Determine Values Of The Variables That Will Make The Following Equation True, If Possible. If Not, State

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When working with algebraic equations, a fundamental skill is to find the values of variables that satisfy the equation or to determine if such values exist at all. This process involves solving for variables within the context of the equation, which can range from simple linear equations to complex polynomial or rational expressions. Sometimes, solutions are straightforward; other times, the equation might have no solutions, or infinitely many solutions. This article aims to thoroughly explore how to determine the values of variables that make an equation true, providing detailed explanations, strategies, and examples to enhance your understanding.

Understanding the Basics of Solving Equations

Before diving into specific types of equations, it's essential to understand the foundational principles of solving equations.

What Is an Equation?

An equation is a mathematical statement that asserts the equality of two expressions, often involving variables. For example:
- Linear equation: $2x + 3 = 7$
- Quadratic equation: $x^2 - 4x + 4 = 0$
- Rational equation: $(x + 1)/(x - 2) = 3$

The goal is to find all possible values of the variables that satisfy the equation—meaning, when substituted back into the equation, the equation holds true.

Solution Strategies

Solutions typically involve:
- Isolating the variable
- Simplifying the equation
- Factoring
- Applying algebraic formulas
- Checking for extraneous solutions, especially in rational or radical equations

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Types of Equations and Methods to Find Variable Values

Different types of equations require different solving methods. Let’s explore the most common types.

1. Linear Equations

Linear equations involve variables to the first power and are generally straightforward to solve.

Form: $ax + b = 0$

Solution:
- Isolate x: $x = -b/a$ (provided $a \neq 0$)
- If $a = 0$, then the equation is either always true (if $b=0$) or has no solution (if $b\neq0$)

Example:
Solve $3x - 6 = 0$

Solution:
$x = 6/3 = 2$

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2. Quadratic Equations

Quadratic equations involve the variable squared, often taking the form $ax^2 + bx + c = 0$.

Methods to Solve:
- Factoring
- Completing the square
- Quadratic formula

Quadratic Formula:
$x = [-b \pm \sqrt{(b^2 - 4ac)}] / (2a)$

Discriminant (D):
$D = b^2 - 4ac$
- If $D > 0$: two real solutions
- If $D = 0$: one real solution (a repeated root)
- If $D < 0$: no real solutions (complex solutions)

Example:
Solve $x^2 - 5x + 6 = 0$

Solution:
Discriminant $D = (-5)^2 - 416 = 25 - 24 = 1$

$x = [5 \pm \sqrt{1}] / 2$
$x = (5 \pm 1) / 2$
Solutions: $x = 3$ or $x = 2$

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3. Rational Equations

Involving fractions, these require careful handling to avoid undefined expressions.

Key Steps:
- Find common denominators
- Cross-multiply when appropriate
- Check for extraneous solutions (values that make denominators zero)

Example:
Solve $(x + 2)/(x - 3) = 4$

Solution:
Cross-multiplied: $x + 2 = 4(x - 3)$
$x + 2 = 4x - 12$
Bring all to one side: $0 = 3x - 14$
$x = 14/3$

Check:
Denominator $x - 3 \neq 0 \Rightarrow x \neq 3$, which is satisfied.

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4. Radical Equations

Equations involving roots require isolating the radical and squaring both sides.

Key Considerations:
- Always check for extraneous solutions after squaring
- Isolate radical before squaring

Example:
Solve $\sqrt{(x + 5)} = x - 1$

Solution:
Isolate radical: $\sqrt{(x + 5)} = x - 1$

Square both sides:
$x + 5 = (x - 1)^2$
$x + 5 = x^2 - 2x + 1$

Bring all to one side:
$x^2 - 3x - 4 = 0$

Factor:
$(x - 4)(x + 1) = 0$

Solutions:
$x = 4$ or $x = -1$

Check solutions:
- For $x=4$:
$\sqrt{(4+5)} = 4 - 1 \Rightarrow \sqrt{9}=3 \Rightarrow 3=3$ (valid)
- For $x=-1$:
$\sqrt{(-1+5)}= -1 -1 \Rightarrow \sqrt{4}= -2 \Rightarrow 2= -2$ (invalid)

Conclusion:
The only solution is $x=4$.

