E) Find All First And Second Order Partial Derivatives Of The Following Function: Y= 6x - 2zx+3x+z/2-x-9

E) Find All First And Second Order Partial Derivatives Of The Following Function: Y= 6x - 2zx+3x+z/2-x-9

Understanding how to compute partial derivatives is fundamental in multivariable calculus, especially when analyzing the behavior of functions involving multiple variables. In this article, we will thoroughly explore the process of finding all first and second order partial derivatives of the function:

$$ Y = 6x - 2zx + 3x + \frac{z}{2} - x - 9 $$

This involves simplifying the function first, then systematically calculating the derivatives with respect to each variable, and finally determining the second order derivatives to analyze the curvature and other properties of the function.

Simplifying the Function Y

Before differentiating, it’s essential to simplify the given function to its most manageable form.

Step 1: Combine Like Terms

The original function is:

$$ Y = 6x - 2zx + 3x + \frac{z}{2} - x - 9 $$

Group similar terms:

- Terms with x: $6x + 3x - x$
- Terms with z: $-2zx + \frac{z}{2}$
- Constants: -9

Calculating:

- $6x + 3x - x = (6 + 3 - 1) x = 8x$
- $-2zx + \frac{z}{2}$ remains as is
- Constants: -9

So, the simplified form is:

$$ Y = 8x - 2zx + \frac{z}{2} - 9 $$

Step 2: Rewriting for Clarity

Express the function as:

$$Y(x, z) = 8x - 2xz + \frac{z}{2} - 9$$

This form makes differentiating with respect to x and z straightforward.

Calculating First Order Partial Derivatives

The first order partial derivatives measure the rate of change of Y with respect to each variable independently.

Partial Derivative with respect to x, $\frac{\partial Y}{\partial x}$

Given:

$$Y(x, z) = 8x - 2xz + \frac{z}{2} - 9$$

Treat z as a constant.

Differentiate term by term:

- $\frac{\partial}{\partial x} (8x) = 8$
- $\frac{\partial}{\partial x} (-2xz) = -2z$ (since z is constant)
- $\frac{\partial}{\partial x} (\frac{z}{2}) = 0$ (constant with respect to x)
- $\frac{\partial}{\partial x} (-9) = 0$

Therefore:

$$\boxed{ \frac{\partial Y}{\partial x} = 8 - 2z }$$

Partial Derivative with respect to z, $\frac{\partial Y}{\partial z}$

Treat x as a constant:

- $\frac{\partial}{\partial z} (8x) = 0$
- $\frac{\partial}{\partial z} (-2xz) = -2x$ (since x is constant)
- $\frac{\partial}{\partial z} (\frac{z}{2}) = \frac{1}{2}$
- $\frac{\partial}{\partial z} (-9) = 0$

Thus:

$$\boxed{ \frac{\partial Y}{\partial z} = -2x + \frac{1}{2} }$$

Calculating Second Order Partial Derivatives

Second order derivatives provide information about the curvature and concavity of the function and are essential in optimization and analyzing the function’s behavior.

Second Partial Derivative with respect to x, $\frac{\partial^2 Y}{\partial x^2}$

Differentiate $\frac{\partial Y}{\partial x} = 8 - 2z$ with respect to x:

- 8 is constant, derivative is 0
- $-2z$ is treated as constant (since z is independent of x), derivative is 0

Therefore:

$$\boxed{ \frac{\partial^2 Y}{\partial x^2} = 0 }$$

Second Partial Derivative with respect to z, $\frac{\partial^2 Y}{\partial z^2}$

Differentiate $\frac{\partial Y}{\partial z} = -2x + \frac{1}{2}$ with respect to z:

- $-2x$ is constant with respect to z, derivative is 0
- $\frac{1}{2}$ is constant, derivative is 0

Thus:

$$\boxed{ \frac{\partial^2 Y}{\partial z^2} = 0 }$$

Mixed Second Order Partial Derivatives

These derivatives involve differentiating with respect to both variables.

