Find An Equation For The Linear Function G(x) Which Is Perpendicular To The Line 3x-8y=24 And Intersects

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Understanding how to find the equation of a line perpendicular to another and intersecting a particular point or line is a fundamental skill in algebra and analytic geometry. This process involves concepts such as the slope of a line, the negative reciprocal of that slope, and the point-slope form of a linear equation. In this comprehensive guide, we will explore each step carefully to help you master how to derive the equation of a linear function G(x) that is perpendicular to the given line 3x - 8y = 24 and intersects a specified point or line.

Breaking Down the Problem

Before diving into calculations, it’s essential to understand what the problem asks:

- Find the equation of a linear function G(x).
- G(x) must be perpendicular to the given line 3x - 8y = 24.
- G(x) must intersect a certain point or line (the problem statement is incomplete; typically, it might specify a point or the intersection with another line).

For this article, we will cover common scenarios:
1. When G(x) is perpendicular to the given line and intersects a specific point.
2. When G(x) is perpendicular and intersects the given line itself at a particular point.

If you have a specific point of intersection, you can substitute it into the equations accordingly. For illustration, we will assume that the linear function G(x) intersects the given line at a point (x_0, y_0), which can be specified later.

Step 1: Convert the Given Line Into Slope-Intercept Form

The first step is to express the given line in slope-intercept form y = mx + b, where m is the slope.

Given line:
$$ 3x - 8y = 24 $$

Solve for y:
$$
-8y = -3x + 24
$$
$$
y = \frac{3}{8}x - 3
$$

Conclusion:
The slope of the given line is:
$$ m_{line} = \frac{3}{8} $$

Step 2: Find the Slope of the Perpendicular Line G(x)

Lines that are perpendicular to each other have slopes that are negative reciprocals.

Negative reciprocal rule:
If the slope of the original line is m, then the slope of the perpendicular line m_{perp} is:
$$
m_{perp} = -\frac{1}{m}
$$

Applying this:
$$
m_{G} = -\frac{1}{\frac{3}{8}} = -\frac{8}{3}
$$

Conclusion:
The slope of the linear function G(x) is:
$$ m_{G} = -\frac{8}{3} $$

Step 3: Find the Equation of G(x)

The general form of a linear function:
$$
G(x) = m_{G}x + b
$$
where b is the y-intercept, which we need to determine.

Scenario 1: G(x) passes through a specific point (x_1, y_1)

Suppose you are given a point (x_1, y_1) through which G(x) passes. To find b:

$$
y_1 = m_{G}x_1 + b
$$
$$
b = y_1 - m_{G}x_1
$$

Therefore, the equation of G(x):
$$
G(x) = -\frac{8}{3}x + \left(y_1 - \left(-\frac{8}{3} \right) x_1 \right)
$$

Scenario 2: G(x) intersects the original line at a point (x_0, y_0)

If G(x) intersects the original line at some point (x_0, y_0), then:

$$
y_0 = -\frac{8}{3} x_0 + b
$$
And since (x_0, y_0) lies on the original line:
$$
y_0 = \frac{3}{8} x_0 - 3
$$

Set equal:
$$
\frac{3}{8} x_0 - 3 = -\frac{8}{3} x_0 + b
$$

Solve for b:
$$
b = \frac{3}{8} x_0 - 3 + \frac{8}{3} x_0
$$

Combine like terms:
$$
b = \left(\frac{3}{8} + \frac{8}{3} \right) x_0 - 3
$$

Find common denominator for the coefficients:
$$
\frac{3}{8} = \frac{9}{24}, \quad \frac{8}{3} = \frac{64}{24}
$$
$$
b = \left(\frac{9}{24} + \frac{64}{24} \right) x_0 - 3 = \frac{73}{24} x_0 - 3
$$

Thus, the equation of G(x) is:
$$
G(x) = -\frac{8}{3} x + \frac{73}{24} x_0 - 3
$$

The specific form depends on the point of intersection (x_0, y_0).

Step 4: Practical Examples

Let’s apply these steps with concrete points to clarify the process.

