

Construct A 90% Confidence Interval To Estimate The Population Proportion With A Sample Proportion Equal

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Understanding how to construct a confidence interval for a population proportion is fundamental in statistics, especially when making inferences based on sample data. When the sample proportion is known, and the goal is to estimate the true proportion in the entire population with a specified confidence level—such as 90%—it is essential to follow systematic steps grounded in statistical theory. This article provides a comprehensive guide on how to construct a 90% confidence interval for a population proportion, explaining key concepts, formulas, assumptions, and practical applications.

What Is a Confidence Interval for a Population Proportion?

A confidence interval offers a range of plausible values for an unknown population parameter based on sample data. For a population proportion (denoted as p), a confidence interval estimates the range within which the true proportion is expected to fall with a certain level of confidence (e.g., 90%, 95%, 99%).

Key components:

- Sample proportion ($\hat{p}$): The proportion in the sample, calculated as the number of successes divided by the sample size.
- Confidence level: The probability that the interval contains the true population proportion, expressed as a percentage.
- Margin of Error (ME): The extent to which the estimated interval may vary from the true population proportion.

Constructing a confidence interval involves calculating the point estimate ($\hat{p}$) and then determining the range around it that accounts for sampling variability, scaled according to the desired confidence level.

Prerequisites and Assumptions

Before constructing the confidence interval, certain assumptions must be verified:

1. Random Sampling

The data should come from a random sample of the population to ensure the results are representative.

2. Sample Size and Normal Approximation

The sample size must be sufficiently large so that the sampling distribution of the sample proportion approximates a normal distribution. Typically, this is checked using the success-failure condition:

- $n\hat{p} \geq 10$
- $n(1 - \hat{p}) \geq 10$

where n is the sample size.

3. Independence

The observations should be independent, which is generally satisfied if the sample size is less than 10% of the population when sampling without replacement.

Steps to Construct a 90% Confidence Interval for a Population Proportion

Constructing a confidence interval involves several systematic steps:

Step 1: Collect and Summarize Sample Data

Obtain the sample size (n) and the number of successes (x). Calculate the sample proportion:

$$\hat{p} = \frac{x}{n}$$

Step 2: Verify Conditions

Ensure the success-failure condition holds:

$$n\hat{p} \geq 10 \quad \text{and} \quad n(1 - \hat{p}) \geq 10$$

If these are not satisfied, the normal approximation may not be appropriate, and alternative methods should be considered.

Step 3: Determine the Critical Z-Value for 90% Confidence

Find the z-score corresponding to the middle 90% of the standard normal distribution. Since confidence levels are symmetric, the critical value is:

$$z_{\alpha/2} = z_{0.05}$$

From standard z-tables, this value is approximately:

$$z_{0.05} \approx 1.645$$

Step 4: Calculate the Standard Error (SE)

The standard error measures the variability of the sample proportion:

$$SE = \sqrt{\frac{\hat{p}(1 - \hat{p})}{n}}$$

Step 5: Compute the Margin of Error (ME)

Multiply the critical z-value by the standard error:

$$ME = z_{\alpha/2} \times SE$$

For a 90% confidence level:

$$ME = 1.645 \times SE$$

Step 6: Construct the Confidence Interval

The confidence interval is:

$$\hat{p} \pm ME$$

which provides the lower and upper bounds:

$$\text{Lower bound} = \hat{p} - ME$$

$$\text{Upper bound} = \hat{p} + ME$$

Practical Example

Suppose a researcher surveys 200 individuals to determine the proportion who favor a new policy. Out of these, 120 support the policy.

Step 1: Calculate the sample proportion:

$$\hat{p} = \frac{120}{200} = 0.6$$

Step 2: Verify conditions:

$$n\hat{p} = 200 \times 0.6 = 120 \geq 10$$

$$n(1 - \hat{p}) = 200 \times 0.4 = 80 \geq 10$$

Conditions satisfied.

