Find A Polynomial P(x) With Realcoefficients Having A Degree 4, Leadingcoefficient3,andzeros5iand3i.Find

Find A Polynomial P(x) With Realcoefficients Having A Degree 4, Leadingcoefficient3,andzeros5iand3i.Find

---

Introduction

Constructing a polynomial with specific properties is a fundamental problem in algebra and polynomial theory. When given certain zeros, degree, and leading coefficient, the task involves determining the polynomial's explicit form. In this article, we will explore how to find a degree 4 polynomial $P(x)$ with real coefficients, a leading coefficient of 3, and zeros at $5i$ and $3i$. We will systematically analyze the problem, consider the implications of the zeros, and derive the polynomial step by step.

Understanding the Problem

The Given Conditions

- Degree of the polynomial: 4
- Leading coefficient: 3
- Zeros: $5i$ and $3i$
- Coefficients: Real numbers

Key Concepts

To solve this problem, it is essential to understand the implications of the zeros and how they relate to the factors of the polynomial:

- Zeros of a polynomial: Values of x where $P(x) = 0$.
- Complex zeros and conjugate pairs: Since the polynomial has real coefficients, complex zeros occur in conjugate pairs. This means if $a + bi$ is a zero, then $a - bi$ must also be a zero.
- Degree and zeros: The degree of the polynomial corresponds to the total number of zeros (counting multiplicities). Since the degree is 4, we expect four zeros.

Zeros and Their Conjugates

Given zeros:

- $5i$ (which is purely imaginary)
- $3i$ (also purely imaginary)

Their conjugates are:

- $-5i$
- $-3i$

Thus, the zeros of the polynomial are:

$$\boxed{
\{ 5i, -5i, 3i, -3i \}
}$$

Step-by-Step Solution

Step 1: Write the Factors Corresponding to the Zeros

Each zero corresponds to a factor of the polynomial:

- Zero at $5i$: $(x - 5i)$
- Zero at $-5i$: $(x + 5i)$
- Zero at $3i$: $(x - 3i)$
- Zero at $-3i$: $(x + 3i)$

Step 2: Form the Polynomial as a Product of Factors

The polynomial $P(x)$ can be expressed as:

$$
P(x) = k \times (x - 5i)(x + 5i)(x - 3i)(x + 3i)
$$

where k is the leading coefficient, which is given as 3.

Therefore,

$$
P(x) = 3 \times (x - 5i)(x + 5i)(x - 3i)(x + 3i)
$$

Step 3: Simplify the Factors Using Difference of Squares

Recall that:

$$
(a - bi)(a + bi) = a^2 + b^2
$$

Applying this to each pair:

- $(x - 5i)(x + 5i) = x^2 + (5)^2 = x^2 + 25$
- $(x - 3i)(x + 3i) = x^2 + (3)^2 = x^2 + 9$

Thus,

$$
P(x) = 3 \times (x^2 + 25)(x^2 + 9)
$$

Step 4: Expand the Polynomial

First, multiply $(x^2 + 25)$ and $(x^2 + 9)$:

$$
(x^2 + 25)(x^2 + 9) = x^2 \times x^2 + x^2 \times 9 + 25 \times x^2 + 25 \times 9
$$

Simplify:

$$
= x^4 + 9x^2 + 25x^2 + 225
$$

$$
= x^4 + (9x^2 + 25x^2) + 225
$$

$$
= x^4 + 34x^2 + 225
$$

Now, multiply by the leading coefficient 3:

$$
P(x) = 3 \times [x^4 + 34x^2 + 225] = 3x^4 + 102x^2 + 675
$$

Final Polynomial Expression

The polynomial $P(x)$ with the specified properties is:

$$
\boxed{
\boxed{P(x) = 3x^4 + 102x^2 + 675}
}
$$

This polynomial has:

- Degree 4
- Leading coefficient 3
- Zeros at $5i, -5i, 3i, -3i$
- Real coefficients

Additional Considerations

Confirming the Zeros

To verify, substitute the zeros into $P(x)$:

- At $x = 5i$:

$$
P(5i) = 3(5i)^4 + 102(5i)^2 + 675
$$

Calculate each term:

- $(5i)^2 = 25i^2 = 25 \times (-1) = -25$

- $(5i)^4 = [(5i)^2]^2 = (-25)^2 = 625$

Substitute:

$$
P(5i) = 3 \times 625 + 102 \times (-25) + 675 = 1875 - 2550 + 675 = 0
$$

Similarly, for $x = 3i$:

- $(3i)^2 = -9$

- $(3i)^4 = (-9)^2 = 81$

Calculate:

$$
P(3i) = 3 \times 81 + 102 \times (-9) + 675 = 243 - 918 + 675 = 0
$$

Thus, the zeros check out, confirming the correctness of the polynomial.