Example 1: G(x) passing through point $(0, y_1)$

Suppose G(x) passes through the point $(0, y_1)$. Then:

$$
b = y_1 - m_{G} \times 0 = y_1
$$

The equation:
$$
G(x) = -\frac{8}{3} x + y_1
$$

For example, if the point is (0, 4):

$$
G(x) = -\frac{8}{3} x + 4
$$

Example 2: G(x) intersects the original line at $x_0 = 2$

Calculate y_0:
$$
y_0 = \frac{3}{8} \times 2 - 3 = \frac{3}{4} - 3 = -\frac{9}{4}
$$

Calculate b:
$$
b = \frac{73}{24} \times 2 - 3 = \frac{73 \times 2}{24} - 3 = \frac{146}{24} - 3 = \frac{73}{12} - 3
$$
Express 3 as $\frac{36}{12}$:
$$
b = \frac{73}{12} - \frac{36}{12} = \frac{37}{12}
$$

Equation of G(x):
$$
G(x) = -\frac{8}{3} x + \frac{37}{12}
$$

Additional Considerations and Applications

In practical problems, you might encounter various scenarios:
- Finding the perpendicular line passing through a specific point.
- Determining the line that is perpendicular to a given line and intersects it at a specific point.
- Analyzing the angle between two lines and their slopes.

Let’s explore some real-world applications:

Applications in Engineering and Physics
- Designing components that are perpendicular to existing structures.
- Calculating the trajectory paths in physics where directions are perpendicular.
- Signal processing where phase lines are orthogonal.

Applications in Computer Graphics
- Creating perpendicular lines for rendering and shading.
- Calculating normal vectors to surfaces.

Applications in Mathematics and Education
- Teaching concepts of perpendicularity, slopes, and equations.
- Developing problem-solving skills related to geometric transformations.

Summary and Key Takeaways

- To find a line perpendicular to a given line, first convert the given line to slope-intercept form to identify its slope.
- The perpendicular line's slope is the negative reciprocal.
- Use the point-slope form or slope-intercept form to derive the equation of the perpendicular line, depending on known points.
- The process involves algebraic manipulation and understanding the geometric relationships between lines.

Final Note

Mastering the process of finding the equation of a perpendicular line involves understanding the relationship between slopes, the ability to manipulate algebraic equations, and applying these concepts to real-world problems. Whether you're tackling homework, preparing for exams, or applying these concepts practically, these steps and explanations serve as a solid foundation.

If you have specific points or additional conditions (such as the line's intersection point), applying the above principles will allow you to derive the precise equation of the line G(x) that fulfills the given criteria.

Remember: Practice with different points and lines to strengthen your understanding and develop confidence in solving similar problems efficiently.

Frequently Asked Questions

What is the slope of the line 3x - 8y = 24?

First, rewrite the equation in slope-intercept form: $3x - 8y = 24 \rightarrow -8y = -3x + 24 \rightarrow y = (3/8)x - 3$. The slope is 3/8.

What is the slope of a line perpendicular to the given line?

The slope of the given line is 3/8, so the perpendicular line will have a slope of -8/3.

How do I write the equation of a line perpendicular to 3x - 8y = 24?

First, find the perpendicular slope (-8/3). Then, use the point-slope form with a given point or general form if the line intersects at a specific point.

What additional information do I need to find the specific equation of G(x)?

You need a point where the line G(x) intersects or additional details about the intersection point.

How do I write the equation of G(x) if it passes through a specific point?

Use the point-slope form: $y - y_1 = m(x - x_1)$, where (x_1, y_1) is the point and m is the slope (-8/3). Then, simplify to slope-intercept form if needed.

Can you provide a general form of the equation for G(x)?

Yes. If G(x) passes through a point (x_0, y_0), its equation is $y - y_0 = -8/3(x - x_0)$. If no point is given, the equation is $y = -8/3\ x + b$, where b is determined by the intersection point.

What is the significance of the intersection point in finding G(x)?

The intersection point determines the specific line G(x). Without it, only the family of lines perpendicular to the given line can be described.

How do I find the equation of G(x) if it intersects the original line at a known point?

Identify the intersection point, then substitute it into the point-slope form with the perpendicular slope (-8/3) to find the equation.

What is the final form of the equation for G(x) if it intersects the line at (x_1, y_1)?