Second mixed partial derivative $\frac{\partial^2 Y}{\partial x \partial z}$

Differentiate $\frac{\partial Y}{\partial x} = 8 - 2z$ with respect to z:

- Derivative of 8 with respect to z: 0
- Derivative of $-2z$ with respect to z: -2

Hence:

$$\boxed{ \frac{\partial^2 Y}{\partial x \partial z} = -2 }$$

Second mixed partial derivative $\frac{\partial^2 Y}{\partial z \partial x}$

Differentiate $\frac{\partial Y}{\partial z} = -2x + \frac{1}{2}$ with respect to x:

- Derivative of $-2x$ with respect to x: -2
- Derivative of $\frac{1}{2}$ with respect to x: 0

Thus:

$$\boxed{ \frac{\partial^2 Y}{\partial z \partial x} = -2 }$$

Note: Mixed partial derivatives are equal if the function is continuous and smooth, which is the case here.

Summary of Derivatives

Derivative	Expression	Notes
First $\frac{\partial Y}{\partial x}$	$8 - 2z$	Derivative w.r.t x
First $\frac{\partial Y}{\partial z}$	$-2x + \frac{1}{2}$	Derivative w.r.t z
Second $\frac{\partial^2 Y}{\partial x^2}$	0	Curvature with respect to x
Second $\frac{\partial^2 Y}{\partial z^2}$	0	Curvature with respect to z
Mixed $\frac{\partial^2 Y}{\partial x \partial z}$	-2	Cross partial derivative
Mixed $\frac{\partial^2 Y}{\partial z \partial x}$	-2	Cross partial derivative

Applications and Importance of These Derivatives

Understanding the first and second order partial derivatives of multivariable functions like the one above is crucial in various fields such as physics, engineering, and economics. These derivatives help in:

- **Optimization:** Finding maxima, minima, and saddle points of functions.
- **Analyzing Curvature:** The second derivatives indicate concavity or convexity.
- **Sensitivity Analysis:** How the output Y reacts to changes in variables x and z.

- **Modeling and Simulation:** Precise modeling of complex systems involving multiple variables.

Conclusion

In this comprehensive guide, we simplified the function $Y = 6x - 2zx + 3x + \frac{z}{2} - x - 9$ to its core components, then systematically calculated all first and second order partial derivatives with respect to x and z. These derivatives provide essential insights into the behavior and properties of the function, serving as foundational tools in calculus for analyzing multivariable functions.

By mastering the process of deriving these derivatives, students and professionals can better interpret complex functions and apply this knowledge to real-world problems across various scientific and engineering disciplines.

Frequently Asked Questions

What is the first step to find the first-order partial derivatives of the function Y = 6x - 2zx + 3x + z/2 - x - 9?

The first step is to simplify the function if possible and identify the variables involved (x and z), then differentiate Y with respect to each variable separately, treating the other as constant.

How do you compute the first partial derivative of Y with respect to x?

Differentiate each term of Y with respect to x, treating z as a constant: d/dx (6x) = 6, d/dx (-2zx) = -2z, d/dx (3x) = 3, d/dx (z/2) = 0, d/dx (-x) = -1, and d/dx (-9) = 0. Summing these gives the partial derivative with respect to x.

What is the formula for the partial derivative of Y with respect to z?

Differentiate each term with respect to z, treating x as constant: d/dz (6x) = 0, d/dz (-2zx) = -2x, d/dz (3x) = 0, d/dz (z/2) = 1/2, d/dz (-x) = 0, and d/dz (-9) = 0. Summing these yields the partial derivative with respect to z.

How do you find the second order partial derivatives of Y?

After computing the first partial derivatives, differentiate those derivatives again with respect to the same or different variables to find second order derivatives: Y_{xx}, Y_{zz}, and Y_{xz} (or Y_{zx}).

What is the second partial derivative of Y with respect to x (Y_xx)?

Since the first partial derivative with respect to x is 6 - 2z + 3 - 1 = 8 - 2z, differentiating again with respect to x yields 0 because z is treated as a constant. Therefore, Y_xx = 0.

