

Consider The Vector

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Understanding vectors is fundamental in physics and engineering, especially when analyzing forces, velocities, and other directional quantities. In this article, we will explore the vector $\mathbf{P} = 110\mathbf{i} + 60\mathbf{j} + 0.58P\mathbf{k}$, focusing on how to determine its magnitude and direction cosines. These concepts are essential for visualizing and quantifying the orientation of vectors in three-dimensional space.

Understanding the Vector $\mathbf{P}$

The given vector is expressed in component form as:

$$\mathbf{P} = 110\mathbf{i} + 60\mathbf{j} + 0.58P\mathbf{k}$$

where:

- $\mathbf{i}, \mathbf{j}, \mathbf{k}$ are the unit vectors along the x, y, and z axes respectively.
- The coefficients (110, 60, 0.58) are the components of vector $\mathbf{P}$ along each axis.
- The units are Newtons (N) for the x and y components, and possibly another unit (or a scalar factor) for the z component, which could denote a different physical quantity, such as pressure or another parameter, depending on context.

Since the problem states "Where $P = P$ ", it indicates that the vector's magnitude is denoted by P , a scalar quantity derived from the components.

Calculating the Magnitude of Vector $\mathbf{P}$

What is the Magnitude?

The magnitude (or length) of a vector in three-dimensional space is a measure of its size, calculated using the Pythagorean theorem extended into three dimensions. For a vector $\mathbf{P} = P_x\mathbf{i} + P_y\mathbf{j} + P_z\mathbf{k}$, the magnitude $|\mathbf{P}|$ is given by:

$$|\mathbf{P}| = \sqrt{P_x^2 + P_y^2 + P_z^2}$$

Calculating the Magnitude of P

Applying this formula to our vector:

```
\[
P_x = 110\, \text{N}
\]
\[
P_y = 60\, \text{N}
\]
\[
P_z = 0.58\, \text{(unit not specified, but assumed consistent with context)}
\]
```

Thus:

```
\[
|\mathbf{P}| = \sqrt{(110)^2 + (60)^2 + (0.58)^2}
\]
```

Calculating each term:

```
- \ ( 110^2 = 12,100 \)
- \ ( 60^2 = 3,600 \)
- \ ( 0.58^2 = 0.3364 \)
```

Adding these:

```
\[
12,100 + 3,600 + 0.3364 = 15,700.3364
\]
```

Taking the square root:

```
\[
|\mathbf{P}| = \sqrt{15,700.3364} \approx 125.28\, \text{N}
\]
```

Therefore, the magnitude of vector $(\mathbf{P})$ is approximately 125.28 N.

Understanding Direction Cosines of Vector P

What are Direction Cosines?

Direction cosines are the cosines of the angles between a vector and the coordinate axes. They provide a way to express the orientation of a vector in space. For a vector $(\mathbf{P})$, the direction cosines $(\cos \alpha, \cos \beta, \cos \gamma)$ are defined as:

```
\[
\cos \alpha = \frac{P_x}{|\mathbf{P}|}, \quad
\cos \beta = \frac{P_y}{|\mathbf{P}|}, \quad
\cos \gamma = \frac{P_z}{|\mathbf{P}|}
\]
```

where:

- (α) is the angle between $(\mathbf{P})$ and the x-axis.
- (β) is the angle between $(\mathbf{P})$ and the y-axis.
- (γ) is the angle between $(\mathbf{P})$ and the z-axis.

Calculating the Direction Cosines of P

Using the components and the magnitude calculated previously:

```
\[
|\mathbf{P}| \approx 125.28\text{ N}
\]
```

The direction cosines are:

```
\[
\cos \alpha = \frac{110}{125.28} \approx 0.878
\]
\[
\cos \beta = \frac{60}{125.28} \approx 0.479
\]
\[
\cos \gamma = \frac{0.58}{125.28} \approx 0.00463
\]
```

Interpretation:

- The vector is predominantly aligned along the x-axis, with a cosine of approximately 0.878.
- It has a moderate component along the y-axis, with a cosine of about 0.479.
- Its projection along the z-axis is very small, indicated by a cosine close to zero (~0.00463), meaning it is nearly lying in the xy-plane.

Summary and Practical Applications

Understanding the magnitude and direction cosines of vectors like $\mathbf{P}$ is crucial in various fields, including physics, engineering, and navigation. These calculations help in:

- Determining the resultant force or velocity in a given direction.
- Analyzing the orientation of objects or forces in space.
- Calculating work done when a force moves an object along a certain path.

By mastering the calculation of magnitude and direction cosines, engineers and scientists can better interpret physical phenomena and design systems with precise directional control.

Conclusion

In this discussion, we:

- Calculated the magnitude of vector $\mathbf{P}$ to be approximately 125.28 N.
- Determined its direction cosines relative to the coordinate axes, which describe its orientation in space.

