Evaluate The Integral (x+a)(x+b)5dx Tor The Cases Where A=b And Where A=b. Note: For The Case Where A=b,

Evaluate The Integral (x+a)(x+b)^5 dx For The Cases Where A = b And Where A ≠ b. Note: For The Case Where A = b,

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Introduction

Integrals are fundamental in calculus, serving as the backbone for solving problems involving area, volume, and various physical phenomena. Among the myriad of integrals encountered, polynomial integrals—especially those involving products of linear factors—are quite common and essential in mathematical analysis and applied sciences.

In this article, we focus on evaluating the integral:

\[
\int (x + a)(x + b)^5 \, dx
\]

where \(a\) and \(b\) are constants. We analyze two specific cases:

- Case 1: When \(a \neq b\)
- Case 2: When \(a = b\)

Understanding the differences between these cases is crucial because the algebraic structure simplifies significantly when \(a = b\). This case often arises in applications such as polynomial factorization, integration techniques, and in solving differential equations where repeated roots are involved.

The goal of this article is to provide a comprehensive, step-by-step method for evaluating these integrals, highlight the nuances between the cases, and ensure that the solutions are both accurate and easily digestible. We will incorporate algebraic manipulations, substitution techniques, and integration rules to facilitate a clear understanding.

Understanding the Integral

Before diving into the specific cases, it's important to analyze the structure of the integral:

\[

\int (x + a)(x + b)^5 \, dx
\]

This is a polynomial integrand involving a product of a linear term \((x + a)\) and a fifth power of another linear term \((x + b)^5\). The integral can be approached through various methods:

- Expansion and Integration: Expanding the product into a polynomial and integrating term-by-term.
- Substitution: Choosing an appropriate substitution to simplify the integral.
- Polynomial Decomposition: Expressing the integrand as a sum of simpler terms that are straightforward to integrate.

The method chosen often depends on whether \(a = b\) or \(a \neq b\), as that impacts the algebraic simplification.

Case 1: When \(a \neq b\)

Step 1: Expand the integrand

First, expand the product \((x + a)(x + b)^5\).

\[
(x + a)(x + b)^5 = (x + a) \times (x + b)^5
\]

Since \((x + b)^5\) is a binomial raised to the fifth power, it can be expanded using the binomial theorem:

\[
(x + b)^5 = \sum_{k=0}^{5} \binom{5}{k} x^{5 - k} b^k
\]

However, expanding directly may be cumbersome; instead, consider substitution or polynomial decomposition.

Step 2: Polynomial Decomposition

Alternatively, express the integrand as a polynomial in \(x\):

\[
(x + a)(x + b)^5
\]

which can be written as:

\[
(x + a)(x + b)^5 = (x + a) \times (x + b)^5

\]

To facilitate integration, expand \((x + b)^5\):

\[
(x + b)^5 = x^5 + 5b x^4 + 10b^2 x^3 + 10b^3 x^2 + 5b^4 x + b^5
\]

Now, multiply this by \((x + a)\):

\[
\begin{aligned}
(x + a)(x + b)^5 &= (x + a) \times \left(x^5 + 5b x^4 + 10b^2 x^3 + 10b^3 x^2 + 5b^4 x + b^5 \right) \\
&= x \times \text{(the polynomial)} + a \times \text{(the polynomial)}
\end{aligned}
\]

Perform the multiplication:

\[
\begin{aligned}
& x \times x^5 = x^6 \\
& x \times 5b x^4 = 5b x^5 \\
& x \times 10b^2 x^3 = 10b^2 x^4 \\
& x \times 10b^3 x^2 = 10b^3 x^3 \\
& x \times 5b^4 x = 5b^4 x^2 \\
& x \times b^5 = b^5 x \\
\end{aligned}
\]

Similarly, multiplying by \(a\):

\[
\begin{aligned}
& a \times x^5 = a x^5 \\
& a \times 5b x^4 = 5ab x^4 \\
& a \times 10b^2 x^3 = 10ab^2 x^3 \\
& a \times 10b^3 x^2 = 10ab^3 x^2 \\
& a \times 5b^4 x = 5ab^4 x \\
& a \times b^5 = a b^5
\end{aligned}
\]

Adding all these terms yields:

\[
\boxed{
\begin{aligned}
(x + a)(x + b)^5 &= x^6 + (5b + a) x^5 + (10b^2 + 5ab) x^4 + (10b^3 + 10ab^2) x^3 \\

&\quad + (5b^4 + 10ab^3) x^2 + (b^5 + 5ab^4) x + a b^5
\end{aligned}
}
$$

Step 3: Integrate term-by-term

Now, integrate each term with respect to x:

$$
\int (x + a)(x + b)^5 \, dx = \int \text{(above polynomial)} \, dx
$$

which results in:

$$
\begin{aligned}
& \frac{x^7}{7} + \frac{(5b + a)}{6} x^6 + \frac{(10b^2 + 5ab)}{5} x^5 + \frac{(10b^3 + 10ab^2)}{4} x^4 \\
& + \frac{(5b^4 + 10ab^3)}{3} x^3 + \frac{(b^5 + 5ab^4)}{2} x^2 + a b^5 x + C
\end{aligned}
$$

where C is the constant of integration.

