Define R As The Region That Is Bounded By The Graph Of The Function F(x)=2e^x, The X-axis, X=0, And X=1.

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Understanding the concept of regions bounded by curves is fundamental in calculus, especially when evaluating areas, volumes, and other related quantities. In this article, we will explore the specific region R defined by the graph of the function $f(x) = 2e^x$, along with the x-axis, and the vertical lines $x=0$ and $x=1$. This detailed examination will include how to visualize the region, calculate its area, and interpret the results within the context of integral calculus.

Defining the Region R in Detail

The Graph of $f(x) = 2e^x$

The function $f(x) = 2e^x$ is an exponential function, characterized by its rapid growth as x increases. Since e^x is always positive for real numbers, multiplying by 2 simply scales the curve vertically.

- Key features of $f(x) = 2e^x$:
- Domain: All real numbers ($-\infty, \infty$)
- Range: $(0, \infty)$
- Behavior: Exponentially increasing, with a y-intercept at $f(0) = 2e^0 = 2$
- Growth rate: Accelerates as x increases

Visualizing this graph between $x=0$ and $x=1$ shows a smooth, increasing curve starting at $f(0) = 2$ and reaching $f(1) = 2e \approx 5.436$.

Boundaries of Region R

The region R is bounded by:

- The graph of $f(x) = 2e^x$: The upper boundary
- The x-axis: The lower boundary at $y=0$
- Vertical boundary at $x=0$: The left boundary
- Vertical boundary at $x=1$: The right boundary

These boundaries create a closed, bounded region in the xy-plane suitable for calculating the area.

Visualizing the Region R

Understanding the shape of R is crucial for setting up the integral calculations.

Graphical Representation

To visualize the region:

- Plot the exponential curve $f(x) = 2e^x$ from $x=0$ to $x=1$
- Draw the x-axis (the line $y=0$)
- Mark the vertical lines at $x=0$ and $x=1$
- Shade the area between the curve and the x-axis within these bounds

This shaded region R resembles a "curved trapezoid" with the top boundary along the exponential curve and the bottom boundary along the x-axis.

Significance of the Region

This region is not just a geometric shape but also a fundamental object in integral calculus for calculating the area under a curve between two points.

Calculating the Area of Region R

The key purpose of defining such a region is often to find its area, which is achieved using definite integrals.

Setting Up the Integral

Since the region R is bounded between $x=0$ and $x=1$, and between the curve $f(x)=2e^x$ and the x-axis, the area A can be expressed as:

$$
A = \int_{0}^{1} f(x) \, dx = \int_{0}^{1} 2e^x \, dx
$$

This integral sums the infinitesimal rectangles under the curve from $x=0$ to $x=1$.

Calculating the Integral

The integral of $2e^x$ is straightforward:

$$
\int 2e^x \, dx = 2 \int e^x \, dx = 2e^x + C
$$

Applying the limits from 0 to 1:

$$
A = \left[2e^x \right]_0^1 = 2e^1 - 2e^0 = 2e - 2
$$

Since $e \approx 2.718$, the numerical value of the area is approximately:

$$
A \approx 2 \times 2.718 - 2 = 5.436 - 2 = 3.436
$$

Thus, the area of the region R is $2e - 2$.

Interpreting the Result

Meaning of the Area

The computed area, $A = 2e - 2$, quantifies the size of the region enclosed by the exponential curve, the x-axis, and the vertical lines at $x=0$ and $x=1$. This result is a fundamental example in integral calculus, demonstrating how to find the area under an exponential curve.

Applications of the Calculated Area

- Probability: If the exponential function models a probability density function, the area could represent the probability of an event.
- Physics: The area might correspond to physical quantities like work done or displacement under an exponential velocity curve.
- Economics: In models where exponential growth applies, such as investments or populations, the area can represent accumulated quantities over specific intervals.

Extensions and Related Concepts

Definite Integrals and Area Calculation

The process illustrated can be extended to other functions and intervals, emphasizing the importance of setting up correct limits and integrals.

Region R for Other Functions

- Different functions: You could replace $f(x) = 2e^x$ with other functions to analyze similar regions.
- Multiple intervals: Calculating areas between different bounds or over multiple intervals involves similar integral setups.

Applications in Advanced Calculus

- Volume calculations: Using the disk or washer method to find volumes generated by revolving the region R around an axis.
- Surface area: Computing the surface area of the surface obtained by revolving R around a line.