These fundamental vector operations are essential tools in analyzing and understanding the behavior of physical systems. Whether in statics, dynamics, or electromagnetism, the ability to decompose vectors into components and interpret their directions forms the backbone of many engineering and physics applications.

Frequently Asked Questions

What is the magnitude of the vector $P = 110\mathbf{i}\text{N} + 60\mathbf{j}\text{N}$?

The magnitude P is calculated using $P = \sqrt{(110^2 + 60^2)} = \sqrt{(12100 + 3600)} = \sqrt{15700} \approx 125.2 \text{ N}$.

How do you find the direction cosines of the vector P ?

The direction cosines are found by dividing each component by the magnitude: $\cos \alpha = 110 / 125.2 \approx 0.878$, $\cos \beta = 60 / 125.2 \approx 0.479$, and $\cos \gamma = 0$ (since P is in the XY -plane).

What is the angle of the vector P with respect to the X -axis?

The angle θ with the X -axis is given by $\theta = \cos^{-1}(110 / 125.2) \approx \cos^{-1}(0.878) \approx 28.4$ degrees.

How can the direction cosines be used to determine the orientation of vector P ?

The direction cosines indicate the angles between the vector and each coordinate axis, helping to understand its spatial orientation. For P , they show it is inclined approximately 28.4° to the X -axis and about 62.3° to the Y -axis.

If the vector P is expressed in unit vector form, what is its representation?

The unit vector in the direction of P is $(110/125.2)\mathbf{i} + (60/125.2)\mathbf{j} \approx 0.878\mathbf{i} + 0.479\mathbf{j}$.

Additional Resources

Comprehensive Analysis of the Vector $P = 110\mathbf{i}\text{N} + 60\mathbf{j}\text{N}$
0.58Pk: Magnitude and Direction Cosines

Introduction

Understanding vectors is fundamental in physics and engineering, providing a means to represent quantities that have both magnitude and direction. The vector in question, $P = 110\mathbf{i}\text{N} + 60\mathbf{j}\text{N}$, encapsulates multiple components across different axes, hinting at a complex spatial orientation and magnitude. This detailed review will dissect the problem into two core parts:

1. Calculating the magnitude of vector P .
2. Determining the direction cosines associated with P .

Throughout this discussion, we will clarify the notation, interpret the components, and perform comprehensive calculations to deepen understanding.

Clarifying the Vector Expression

Before diving into calculations, it's essential to interpret the given vector:

$$\mathbf{P} = 110\hat{i}\text{N} + 60\hat{j}\text{N} \cdot 0.58\text{Pk}$$

At first glance, this appears to be a vector expressed in component form, but the notation is somewhat ambiguous, especially concerning the term '60jN0.58Pk'.

Possible interpretations:

- The vector components could be expressed in Cartesian form with the following breakdown:
- 110iN: A component along the x-axis of 110 Newtons.
- 60jN0.58Pk: Potentially a component along the y-axis involving the scalar 60N multiplied by 0.58 and possibly the unit vector Pk.

However, the inclusion of Pk suggests the vector might have a component along a third axis (k), or perhaps it's a scalar projection involving a scalar multiple of another vector.

Given the notation, the most reasonable interpretation is:

```
\[
\mathbf{P} = 110\hat{i}\text{N} + 60\hat{j}\text{N} \cdot 0.58\text{Pk} + \text{(possibly a component along } \hat{k})
\]
```

But since Pk is written as part of the component, it indicates the vector component along the k-direction is scaled by this factor.

Assumption for Simplification

Assuming the vector P is three-dimensional, with components:

- Along x: $P_x = 110\text{N}$
- Along y: $P_y = 60\text{N} \cdot 0.58 = 34.8\text{N}$
- Along z: $P_z = P_k$, which seems to be a scalar notation, possibly a variable or a scalar multiple of some unit vector.

Given the context, and typical notation, perhaps Pk is a scalar projection or a scalar component along the z-axis, with P possibly being a scalar or vector quantity.

Important Note: The statement "Where $P = P$ " hints at P being a scalar, but since the question asks for the magnitude and direction cosines, P is a vector quantity.

Final Assumption:

Based on typical vector notation and the form, the most logical assumption is:

$$\mathbf{P} = 110 \hat{i} \text{ N} + 34.8 \hat{j} \text{ N} + P_z \hat{k} \text{ N}$$

Where:

- $P_x = 110 \text{ N}$
- $P_y = 34.8 \text{ N}$ (from 60×0.58)
- $P_z = P_k \text{ N}$ (unknown scalar component)

However, since P_z is not explicitly provided, but the question mentions $P = P$, perhaps the scalar P is the magnitude of the vector, and P_k is a component along the z -axis.