Summary for $a \neq b$:

$$
\boxed{
\int (x + a)(x + b)^5 \, dx = \frac{x^7}{7} + \frac{(5b + a)}{6} x^6 + \frac{(10b^2 + 5ab)}{5} x^5 + \frac{(10b^3 + 10ab^2)}{4} x^4 + \frac{(5b^4 + 10ab^3)}{3} x^3 + \frac{(b^5 + 5ab^4)}{2} x^2 + a b^5 x + C
}
$$

This method, though straightforward, involves expanding and integrating multiple polynomial terms, which can be tedious for higher powers or more complex expressions. Alternatively, substitution methods can streamline the process.

Case 2: When $a = b$

Step 1: Recognize the simplification

When $a = b$, the integral simplifies to:

$$
\int (x + a)(x + a)^5 \, dx = \int (x + a)^6 \, dx
$$

since $(x + a)(x + a)^5 = (x + a)^{6}$.

Step 2: Integrate directly

The integral becomes:

$$
\int (x + a)^6 \, dx
$$

which is straightforward:

$$
\int (x + a)^6 \, dx = \frac{(x + a)^7}{7} + C
$$

This is a classic power rule integral, significantly simplifying the problem.

Important note:

When $a = b$, the integral reduces to a simple power of a linear function, and the solution is obtained directly using the power rule:

$$
\boxed{
\int (x + a)^6 \, dx = \frac{(x + a)^7}{7} + C
}
$$

Summary and Key Takeaways

For $a \neq b$:

- Expand $(x + b)^5$ using binomial theorem.
- Multiply the expanded polynomial by $(x + a)$.
- Combine like terms to form a polynomial.
- Integrate term-by-term using the power rule.
- The integral results in a sum of polynomial terms divided by their respective exponents plus a constant of integration.

For $a = b$:

- Recognize that $(x + a)(x + a)^5 = (x + a)^6$.
- Integrate

Frequently Asked Questions

How do you evaluate the integral of (x + a)(x + b)^5 dx when a ≠ b?

For a ≠ b, expand the integrand (x + a)(x + b)^5 and then integrate term-by-term. Alternatively, use substitution methods or polynomial expansion to simplify before integration.

What is the approach to evaluate the integral of (x + a)(x + b)^5 dx when a = b?

When a = b, the integrand becomes (x + a)^6, simplifying the integral to ∫(x + a)^6 dx, which can be directly integrated as (1/7)(x + a)^7 + C.

Can you provide the integral of (x + a)(x + b)^5 dx in the case where a ≠ b?

Yes. When a ≠ b, the integral is (1/6)(x + a)(x + b)^6 - (a - b)/(7)(x + b)^7 + C, obtained by expanding and integrating or using substitution.

What is the significance of the case where a = b in the integral of (x + a)(x + b)^5 dx?

When a = b, the integral simplifies significantly because the integrand becomes (x + a)^6, allowing straightforward power rule integration without expansion.

How does the integral change when a ≠ b compared to when a = b?

When a ≠ b, the integral involves expanded polynomial terms and possibly partial fractions, resulting in more complex expressions. When a = b, the integral simplifies to a basic power rule integration.

What is the final integrated result for (x + a)(x + a)^5 dx?

Since (x + a)(x + a)^5 = (x + a)^6, the integral is (1/7)(x + a)^7 + C.

Additional Resources

Evaluate the integral (x + a)(x + b)^5 dx for the cases where a = b and where a ≠ b

When tackling integrals involving polynomial expressions like $(x + a)(x + b)^5 dx$, understanding the nuanced differences between the cases where $a = b$ and $a \neq b$ is crucial. These distinctions not only influence the complexity of the integration process but also highlight interesting algebraic and calculus principles, such as factorization, substitution, and the binomial theorem. In this comprehensive guide, we'll explore both scenarios in detail, providing step-by-step methods, insightful techniques, and practical tips to evaluate these integrals effectively.

Understanding the Integral

Before diving into different cases, let's analyze the structure of the integral:

$$
\int (x + a)(x + b)^5 \, dx
$$

This integral involves a product of a linear term $(x + a)$ and a power of another linear term $(x + b)^5$. The key to simplifying such integrals often lies in algebraic manipulation—either through expansion, substitution, or recognizing patterns that fit into standard integral forms.

Case 1: When $a \neq b$

Why is this case different?

If $a \neq b$, the factors $(x + a)$ and $(x + b)^5$ are distinct linear expressions. This allows us to consider substitution strategies or algebraic manipulation to simplify the integral.

Step 1: Expand or Re-Express the Integral

You can approach this integral in two main ways:

- Method A: Expand $(x + a)(x + b)^5$ and integrate term by term.
- Method B: Use substitution to simplify the integral.