Conclusion

Defining the region R as the bounded area between the graph $f(x) = 2e^x$, the x-axis, and the vertical lines $x=0$ and $x=1$ provides a clear example of how to apply integral calculus to find areas under curves. The integral setup and calculation reveal that the area is $2e - 2$, approximately 3.436 square units. Such exercises are foundational in understanding how calculus quantifies geometric regions and applies to various scientific and engineering disciplines. Whether for theoretical insight or practical application, mastering the process of defining regions and calculating their areas is essential for advanced mathematical problem-solving.

Frequently Asked Questions

What is the region R in the given problem?

Region R is the area bounded by the graph of the function f(x) = 2e^x, the x-axis, the line x=0, and the line x=1.

How do you set up the integral to find the area of region R?

The area is found by integrating the function f(x)=2e^x from x=0 to x=1: Area = $\int_0^1$ 2e^x dx.

What is the value of the integral ∫ 2e^x dx?

The integral of 2e^x with respect to x is 2e^x + C.

How do you evaluate the definite integral $\int_0^1$ 2e^x dx?

Evaluate 2e^x from x=0 to x=1: 2e^1 - 2e^0 = 2e - 2.

What is the numerical value of the area of region R?

The area of region R is 2e - 2, which is approximately 4.436.

Why is the function f(x)=2e^x relevant in determining the region R?

Because f(x)=2e^x defines the upper boundary of the region R, which is bounded below by the x-axis.

What role do the lines x=0 and x=1 play in defining region R?

They serve as the left and right boundaries of the region, delimiting the interval over which we integrate.

Can the area of region R be interpreted geometrically?

Yes, it represents the total area under the curve f(x)=2e^x from x=0 to x=1, above the x-axis.

How does the exponential function influence the shape of the region R?

Since 2e^x is an increasing exponential function, the region's upper boundary rises rapidly from x=0 to x=1.

How would the calculation change if the limits were from x=0 to x=2?

You would evaluate the integral $\int_0^2$ 2e^x dx, which equals 2e^2 - 2e^0 = 2e^2 - 2, giving the area over the interval from 0 to 2.

Additional Resources

Define R As The Region That Is Bounded By The Graph Of The Function F(x)=2e^x, The X-axis, X=0, And X=1

Introduction

In the realm of calculus and mathematical analysis, the concept of regions bounded by curves and lines plays a pivotal role in understanding the geometric interpretation of functions, as well as in computing areas, volumes, and other integral-based measures. Among these, the region R defined by specific boundaries offers a concrete example of how functions translate into tangible geometrical shapes.

This article delves into the detailed exploration of the region R, bounded by the graph of the function

$f(x) = 2e^x$, the x-axis, and the vertical lines $x=0$ and $x=1$. We will analyze the properties of this region, interpret its significance, and explore the methods to compute its area, providing a comprehensive understanding suitable for students, educators, and enthusiasts of calculus alike.

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1. Understanding the Boundaries of Region R

1.1 The Graph of $f(x) = 2e^x$

The function $f(x) = 2e^x$ is an exponential function scaled vertically by a factor of 2. Its key characteristics include:

- Domain: $(-\infty, +\infty)$
- Range: $(0, +\infty)$
- Behavior: The function is increasing exponentially, with a base of Euler's number $e \approx 2.718$. As $x \to +\infty$, $f(x) \to +\infty$; as $x \to -\infty$, $f(x) \to 0$.

Graphically, the curve starts near the x-axis for large negative x, rises slowly initially, then accelerates exponentially as x increases.

1.2 The X-axis

The x-axis, defined by $y=0$, serves as a baseline for the region. Since $f(x)$ is positive for all real x, the region of interest lies above the x-axis.

1.3 The Vertical Lines $x=0$ and $x=1$

These lines serve as the vertical boundaries of the region. Specifically:

- At $x=0$: $f(0) = 2e^0 = 2$
- At $x=1$: $f(1) = 2e^1 = 2e \approx 5.436$

The segment of the graph between these x-values forms the upper boundary of the region.

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2. Defining the Region R

2.1 Formal Description

The region R can be described as:

$$
R = \left\{ (x, y) \mid x \in [0, 1], \quad 0 \leq y \leq 2e^x \right\}
$$

This means that for every x between 0 and 1, y ranges from the x-axis ($y=0$) up to the curve $y=2e^x$.