Determining If Solutions Exist and When Equations Are Impossible to Solve

In some cases, an equation has no solutions or infinitely many solutions. Recognizing these situations is crucial.

No Solution Scenarios

- Contradictory equations, e.g., $0x + 3 = 0x + 5$, which simplifies to $3=5$, a contradiction.
- Equations that involve square roots or other radicals leading to a negative value under the radical.
- Rational equations where the solution makes the denominator zero.

Example:
Solve $x + 3 = x + 5$

Analysis:
Subtract x from both sides:
$3 = 5$

Contradiction—no solution.

Infinitely Many Solutions

- When both sides are identical after simplification, such as $2x + 4 = 2x + 4$, every value of x satisfies the equation.

Example:
Solve $3x - 2 = 3x - 2$

Solution:
Subtract 3x from both sides:
-2 = -2

Always true; infinitely many solutions.

Step-by-Step Approach to Solving Equations and Finding Variable Values

To systematically determine the values of variables that satisfy an equation:

1. **Simplify the equation:** Expand, combine like terms, and reduce to simplest form.
2. **Isolate the variable:** Use inverse operations to get the variable alone on one side.
3. **Factor when necessary:** For polynomial equations, factor to find roots.
4. **Apply relevant formulas:** Use quadratic formula or other methods for specific types.
5. **Check solutions:** Substitute solutions back into the original equation to verify legitimacy, especially for radical or rational equations.
6. **Identify extraneous solutions:** Discard any solutions that make denominators zero or produce invalid radicals.

Examples of Solving Equations and Determining Variable Values

Example 1: Linear Equation

Solve 5x + 7 = 2x + 16

Solution:
Subtract 2x from both sides:
3x + 7 = 16
Subtract 7:
3x = 9
Divide by 3:

$x = 3$

Check:
Left side: $53 + 7 = 15 + 7 = 22$
Right side: $23 + 16 = 6 + 16 = 22$
Equal; solution verified.

Example 2: Quadratic Equation

Solve $2x^2 - 8x + 6 = 0$

Solution:
Discriminant:
$D = (-8)^2 - 426 = 64 - 48 = 16$
$x = [8 \pm \sqrt{16}] / (22) = (8 \pm 4)/4$

Solutions:
$x = (8 + 4)/4 = 12/4=3$
$x = (8 - 4)/4= 4/4=1$

Verify solutions are valid in original equation:
- For $x=3$:
$2(3)^2 - 8(3) + 6 = 18 - 24 + 6=0$
- For $x=1$:
$2(1)^2 - 8(1) + 6= 2 - 8 + 6=0$

Both solutions are valid.

Example 3: Rational Equation

Solve $(x - 1)/(x + 2) = 3$

Solution:
Cross-multiplied:
$x - 1 = 3(x + 2)$
$x - 1 = 3x + 6$
Bring all to one side:
$x - 3x = 6 + 1$
$-2x=7$
$x=-7/2$

Check for extraneous solutions:
Denominator $x+2 \neq 0 \Rightarrow x \neq -$

Frequently Asked Questions

Given the equation $2x + 3 = 7$, what is the value of x?

$x = 2$

For the equation $5y - 10 = 0$, what value of y makes the equation true?

$y = 2$

Determine the value of z that satisfies the equation $4z/2 = 8$.

$z = 4$

Solve for a in the equation $3a + 4 = 2a + 7$.

$a = 3$

Find the value of x in the equation $x^2 - 4 = 0$.

$x = 2$ or $x = -2$

Is there a value of t that satisfies the equation $0t = 5$? If yes, what is it?

No, there is no such t because 0 times any number is 0, not 5.

Determine if the equation $y/0 = 3$ has a solution, and if so, what is it?