Applications of the Polynomial

Understanding how to construct such polynomials is important in various mathematical and applied fields, including:

- Complex analysis: Studying polynomials with complex zeros
- Control systems: Designing characteristic equations with specific zeros
- Signal processing: Analyzing filters with certain frequency zeros
- Algebraic factorization: Factoring polynomials with complex zeros

---

Summary

In this article, we have demonstrated how to find a degree 4 polynomial with real coefficients, a leading coefficient of 3, and zeros at $5i$ and $3i$. The key steps involved:

1. Recognizing the conjugate zeros due to real coefficients
2. Forming factors from the zeros
3. Simplifying the factors using the difference of squares
4. Multiplying the factors and incorporating the leading coefficient

The resulting polynomial is:

$$
\boxed{
P(x) = 3x^4 + 102x^2 + 675
}
$$

This construction exemplifies the process of deriving polynomials with specified complex zeros and coefficients, a vital skill in algebra and related mathematical disciplines.

SEO Keywords

- Polynomial with complex zeros
- Degree 4 polynomial with real coefficients
- Construct polynomial from zeros
- Find polynomial with given zeros
- Polynomial with leading coefficient 3
- Complex conjugate zeros polynomial
- How to find polynomial with specified zeros
- Polynomial factorization with imaginary zeros

- Algebraic polynomial construction
- Zeroes of polynomial with real coefficients

Conclusion

Constructing a polynomial with specific zeros, degree, and coefficients is a fundamental skill in algebra. By understanding the relationships between zeros and factors, especially for complex conjugate zeros, one can systematically derive the polynomial's explicit form. The process illustrated here provides a clear methodology applicable to similar problems involving polynomial construction with complex zeros, reinforcing core algebraic concepts and expanding problem-solving skills.

Frequently Asked Questions

What are the zeros of the polynomial P(x) with real coefficients, degree 4, leading coefficient 3, and zeros at 5i and 3i?

The zeros are at 5i, -5i, 3i, and -3i, since complex zeros with real coefficients come in conjugate pairs.

How do you construct the polynomial P(x) given the zeros 5i and 3i for a degree 4 polynomial with leading coefficient 3?

Form the factors (x - 5i), (x + 5i), (x - 3i), and (x + 3i), then multiply all factors and multiply the resulting polynomial by 3 to set the leading coefficient.

What is the general form of the factors for zeros at 5i and 3i?

The factors are (x^2 + 25) for the zeros ±5i and (x^2 + 9) for the zeros ±3i.

How do you find the polynomial P(x) given the factors from the zeros 5i and 3i?

Multiply the factors: P(x) = 3 (x^2 + 25) (x^2 + 9).

What is the expanded form of the polynomial P(x) with the given zeros and leading coefficient?

Expanding, P(x) = 3(x^4 + 34x^2 + 225).

Why do the conjugate zeros 5i and -5i, 3i and -3i, lead to quadratic factors in the polynomial?

Because complex conjugate zeros produce quadratic factors with real coefficients, ensuring the polynomial remains real.

How do you verify that the polynomial P(x) has the specified zeros and leading coefficient?

By plugging in the zeros into P(x) and confirming it equals zero, and checking the leading term of the expanded polynomial is 3x^4.

What is the importance of the leading coefficient being 3 in the polynomial construction?

It determines the scale of the polynomial; after multiplying the factors, we multiply the entire polynomial by 3 to ensure the leading coefficient is 3.

Additional Resources

Find A Polynomial P(x) With Real Coefficients Having A Degree 4, Leading Coefficient 3, and Zeros 5i and 3i

When tasked with finding a polynomial P(x) with real coefficients, degree 4, leading coefficient 3, and zeros at 5i and 3i, it's important to understand the underlying principles that guide polynomial construction, especially when complex zeros are involved. This problem combines knowledge of complex conjugate roots, polynomial degree, and leading coefficient determination, making it a rich example of algebraic reasoning and polynomial theory in action.

Understanding the Problem: Key Components and Constraints

Before diving into the construction process, let's clarify what the problem asks:

- Degree of the polynomial: 4
- Leading coefficient: 3
- Zeros (roots): 5i and 3i
- Coefficients: Real (meaning the polynomial must have real coefficients, which governs the nature of complex roots)

Why Focus on These Components?

Understanding each component helps us build a step-by-step approach:

- The degree (4) indicates the polynomial is a quartic, which has four roots (counting multiplicities).
- The zeros at 5i and 3i are complex roots, and because the coefficients are real, their conjugates must also be roots.
- The leading coefficient (3) influences the polynomial's overall scale but does not affect the roots' positions.

Step 1: Identifying the Roots and Their Conjugates

Since the polynomial has real coefficients, any complex roots must come in conjugate pairs. This is a fundamental property:

- If a + bi is a root, then a - bi is also a root.
- For zeros at 5i and 3i, the conjugates are -5i and -3i respectively.

Roots Summary:
- Given roots: 5i, 3i
- Conjugate roots: -5i, -3i

Roots List:
1. 5i
2. -5i
3. 3i
4. -3i

Since the degree is 4, and all four roots are accounted for, the polynomial can be constructed by combining these roots.

Step 2: Constructing the Polynomial From Roots

A polynomial with roots r_1, r_2, r_3, r_4 can be expressed as:

$$P(x) = a (x - r_1)(x - r_2)(x - r_3)(x - r_4)$$

where a is the leading coefficient.