Step 3: Find the z-value for 90% confidence:

$$\lceil z_{(0.05)} \approx 1.645 \rceil$$

Step 4: Calculate standard error:

$$\lceil SE = \sqrt{\frac{0.6 \times 0.4}{200}} = \sqrt{\frac{0.24}{200}} = \sqrt{0.0012} \approx 0.0346 \rceil$$

Step 5: Compute margin of error:

$$\lceil ME = 1.645 \times 0.0346 \approx 0.057 \rceil$$

Step 6: Determine confidence interval:

$$\lceil \text{Lower bound} = 0.6 - 0.057 = 0.543 \rceil$$

$$\lceil \text{Upper bound} = 0.6 + 0.057 = 0.657 \rceil$$

Interpretation: With 90% confidence, the true proportion of the population supporting the policy is between approximately 54.3% and 65.7%.

Interpreting the Results

The confidence interval provides a range that, based on the sample data, is likely to contain the true population proportion. It is crucial to remember that:

- The specified confidence level (90%) indicates that if we were to take many samples and construct a confidence interval from each, approximately 90% of these intervals would contain the actual population proportion.
- The interval does not specify the probability that the particular interval calculated from this sample contains the true proportion; rather, it reflects the long-term success rate of the method.

Limitations and Considerations

While constructing confidence intervals is a powerful statistical tool, some limitations should be acknowledged:

- Sample Size: Small samples may lead to inaccurate intervals, especially if the success-failure condition isn't met.
- Bias: Non-random sampling or biased data collection methods can distort the results.
- Assumption of Normality: If the sample size is insufficient, the normal approximation may not hold, and alternative methods such as the Wilson score interval or bootstrap methods may be needed.
- Confidence Level Choice: Higher confidence levels (e.g., 99%) produce wider intervals, reflecting increased uncertainty containment.

Extensions and Variations

Beyond the basic method described, several advanced techniques can enhance the accuracy and applicability of confidence interval estimates:

1. Wilson Score Interval

Provides better coverage probabilities, especially with small samples or proportions near 0 or 1.

2. Clopper-Pearson Exact Interval

A conservative method based on the exact binomial distribution, suitable for small samples.

3. Adjustments for Finite Populations

When sampling without replacement from a finite population, the finite population correction factor should be applied.

Conclusion

Constructing a 90% confidence interval to estimate the population proportion involves a clear understanding of sample data, assumptions, and statistical formulas. By following structured steps—calculating the sample proportion, verifying conditions, determining the critical z-value, computing the standard error, and finally deriving the interval—researchers and analysts can make informed inferences about the true proportion in the population. This process is vital across numerous fields, including public health, social sciences, marketing, and political polling, enabling data-driven decisions grounded in statistical confidence.

Remember: Always verify assumptions before constructing confidence intervals, and interpret the results within the context of your specific study, considering potential biases and limitations.

Frequently Asked Questions

What is the formula for constructing a 90% confidence interval for a

population proportion?

The formula is: $\hat{p} \pm Z(\sqrt{\hat{p}(1 - \hat{p})/n})$, where $\hat{p}$ is the sample proportion, n is the sample size, and Z is the Z-value corresponding to the 90% confidence level (approximately 1.645).

How do I interpret a 90% confidence interval for a population proportion?

It means that we are 90% confident that the true population proportion lies within the calculated interval based on our sample data.

What steps are involved in constructing a 90% confidence interval for a population proportion?

First, calculate the sample proportion $\hat{p}$; then find the Z-value for 90% confidence; next, compute the standard error $\sqrt{\hat{p}(1 - \hat{p})/n}$; finally, apply the formula $\hat{p} \pm Z \times \text{standard error}$ to find the interval.

What sample size is needed to construct a reliable 90% confidence interval for a population proportion?

The sample size should be large enough so that $n\hat{p}$ and $n(1 - \hat{p})$ are both at least 10 to ensure the normal approximation is valid for the confidence interval.