2.2 Geometrical Interpretation

Visually, R resembles a "curved strip" above the x-axis, confined between the vertical lines $x=0$ and $x=1$. The area of this region provides insights into integral calculus, as it can be computed via definite integrals.

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3. Calculating the Area of Region R

3.1 Motivation

The primary application of defining such a region is to compute its area. In calculus, the area A of R bounded by the given curves and lines is:

$$
A = \int_{x=0}^{x=1} \text{(top boundary)} - \text{(bottom boundary)} \, dx
$$

Since the bottom boundary is the x-axis ($y=0$) and the top boundary is $y=2e^x$, the integral simplifies to:

$$
A = \int_0^1 2e^x \, dx
$$

3.2 Computing the Integral

The integral:

$$
A = 2 \int_0^1 e^x \, dx
$$

is straightforward to evaluate:

1. Recall that $\int e^x \, dx = e^x + C$.
2. Applying the bounds:

$$
A = 2 [e^x]_0^1 = 2 (e^1 - e^0) = 2 (e - 1)
$$

3. Substituting $e \approx 2.718$:

$$
A \approx 2 (2.718 - 1) = 2 \times 1.718 = 3.436
$$

Thus, the area of the region R is exactly:

$$
\boxed{
A = 2 (e - 1)
}
$$

which numerically approximates to approximately 3.436 square units.

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4. Significance and Applications

4.1 Geometric Insights

Understanding the area of R offers a concrete interpretation of exponential functions in geometric terms. It demonstrates how exponential growth, when confined within specific bounds, can be quantified precisely using integrals.

4.2 Real-World Applications

The region R and its area are not merely abstract constructs; they mirror many real-world phenomena:

- Population Dynamics: The exponential growth of populations over time, with specific initial conditions, can be modeled similarly.
- Radioactive Decay and Compounding: The exponential behavior encapsulates decay and growth processes.
- Economics and Finance: Calculating accumulated interest or investment growth over intervals.

4.3 Teaching and Learning

This example serves as an excellent teaching tool for illustrating the connection between algebraic functions, their graphs, and the calculation of areas under curves—a foundational concept in integral calculus.

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5. Extending the Analysis

5.1 Area Under the Curve Beyond $x=1$

While the focus here is on $x \in [0, 1]$, mathematicians often explore similar regions over different intervals, leading to more complex integrations or applications.

5.2 Volume Generation via Revolution

If the region R were revolved around the x-axis, it would generate a three-dimensional volume, opening avenues for further calculus explorations such as the disk or shell methods.

5.3 Variations with Different Functions

Replacing $f(x) = 2e^x$ with other functions—polynomials, trigonometric, or logarithmic—would alter the shape and area, showcasing the versatility and depth of integral calculus.

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6. Visual Representation

While textual descriptions provide clarity, graphical visualization enhances understanding:

- The graph of $y=2e^x$ over $[0, 1]$ can be plotted to show the exponential increase.
- The bounded region R appears as a curved strip, starting at $(0, 2)$ and ending at $(1, 2e)$.
- Shade the area under $y=2e^x$ between $x=0$ and $x=1$ to visually represent the computed area.

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7. Conclusion

The region R, bounded by the graph of $f(x) = 2e^x$, the x-axis, and the vertical lines $x=0$ and $x=1$, encapsulates fundamental principles of calculus. Its area, meticulously calculated as $2(e - 1)$, exemplifies how exponential functions translate into measurable geometric quantities. This exploration not only reinforces the core concepts of integration but also highlights the interconnectedness of algebra, geometry, and analysis.

Understanding such regions enhances our grasp of mathematical modeling, enabling us to interpret complex systems through the lens of geometric intuition and integral calculus. Whether in academic contexts or real-world applications, the ability to define, analyze, and compute areas of bounded regions remains a cornerstone of mathematical literacy and analytical prowess.

References and Further Reading

- Stewart, J. (2015). Calculus: Early Transcendentals. Cengage Learning.
- Thomas, G. B., & Finney, R. L. (2000). Calculus and Analytic Geometry. Pearson.
- Online resources such as Khan Academy and Paul's Online Math Notes offer interactive tutorials on integration and area calculations.

End of article.

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