No, division by zero is undefined, so no solution exists.

Given the equation $3(x - 2) = 3x - 6$, is any value of x possible? If yes, what are they?

Yes, all real numbers x satisfy the equation because both sides simplify to the same expression.

Additional Resources

Determine Values Of The Variables That Will Make The Following Equation True, If Possible. If Not, State

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Introduction

Mathematics often presents us with equations that challenge our understanding of variables and their relationships. The core task in such problems is to determine whether specific values of variables can satisfy a given equation, and if so, to identify those values explicitly. When approaching these problems, mathematicians and students alike must analyze the structure of the equations, assess possible constraints, and explore solutions systematically. This article delves into the methodology of determining variable values that satisfy equations, discusses common scenarios, and offers a comprehensive framework for solving such problems.

Understanding the Nature of Equations

What Is an Equation?

An equation is a mathematical statement asserting the equality of two expressions, often involving variables. Its general form can be represented as:

$$
f(x_1, x_2, \ldots, x_n) = g(x_1, x_2, \ldots, x_n)
$$

where f and g are expressions involving the variables $x_1, x_2, \ldots, x_n$.

The goal is to find all possible values or sets of values of these variables that make the equation true. Depending on the form and complexity of the equation, solutions may range from unique to infinitely many, or none at all.

Types of Equations and Their Characteristics

1. Linear Equations: Equations where variables are raised to the first power, such as $ax + by = c$. These are generally straightforward to solve.

2. Quadratic and Higher-Degree Equations: Involving variables raised to powers greater than one, requiring methods like factoring or quadratic formulas.

3. Algebraic Equations: Combinations of polynomials, rational expressions, radicals, etc.

4. Transcendental Equations: Contain transcendental functions such as exponential, logarithmic, trigonometric, or hyperbolic functions.

Understanding the type of equation is crucial because it influences the solution approach.

Methodologies for Determining Variable Values

Step 1: Simplify the Equation

Before attempting to find solutions, simplify the equation as much as possible:

- Combine like terms.
- Clear denominators (if rational expressions are involved).
- Rationalize radicals or complex fractions.
- Use algebraic identities to factor or expand expressions.

Simplification often reveals the structure more clearly and can reduce the problem to a more manageable form.

Step 2: Analyze the Domain and Constraints

Variables may have inherent restrictions based on the equation:

- Domain considerations: For example, square roots require the radicand to be non-negative; denominators cannot be zero.
- Physical constraints: If the variables represent real-world quantities (length, time, probability), they often need to be within specific ranges (e.g., non-negative, less than or equal to 1).

Explicitly stating these constraints helps avoid extraneous solutions and focuses the search.

Step 3: Isolate Variables and Solve Step-by-Step

Depending on the equation's form:

- Linear equations: Use algebraic manipulations to isolate variables directly.
- Quadratic equations: Use factoring, completing the square, or the quadratic formula.
- Systems of equations: Use substitution, elimination, or matrix methods.
- Higher-degree or complex equations: Factorization, substitution, or numerical methods may be necessary.

Step 4: Check for Extraneous or Invalid Solutions

Solutions derived algebraically must be verified within the original equation, especially when dealing with radicals or denominators that could restrict the domain.

Step 5: Interpret the Solutions in Context

Once potential solutions are identified, interpret or analyze them to determine their validity and relevance based on the original problem statement.

Cases When No Solutions Are Possible

There are scenarios where equations have no solutions:

1. Contradictory equations: Simplify to a false statement like \(0=5\).

2. Domain restrictions: The algebraic manipulations suggest solutions outside the

permissible domain.

3. Incompatibility of conditions: Multiple equations or inequalities that cannot be satisfied simultaneously.

Example:

$$
x^2 + 4x + 5 = 0
$$

Discriminant $D = 4^2 - 4 \times 1 \times 5 = 16 - 20 = -4 < 0$. Since the discriminant is negative, there are no real solutions.