How does the sample proportion $\hat{p}$ affect the width of the 90% confidence interval?

A larger $\hat{p}$ (closer to 0.5) generally results in a wider interval, while a $\hat{p}$ near 0 or 1 produces a narrower interval, reflecting the variability in the estimate.

Can I construct a confidence interval for a population proportion if my sample size is small?

For small sample sizes, the normal approximation may not be appropriate. In such cases, consider using exact methods like the Clopper-Pearson interval or Wilson score interval for better accuracy.

What are common pitfalls to avoid when constructing a 90% confidence interval for a population proportion?

Avoid using the normal approximation with small sample sizes, neglecting to check that $n\hat{p}$ and $n(1 - \hat{p})$ are at least 10, and misinterpreting the confidence level as the probability that the interval contains the true proportion in any given sample.

Additional Resources

Constructing a 90% Confidence Interval to Estimate the Population Proportion With a Sample Proportion

Understanding how to estimate a population proportion accurately is fundamental in statistics, especially when conducting surveys or studies involving categorical data. Constructing a 90% confidence interval around a sample proportion provides a range within which we expect the true population proportion to lie with 90% certainty. This process combines theoretical concepts with practical application, ensuring data-driven decision-making. In this comprehensive review, we will explore every facet of constructing a 90% confidence interval for a population proportion, from foundational ideas to detailed calculations and interpretation.

Introduction to Confidence Intervals and Population Proportions

What is a Population Proportion?

The population proportion (denoted as p) refers to the fraction of the entire population possessing a particular characteristic. For example, if a city's residents are surveyed about support for a new policy, the proportion who favor it represents the population proportion.

Key points:

- It is a parameter, meaning it describes the entire population.
- Usually unknown and estimated via sample data.
- Denoted as p , with the sample proportion denoted as $\hat{p}$.

Why Use Confidence Intervals?

Since it's often impractical or impossible to survey the entire population, statisticians rely on samples. A point estimate, like $\hat{p}$, provides a single best guess of p . However, it does not account for sampling variability. Confidence intervals address this by providing a range of plausible values for p , along with a confidence level indicating the probability that this interval captures the true population proportion.

In essence:

- Confidence intervals quantify uncertainty.
- They are constructed to contain the true p with a specified confidence level (e.g., 90%).

Fundamental Concepts for Constructing a Confidence Interval for a Proportion

Sampling Distribution of the Sample Proportion

When sampling repeatedly from a population, the distribution of the sample proportion $\hat{p}$ tends to be approximately normal, given certain conditions. This normal approximation is vital for constructing confidence intervals.

Conditions for normal approximation:

- $np \geq 10$
- $n(1 - p) \geq 10$

where:

- n is the sample size.
- p is the true population proportion (unknown, but estimated via $\hat{p}$).

Standard Error of the Sample Proportion

The standard error (SE) measures the variability of $\hat{p}$:

$$SE = \sqrt{\frac{p(1 - p)}{n}}$$

Since p is unknown, we replace it with $\hat{p}$ to estimate SE:

$$SE \approx \sqrt{\frac{\hat{p}(1 - \hat{p})}{n}}$$

This estimate reflects the sampling variability and is central to constructing the confidence interval.

Z-Score for a 90% Confidence Level

The confidence level determines the critical value, often denoted as (z^*) , corresponding to the middle 90% of the standard normal distribution.

For 90% confidence:

- $(z^* \approx 1.645)$

This value is obtained from standard normal distribution tables or statistical software.

Step-by-Step Process to Construct the 90% Confidence Interval

Constructing the interval involves several steps:

1. Collect Sample Data

- Obtain a random sample of size (n) .
- Calculate the sample proportion $(\hat{p} = \frac{x}{n})$, where (x) is the number of successes in the sample.

2. Check Conditions

- Ensure $(n \times \hat{p} \geq 10)$ and $(n \times (1 - \hat{p}) \geq 10)$.
- If conditions are not met, consider alternative methods or larger samples.