Given the roots, the factors are:

$$
P(x) = a (x - 5i)(x + 5i)(x - 3i)(x + 3i)
$$

Note: We include both roots and their conjugates to ensure real coefficients.

Step 3: Simplify the Factors Using Difference of Squares

Recall that:

$$
(x - bi)(x + bi) = x^2 + b^2
$$

Applying this to each pair:

- $(x - 5i)(x + 5i) = x^2 + (5)^2 = x^2 + 25$
- $(x - 3i)(x + 3i) = x^2 + (3)^2 = x^2 + 9$

This simplifies the polynomial to:

$$
P(x) = a (x^2 + 25)(x^2 + 9)
$$

Step 4: Incorporate the Leading Coefficient

The problem states that the leading coefficient is 3. Since the current product has a leading coefficient of 1 (as the factors are monic quadratic polynomials), we set:

$$
a = 3
$$

Thus, the polynomial is:

$$
P(x) = 3 (x^2 + 25)(x^2 + 9)
$$

Step 5: Expand the Polynomial

To find the explicit form of $P(x)$, expand the factors:

$$(x^2 + 25)(x^2 + 9)$$

Applying the distributive property:

$$x^2 \times x^2 = x^4$$

$$x^2 \times 9 = 9x^2$$

$$25 \times x^2 = 25x^2$$

$$25 \times 9 = 225$$

Adding these:

$$x^4 + 9x^2 + 25x^2 + 225 = x^4 + (9x^2 + 25x^2) + 225 = x^4 + 34x^2 + 225$$

Now, multiply by 3:

$$P(x) = 3 \times (x^4 + 34x^2 + 225) = 3x^4 + 102x^2 + 675$$

Final Polynomial:

$$\boxed{\textbf{P(x) = 3x^4 + 102x^2 + 675}}$$

This polynomial has:

- Degree 4
- Leading coefficient 3
- Zeros at $5i, -5i, 3i, -3i$
- Real coefficients

Additional Insights and Checks

Verifying the Roots:
- Substituting $x = 5i$:

$$
P(5i) = 3(5i)^4 + 102(5i)^2 + 675
$$

Calculate each term:

$$
(5i)^2 = 25i^2 = 25(-1) = -25
$$
$$
(5i)^4 = [(5i)^2]^2 = (-25)^2 = 625
$$

Plug back in:

$$
P(5i) = 3 \times 625 + 102 \times (-25) + 675 = 1875 - 2550 + 675 = 0
$$

Similarly, for $x = 3i$:

$$
(3i)^2 = -9
$$
$$
(3i)^4 = (-9)^2 = 81
$$

Then:

$$
P(3i) = 3 \times 81 + 102 \times (-9) + 675 = 243 - 918 + 675 = 0
$$

Thus, the roots check out.

Graphical Interpretation:
The polynomial's graph will reflect symmetry about the y-axis (even function), given the only even powers of x. The roots at purely imaginary points indicate the polynomial crosses the x-axis at these complex conjugate points in the complex plane, but does not have real roots.

Summary: Constructing the Polynomial Step-by-Step

1. Identify the roots: 5i and 3i.
2. Determine conjugate roots: -5i and -3i.
3. Express the polynomial as a product of factors: $(x - 5i)(x + 5i)(x - 3i)(x + 3i)$.
4. Simplify using difference of squares: $(x^2 + 25)(x^2 + 9)$.
5. Incorporate the leading coefficient: multiply by 3.
6. Expand to obtain the explicit polynomial: $P(x) = 3x^4 + 102x^2 + 675$.

Final Thoughts and Broader Applications

This problem exemplifies how complex roots influence polynomial structure, especially under the constraint of real coefficients. The key steps involve recognizing conjugate pairs, leveraging algebraic identities to simplify, and ensuring the leading coefficient aligns with the problem's specifications.

In real-world applications, such polynomials can model systems with oscillatory or wave-like behavior, where complex roots correspond to oscillations, damping, or resonance phenomena. Understanding how to construct such polynomials is fundamental in fields like engineering, physics, and applied mathematics.

In conclusion, by following a systematic approach—identifying roots, leveraging conjugate relationships, simplifying, and expanding—you can construct a polynomial that satisfies all the conditions: degree 4, real coefficients, leading coefficient 3, and zeros at the specified complex points.

Find A Polynomial P X With Realcoefficients Having A Degree 4 Leadingcoefficient3 Andzeros5iand3i Find

Find A Polynomial P X With Realcoefficients Having A Degree 4 Leadingcoefficient3 Andzeros5iand3i Find

Related Articles

- Emek Is Planning To Retire In 15 Years. She Decides To Start Saving Toward Building Up A Retirement Fund
- Based On The Passage, Which Choice Best Describes The Narrator's Reaction When, In The Morning, Hethinks
- ILL BRAINLIST RIGHT NOW IF CORRECT Which Statement Best Describes A Representative Democracy?A. Citizens

Back to Home