

The inconsistency in the z-coordinate equations implies that the lines do not intersect at a common point when considering all three dimensions simultaneously.

Part (b): Is There a Point of Intersection?

Despite the apparent contradiction in the z-coordinate, it's essential to verify whether the lines are skew or parallel, or if they intersect in some projection.

Alternative Approach: Check for Intersection in a 2D Projection

Since the x and y equations provided a consistent solution for t and s, but the z-coordinate did not, the lines do not intersect in three-dimensional space. They may be skew lines, which do not meet but are not parallel.

Geometric Interpretation:

- The lines are not parallel, as their direction vectors are not scalar multiples.
- The lines are not intersecting, as demonstrated by the inconsistency in the z-coordinate.

Final Conclusion:

- The lines L1 and L2 do not intersect in three-dimensional space.
- They are skew lines, meaning they are neither parallel nor intersecting but exist in the same space.

Deep Dive: Why the Discrepancy Occurred

The algebraic process reveals a crucial aspect of spatial line analysis: the necessity to verify all three coordinates simultaneously. The initial solution suggested a potential intersection based on the x and y coordinates, but the inconsistent z-coordinate indicates that the lines do not share a common point.

This discrepancy underscores an important principle:

> In three-dimensional analysis, satisfying two coordinate equations does not guarantee an intersection unless all three are satisfied simultaneously.

Additional Considerations and Generalizations

When Do Lines Intersect?

Lines in space intersect if and only if:

- There exist parameters t and s satisfying all three parametric equations simultaneously.
- The system of equations is consistent and yields a unique solution.

When Are Lines Skew?

Lines are skew if:

- They are not parallel.
- They do not intersect.
- They are not coplanar.

The algebraic analysis confirms the skew nature of L1 and L2 in this scenario.

Broader Implications and Applications

Understanding the intersection properties of lines has significant implications across various fields:

- Computer Graphics: Determining visibility, rendering, and collision detection.
- Robotics: Path planning and obstacle avoidance.
- Structural Engineering: Analyzing joints and connections.
- Physics: Trajectory analysis and field line interactions.

The methodology demonstrated here—equating parametric equations, solving systems, and verifying consistency—is foundational in these disciplines, emphasizing the importance of rigorous algebraic verification.

Conclusion

The comprehensive investigation into the lines L1 and L2 reveals that, despite initial indications from two coordinates, the lines do not intersect in three-dimensional space. The algebraic process underscores the necessity for consistency across all coordinates to establish intersection points conclusively. This exercise exemplifies the critical reasoning and analytical skills required in spatial geometry and highlights the importance of thorough verification when analyzing geometric configurations.

Understanding such principles not only advances theoretical knowledge but also enhances practical problem-solving skills relevant across science and engineering fields. The detailed approach showcased here provides a template for similar analyses involving parametric lines, offering clarity and precision in complex spatial reasoning tasks.

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Note: For further exploration, readers are encouraged to investigate the conditions under which lines are coplanar or parallel, and to extend these techniques to lines in higher dimensions or in non-Euclidean geometries.

Consider The Lines L1 2 4t 1 3t 2t And L2 S 5 S 3 2 4s A Show That The Lines Intersect B

Consider The Lines L1 2 4t 1 3t 2t And L2 S 5 S 3 2 4s A Show That The Lines Intersect B

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