3. Calculate the Standard Error (SE)

$$SE = \sqrt{\frac{\hat{p}(1 - \hat{p})}{n}}$$

4. Determine the Critical Z-Value (z^*)

- For 90% confidence, $(z^* \approx 1.645)$.

5. Construct the Confidence Interval

$$\boxed{\hat{p} \pm z^* \times SE}$$

This yields the lower and upper bounds of the interval:

- Lower bound: $\hat{p} - z^* \times SE$

- Upper bound: $\hat{p} + z^* \times SE$

Practical Example with Detailed Calculations

Suppose a researcher surveys 200 voters about their support for a new policy, and 110 express support. Constructing a 90% confidence interval for the true proportion of supporters involves the following:

Step 1: Calculate $\hat{p}$:

$$\hat{p} = \frac{110}{200} = 0.55$$

Step 2: Check conditions:

$$n \times \hat{p} = 200 \times 0.55 = 110 \geq 10$$
$$n \times (1 - \hat{p}) = 200 \times 0.45 = 90 \geq 10$$

Conditions are satisfied.

Step 3: Calculate Standard Error:

$$SE = \sqrt{\frac{0.55 \times 0.45}{200}} = \sqrt{\frac{0.2475}{200}} \approx \sqrt{0.0012375} \approx 0.0352$$

Step 4: Find z^* for 90% confidence:

$$z^* \approx 1.645$$

Step 5: Compute margin of error:

$$ME = 1.645 \times 0.0352 \approx 0.0579$$

Step 6: Calculate confidence interval:

- Lower bound:

$$0.55 - 0.0579 \approx 0.4921$$

- Upper bound:

$$0.55 + 0.0579 \approx 0.6079$$

Final Result:

$$\boxed{\text{The 90\% confidence interval for the population proportion is approximately } (0.492, 0.608)}$$

This means we are 90% confident that between 49.2% and 60.8% of all voters support the policy.

Interpreting the Confidence Interval

Understanding what the interval means is crucial:

- Probabilistic interpretation: “We are 90% confident that the interval (0.492, 0.608) contains the true population proportion.”
- Not a probability statement about the parameter: The true proportion (p) is fixed; the confidence level reflects the method's long-term success rate over many samples.

Important notes:

- A wider interval indicates more uncertainty.
- A narrower interval implies greater precision.
- The confidence level (here, 90%) indicates the proportion of such intervals that would contain (p) if the process were repeated many times.

Factors Affecting Confidence Interval Width

Several elements influence the size of the confidence interval:

- Sample size (n) : Larger samples reduce the standard error, resulting in narrower intervals.
- Sample proportion $(\hat{p})$: When $(\hat{p})$ is close to 0.5, the standard error is maximized, leading to wider intervals.
- Confidence level: Higher confidence levels (e.g., 95%, 99%) require larger z-scores, increasing the margin of error.

Limitations and Considerations

While constructing confidence intervals is powerful, some limitations and considerations include:

- Sample randomness: The sample must be random and representative to ensure validity.
- Normal approximation validity: For small samples or proportions near 0 or 1, the normal approximation may be invalid. Alternative methods (like the Wilson score interval) are recommended.
- Misinterpretation: Confidence intervals do not provide the probability that (p) lies within the specific interval calculated from a single sample; instead, they relate to the long-term success rate of the method.

Alternative Methods for Confidence Intervals

In cases where the normal approximation assumptions are not met, or for more accurate results, consider:

- Wilson Score Interval: Offers better coverage properties, especially with small samples or extreme proportions.
- Clopper-Pearson Interval (Exact Binomial): Provides exact confidence intervals based on binomial distribution, often used for small samples.
- Bootstrapping: Resampling methods to estimate the interval without distributional assumptions.

Summary and Best Practices

Constructing a 90% confidence interval for a population proportion involves:

- Collecting a random, sufficiently large sample.
- Computing the sample proportion $\hat{p}$

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