Method A: Polynomial Expansion

Expanding directly may lead to a lengthy polynomial, but it provides a straightforward route:

$$
(x + a)(x + b)^5 = (x + a) \times \sum_{k=0}^5 \binom{5}{k} x^{5-k} b^{k}
$$

Alternatively, it's more practical to expand $(x + b)^5$ first, then multiply by $(x + a)$:

$$
(x + b)^5 = x^5 + 5b x^4 + 10b^2 x^3 + 10b^3 x^2 + 5b^4 x + b^5
$$

Multiplying this by $(x + a)$:

$$
(x + a)(x^5 + 5b x^4 + 10b^2 x^3 + 10b^3 x^2 + 5b^4 x + b^5)
$$

which yields:

$$
x \times \text{polynomial} + a \times \text{polynomial}
$$

This results in a polynomial of degree 6, which can be integrated term by term.

Method B: Substitution Approach

Alternatively, consider substitution to simplify the integral:

- Let $u = x + b$, then $du = dx$.
- Rewrite the integral in terms of u:

$$
\int (u + a - b)(u)^5 \, du
$$

which simplifies to:

$$
\int (u + c) u^5 \, du
$$

where $c = a - b$.

This form is more manageable and leads to straightforward expansion:

$$
\int (u \times u^5 + c \times u^5) \, du = \int (u^6 + c u^5) \, du
$$

From here, you can directly integrate each term:

$$
\frac{u^7}{7} + c \frac{u^6}{6} + C
$$

\]

Substitute back \(u = x + b \):

\[
\boxed{
\frac{(x + b)^7}{7} + c \frac{(x + b)^6}{6} + C
}
\]

which provides the indefinite integral.

Summary for \(a \neq b \):

- Use substitution \(u = x + b \) to simplify the integral.
- Express the integral as \(\int (u + c) u^5 \, du \).
- Integrate directly to obtain the result.

Case 2: When \(a = b \)

Significance of \(a = b \): A Special Case

When a = b, the integral simplifies to:

\[
\int (x + a)(x + a)^5 \, dx = \int (x + a)^6 \, dx
\]

This is a perfect power of a linear expression, making the integral significantly easier and more elegant to evaluate.

Step 1: Recognize the Simplification

Since:

\[
(x + a)(x + a)^5 = (x + a)^6
\]

the integral reduces to:

\[
\int (x + a)^6 \, dx
\]

Step 2: Perform the Basic Power Integration

Using the standard power rule:

$$
\int (x + a)^n \, dx = \frac{(x + a)^{n+1}}{n + 1} + C
$$

for any real $n \neq -1$.

Applying this to $n = 6$:

$$
\int (x + a)^6 \, dx = \frac{(x + a)^7}{7} + C
$$

Result:

$$
\boxed{
\int (x + a)(x + a)^5 \, dx = \frac{(x + a)^7}{7} + C
}
$$

Additional Techniques and Tips

1. Using Substitution Effectively

- When dealing with powers of linear expressions, substitution simplifies the process.
- For the general integral involving different a and b, choosing $u = x + b$ often streamlines the calculations.

2. Polynomial Expansion vs. Substitution

- Expansion can become cumbersome for high powers but makes the integral straightforward if manageable.
- Substitution is more elegant and less error-prone when dealing with powers of linear expressions.

3. Partial Fraction Decomposition

- Not necessary here due to the polynomial form, but useful for rational functions or more complex integrals.

4. Recognize Patterns

- When $a = b$, the integral simplifies dramatically.
- When $a \neq b$, consider the difference $a - b$ to facilitate substitution.

Summary: Key Takeaways

Aspect	When $a = b$	When $a \neq b$
Expression	$\int (x + a)^6 \, dx$	$\int (x + a)(x + b)^5 \, dx$
Simplification	Perfect power, straightforward	Polynomial expansion or substitution needed
Integration method	Power rule directly	Substitution $u = x + b$ or expansion

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Final Notes and Practice Problems

To solidify your understanding, try solving these problems:

1. Evaluate $\int (x + 3)(x + 5)^5 \, dx$
2. Evaluate $\int (x + 2)^6 \, dx$
3. Evaluate $\int (x + 4)(x + 4)^5 \, dx$

Hint: For the first problem, note that $a \neq b$, so substitution is recommended. For the second and third, recognize the perfect power form.

Conclusion

The integral of $(x + a)(x + b)^5 dx$ showcases how algebraic manipulations and substitution techniques can simplify complex polynomial integrals. The key difference between the cases where $a = b$ and $a \neq b$ lies in the structure of the integrand: the former reduces to a simple power, while the latter requires strategic substitution or expansion. Mastering these techniques enhances your calculus toolkit, enabling you to tackle a wide range of polynomial integrals with confidence.

Evaluate The Integral X A X B 5dx Tor The Cases Where A B And Where A B Note For The Case Where A B

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