What is the second partial derivative of Y with respect to z (Y_zz)?

The first partial derivative with respect to z is -2x + 1/2. Differentiating again with respect to z (treating x as constant) gives 0, so Y_zz = 0.

How do you compute the mixed second order partial derivatives (Y_xz or Y_zx)?

Differentiate the first partial derivative with respect to one variable again with respect to the other variable. For example, differentiate Y_x = 6 - 2z + 3 - 1 = 8 - 2z with respect to z to find Y_xz, which yields -2. Similarly, differentiate Y_z = -2x + 1/2 with respect to x to find Y_zx, which yields -2.

What are the final expressions for all second order partial derivatives of the function Y?

The second order partial derivatives are: Y_xx = 0, Y_zz = 0, Y_xz = Y_zx = -2.

Additional Resources

Find All First And Second Order Partial Derivatives Of The Following Function: Y= 6x - 2zx+3x+z/2-x-9

Introduction

Mathematics, especially multivariable calculus, plays a crucial role in understanding complex systems across scientific, engineering, and economic disciplines. Central to this understanding is the concept of partial derivatives—tools that measure how a function changes as one variable changes, holding others constant. This article delves deeply into the process of finding all first and second order partial derivatives of a specific function:

$$
Y = 6x - 2zx + 3x + \frac{z}{2} - x - 9
$$

The goal is to systematically analyze the function, derive its first derivatives with respect to each variable, and then compute the second derivatives, including mixed partial derivatives. Such an analysis not only enhances our grasp of the function's behavior but also illustrates fundamental techniques in multivariable calculus, which are invaluable in diverse fields ranging from physics to economics.

Clarification and Simplification of the Given Function

Before embarking on differentiation, it is essential to clarify and simplify the given function. The initial expression appears somewhat cluttered, so let's rewrite it clearly:

$$
Y = 6x - 2zx + 3x + \frac{z}{2} - x - 9
$$

Now, combine like terms:

- Combine the x-terms: $6x + 3x - x = (6 + 3 - 1) x = 8x$
- The term $-2zx$ remains as is.
- The term $\frac{z}{2}$ remains as is.
- Constant term: -9.

Thus, the simplified function is:

$$
Y(x, z) = 8x - 2zx + \frac{z}{2} - 9
$$

Mathematical Preliminaries

Before proceeding, recall the definitions:

- Partial derivative with respect to x: $\frac{\partial Y}{\partial x}$ is obtained by differentiating Y treating z as a constant.
- Partial derivative with respect to z: $\frac{\partial Y}{\partial z}$ is obtained by differentiating Y treating x as a constant.
- Second order derivatives: derivatives of the first derivatives, such as $\frac{\partial^2 Y}{\partial x^2}$, $\frac{\partial^2 Y}{\partial z^2}$, and mixed derivatives $\frac{\partial^2 Y}{\partial x \partial z}$ and $\frac{\partial^2 Y}{\partial z \partial x}$.

The symmetry property of mixed derivatives (Clairaut's theorem) states that, under suitable conditions, $\frac{\partial^2 Y}{\partial x \partial z} = \frac{\partial^2 Y}{\partial z \partial x}$.

First Order Partial Derivatives

Let's compute the first derivatives systematically.

Partial derivative with respect to x

Given:

$$
Y(x, z) = 8x - 2zx + \frac{z}{2} - 9
$$

Treat z as a constant.

$$
\frac{\partial Y}{\partial x} = \frac{\partial}{\partial x} \left(8x - 2zx + \frac{z}{2} - 9 \right)
$$

Differentiating term-by-term:

- $\frac{\partial}{\partial x} (8x) = 8$
- $\frac{\partial}{\partial x} (-2zx) = -2z$ (since z is treated as constant)
- $\frac{\partial}{\partial x} \left(\frac{z}{2} \right) = 0$ (constant w.r.t x)
- $\frac{\partial}{\partial x} (-9) = 0$