Analytical Techniques and Tools

Graphical Analysis

Graphing the functions involved can visually reveal intersections (solutions). For example, solving $f(x) = g(x)$ involves plotting both functions and identifying points of intersection.

Numerical Methods

When algebraic solutions are complex or infeasible, numerical algorithms such as:

- Newton-Raphson method
- Bisection method
- Secant method

are employed to approximate solutions within desired accuracy.

Software and Computational Tools

Modern tools like WolframAlpha, MATLAB, GeoGebra, and Python libraries (SymPy, NumPy) facilitate solving equations symbolically or numerically, especially for complex cases.

Examples and Case Studies

Example 1: Linear Equation

Solve for x:

$$
3x - 7 = 2x + 5
$$

Solution:

Subtract $2x$ from both sides:

$$
3x - 2x - 7 = 5
$$

which simplifies to:

$$
x - 7 = 5
$$

Add 7 to both sides:

$$
x = 12
$$

Verification:

Substituting $x=12$:

$$
3(12) - 7 = 36 - 7 = 29
$$
$$
2(12) + 5 = 24 + 5 = 29
$$

Equality holds; thus, the solution is $x=12$.

Example 2: Quadratic Equation

Solve for x:

$$
x^2 - 4x + 3 = 0
$$

Solution:

Factor:

$$
(x - 1)(x - 3) = 0
$$

Solutions:

$$
x=1 \quad \text{or} \quad x=3
$$

\]

Verification:

- For $x=1$:

$$
1 - 4 + 3 = 0 \quad \checkmark
$$

- For $x=3$:

$$
9 - 12 + 3 = 0 \quad \checkmark
$$

Both solutions are valid.

Example 3: No Real Solutions

Solve:

$$
x^2 + 4x + 8 = 0
$$

Discriminant:

$$
D = 4^2 - 4 \times 1 \times 8 = 16 - 32 = -16 < 0
$$

Conclusion: No real solutions exist because the quadratic does not intersect the x-axis.

Special Considerations in Variable Determination

Equations Involving Multiple Variables

When multiple variables are involved, solutions depend on the number of equations and their relationships:

- Single equation with multiple variables: Usually, infinitely many solutions forming a geometric object (line, plane).

- System of equations: Can be solved using substitution, elimination, or matrix methods, leading to specific solutions, parametric solutions, or no solutions.

Parameterized Solutions

Sometimes, equations yield solutions expressed in terms of a parameter:

$$
x = t, \quad y = 2t + 1
$$

where t is any real number, indicating infinitely many solutions.

Constraints and Real-World Contexts

In applied problems, solutions must make sense within the context, such as:

- Lengths or quantities being non-negative.
- Probabilities between 0 and 1.
- Time variables being positive.

Summary and Final Remarks

Determining the values of variables that satisfy an equation is a fundamental task in mathematics, involving a combination of algebraic manipulations, analytical reasoning, and sometimes numerical approximation. The process begins with understanding the structure and constraints of the equation, simplifying it, and then systematically solving or analyzing the solutions.

When solutions are found, they must be verified within the original context, considering domain restrictions and physical plausibility. If no solutions exist, it indicates the equation is inherently inconsistent or restricted by conditions that cannot be simultaneously satisfied.

In conclusion, solving for variable values in equations is both an art and a science, requiring careful analysis, logical reasoning, and sometimes creative approaches. Whether dealing with simple linear equations or complex transcendental relationships, the core principles remain consistent: simplify, analyze, solve, verify, and interpret.

References and Further Reading

- Stewart, J. (2012). Calculus: Early Transcendentals. Brooks Cole.
- Lay, D. C. (2012). Linear Algebra and Its Applications. Pearson.
- Anton, H., Bivens, I., & Davis, S. (2013). Calculus: Early Transcendentals. Wiley.
- Online resources: Khan Academy, Paul's Online Math Notes, WolframAlpha tutorials.

This comprehensive review aims to equip readers with a solid understanding of how to determine variable values satisfying equations, emphasizing the importance of methodical problem-solving and critical analysis.

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