Clarification from the Problem Statement

The question states:

- "Consider the vector $\mathbf{P} = 110\hat{i}\text{N} + 60\hat{j}\text{N} + 0.58P_k\hat{k}$, where $P = P$ "
- Part A: The magnitude of $\mathbf{P}$ is $P = N$
- Part B: The direction cosines

This suggests that:

- The vector has components: $(P_x = 110, \text{N})$, $(P_y = 60 \times 0.58, \text{N}) = 34.8, \text{N})$, and perhaps the component along k is $0.58 P_k$, which is proportional to some scalar P_k .

Given the ambiguity, for clarity and simplicity, we will interpret:

$$\boxed{\mathbf{P} = 110\hat{i} + 34.8\hat{j} + 0.58P_k\hat{k}}$$

with the understanding that P_k is a scalar, perhaps 1, or the problem intends for us to find the magnitude and direction cosines based on the first two components, and P_k is a scalar unknown.

Part A: Calculating the Magnitude of $\mathbf{P}$

Step 1: Recognize the component form

$$\mathbf{P} = P_x\hat{i} + P_y\hat{j} + P_z\hat{k}$$

with:

- \(\ P_x = 110\), \(\mathrm{N}\) \)

- \(\ P_y = 34.8\), \(\mathrm{N}\) \)

and

- \(\ P_z = 0.58\), P_k \)

Step 2: Define the magnitude formula

\[
 $|\mathbf{P}| = \sqrt{P_x^2 + P_y^2 + P_z^2}$
\]

Step 3: Calculate the magnitude

- For the known components:

\[
 $|\mathbf{P}| = \sqrt{(110)^2 + (34.8)^2 + (0.58 P_k)^2}$
\]

which simplifies to:

\[
 $|\mathbf{P}| = \sqrt{12100 + 1211.04 + 0.3364 P_k^2}$
\]

This means:

\[
 $|\mathbf{P}| = \sqrt{13311.04 + 0.3364 P_k^2}$
\]

Step 4: Use the given condition that the magnitude of P is P (a scalar)

The problem states: "Where $P = P$," and in Part A, it says "The Magnitude of P is $P = N$." This implies:

\[
 $|\mathbf{P}| = P$
\]

and the numerical value of P is in Newtons.

Given the notation, perhaps the final magnitude is simply $P = 125\text{ N}$ (as a typical magnitude for such a vector).

Alternatively, if the problem states "The magnitude of P is $P = N$," perhaps P is a scalar magnitude in Newtons.

Conclusion: Without explicit value for P_k , and given the problem states the magnitude is some scalar P, we can express the magnitude as a function of P_k or assume P_k is such that the total magnitude equals a specific value.

Part B: Determining the Direction Cosines

Step 1: Understanding direction cosines

Direction cosines are the cosines of the angles between the vector and the coordinate axes:

$$\begin{aligned} \cos \alpha &= \frac{P_x}{|\mathbf{P}|} \\ \cos \beta &= \frac{P_y}{|\mathbf{P}|} \\ \cos \gamma &= \frac{P_z}{|\mathbf{P}|} \end{aligned}$$

where:

- α : angle with the x-axis
- β : angle with the y-axis
- γ : angle with the z-axis

Step 2: Calculations

Using the components (assuming $P_z = 0.58 P_k$):

$$\begin{aligned} \cos \alpha &= \frac{110}{|\mathbf{P}|} \\ \cos \beta &= \frac{34.8}{|\mathbf{P}|} \\ \cos \gamma &= \frac{0.58 P_k}{|\mathbf{P}|} \end{aligned}$$

Step 3: Conditions for unit direction cosines

Since the direction cosines are cosines of angles, they satisfy the relation:

$$\cos^2 \alpha + \cos^2 \beta + \cos^2 \gamma = 1$$

which leads to:

$$\left(\frac{110}{|\mathbf{P}|} \right)^2 + \left(\frac{34.8}{|\mathbf{P}|} \right)^2 + \left(\frac{0.58 P_k}{|\mathbf{P}|} \right)^2 = 1$$

or:

$$\frac{110^2 + 34.8^2 + (0.58 P_k)^2}{|\mathbf{P}|^2} = 1$$

which simplifies to:

$$\frac{12100 + 1211.04 + 0.3364 P_k^2}{|\mathbf{P}|^2} = 1$$

\]

and thus:

$$|\mathbf{P}|^2 = 13311.04 + 0.3364 P_k^2$$

which matches our earlier magnitude calculation.

Step 4: Final expression for direction cosines

$$\cos \alpha = \frac{110}{\sqrt{13311.04 + 0.3364 P_k^2}}$$

Consider The Vector $\mathbf{P} = 110\mathbf{i} + 60\mathbf{j} + 58\mathbf{k}$ Where P Is The Magnitude Of $\mathbf{P}$ Is P The Direction Cosine

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