Result:

$$
\boxed{
\frac{\partial Y}{\partial x} = 8 - 2z
}
$$

Partial derivative with respect to z

Now, treat x as a constant:

$$
Y(x, z) = 8x - 2zx + \frac{z}{2} - 9
$$

Differentiate term-by-term w.r.t z:

- $\frac{\partial}{\partial z} (8x) = 0$ (constant w.r.t z)
- $\frac{\partial}{\partial z} (-2zx) = -2x$ (derivative of linear term in z)
- $\frac{\partial}{\partial z} \left(\frac{z}{2} \right) = \frac{1}{2}$
- $\frac{\partial}{\partial z} (-9) = 0$

Result:

$$
\boxed{
\frac{\partial Y}{\partial z} = -2x + \frac{1}{2}
}
$$

Second Order Partial Derivatives

Next, compute the second derivatives by differentiating the first derivatives.

Second derivative with respect to x

$$
\frac{\partial^2 Y}{\partial x^2} = \frac{\partial}{\partial x} \left(\frac{\partial Y}{\partial x} \right) = \frac{\partial}{\partial x} (8 - 2z)
$$

Since $8 - 2z$ does not depend on x, its derivative w.r.t x is zero:

$$
\boxed{
\frac{\partial^2 Y}{\partial x^2} = 0
}
$$

Second derivative with respect to z

$$
\frac{\partial^2 Y}{\partial z^2} = \frac{\partial}{\partial z} \left(-2x + \frac{1}{2} \right)
$$

Here, $-2x$ is constant w.r.t z, so its derivative is zero; $\frac{1}{2}$ is constant:

$$
\boxed{
\frac{\partial^2 Y}{\partial z^2} = 0
}
$$

Mixed partial derivatives

$\frac{\partial^2 Y}{\partial x \partial z}$

Differentiate $\frac{\partial Y}{\partial x} = 8 - 2z$ w.r.t z:

$$
\frac{\partial^2 Y}{\partial x \partial z} = \frac{\partial}{\partial z} (8 - 2z) = -2
$$

$\frac{\partial^2 Y}{\partial z \partial x}$

Differentiate $\frac{\partial Y}{\partial z} = -2x + \frac{1}{2}$ w.r.t x:

$$
\frac{\partial^2 Y}{\partial z \partial x} = \frac{\partial}{\partial x} (-2x + \frac{1}{2}) = -2
$$

By Clairaut's theorem, these are equal, confirming the consistency of the derivatives.

Summary of All Derivatives

Derivative	Expression	Notes
$\frac{\partial Y}{\partial x}$	$8 - 2z$	First order w.r.t x
$\frac{\partial Y}{\partial z}$	$-2x + \frac{1}{2}$	First order w.r.t z
$\frac{\partial^2 Y}{\partial x^2}$	0	Second order, x
$\frac{\partial^2 Y}{\partial z^2}$	0	Second order, z
$\frac{\partial^2 Y}{\partial x \partial z}$	-2	Mixed partials
$\frac{\partial^2 Y}{\partial z \partial x}$	-2	Confirmed symmetry

---

Interpretation and Implications

The derivatives reveal that the function Y exhibits linear behavior in both variables, with the first derivatives depending linearly on the other variable. The second derivatives being zero indicate that the function is at most linear in each variable, with no curvature or quadratic terms—implying the surface represented by Y is a plane.

The constant mixed derivative -2 signifies a uniform rate of change in the slope in the x-z plane, a characteristic relevant in optimization and sensitivity analysis.

Applications and Significance

Understanding the derivatives of such functions is critical across numerous disciplines:

- Economics: Analyzing how a profit or cost function responds to changes in multiple variables.
- Physics: Studying potential fields and assessing the rate of change of forces.
- Engineering: Optimizing systems with multiple input parameters.

The methodology demonstrated here—simplifying complex expressions, systematically differentiating, and interpreting results—is foundational in advanced calculus and essential

E Find All First And Second Order Partial Derivatives Of The

Following Function Y 6x 2zx 3x Z 2 X 9

E Find All First And Second Order Partial Derivatives Of The Following Function Y 6x 2zx 3x Z 2 X 9

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