The equation is $y - y_1 = -8/3(x - x_1)$, which can be expanded to $y = -8/3\ x + (y_1 + 8/3\ x_1)$.

How do I verify that the line G(x) is perpendicular to the original line?

Check that the slopes are negative reciprocals: if the original line's slope is 3/8, the perpendicular line's slope should be -8/3. Confirm this in your equation.

Additional Resources

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Introduction

Mathematics, particularly algebra and analytic geometry, provides foundational tools for understanding the relationships between lines and functions. Among these, the concept of perpendicular lines holds special significance, especially in applications ranging from engineering to physics. In this detailed exploration, we will examine how to find the equation of a linear function $G(x)$ that is perpendicular to a given line, specifically $3x - 8y = 24$, and intersects a certain point or line. This investigative journey will involve analyzing the properties of lines, understanding slopes and their relationships, and deriving the required function through systematic calculation.

Understanding the Given Line

Standard Form and Slope of the Line $3x - 8y = 24$

The initial step in solving such problems involves rewriting the given line in a more manageable form, usually slope-intercept form $y = mx + b$, where m denotes the slope.

Starting with the original equation:

$$
3x - 8y = 24
$$

Rearranged to solve for y:

$$
-8y = -3x + 24
$$
$$
y = \frac{3x - 24}{8}
$$
$$
\boxed{ y = \frac{3}{8}x - 3 }
$$

This reveals that the line's slope m_1 is:

$$
m_1 = \frac{3}{8}
$$

Understanding this slope is crucial because the relationship between perpendicular lines is directly related to their slopes.

The Perpendicular Slope

The Negative Reciprocal Relationship

Two lines are perpendicular if and only if their slopes are negative reciprocals. Mathematically, if one line has slope m_1, then the line perpendicular to it has slope m_2:

$$
m_2 = -\frac{1}{m_1}
$$

Applying this to our line:

$$
m_2 = -\frac{1}{\frac{3}{8}} = -\frac{8}{3}
$$

$$

Thus, the slope of the line $G(x)$, which is perpendicular to the original line, is:

$$
\boxed{ m_G = -\frac{8}{3} }
$$

Deriving the Equation of $G(x)$

General Form

The equation of a line with slope m, passing through a point (x_0, y_0), is given by the point-slope form:

$$
y - y_0 = m (x - x_0)
$$

However, the problem statement indicates that $G(x)$ intersects the original line or a specified point. For the purpose of this analysis, we'll assume that the line $G(x)$ intersects the original line at a specified point, say (x_1, y_1), or at least that the line passes through a particular point (x_0, y_0).

Key Point: If the problem specifies a particular intersection point, we can substitute that into the equation. If not, the problem generally seeks the family of all lines with slope $-8/3$.

Case 1: Intersection at a Specific Point

Suppose $G(x)$ intersects the original line $3x - 8y = 24$ at a point (x_0, y_0). To find $G(x)$, we proceed as follows:

1. Find the intersection point (x_0, y_0).

2. Use the point-slope form to write the equation of $G(x)$.

Finding the Intersection Point

Solving for the Point of Intersection

The original line is:

$$
y = \frac{3}{8}x - 3
$$

Let (x_0, y_0) be an intersection point. Since $G(x)$ has slope $-8/3$, and passes through (x_0, y_0), its equation is:

$$
y - y_0 = -\frac{8}{3}(x - x_0)
$$

But we need to find the point where $G(x)$ intersects the original line. To do this, we set the equations equal:

$$
\text{Equation of } G(x): y = y_0 - \frac{8}{3}(x - x_0)
$$

$$
\text{Original line: } y = \frac{3}{8}x - 3
$$

At the intersection point:

$$
y_0 - \frac{8}{3}(x - x_0) = \frac{3}{8}x - 3
$$

Solving for the Intersection Coordinates

Rearranged:

$$
y_0 - \frac{8}{3}x + \frac{8}{3}x_0 = \frac{3}{8}x - 3
$$

Bring all x-terms to one side:

$$
-\frac{8}{3}x - \frac{3}{8}x = -3 - y_0 - \frac{8}{3}x_0
$$

Find a common denominator for the x-terms:

$$
-\frac{8}{3}x = -\frac{64}{24}x
$$
$$
-\frac{3}{8}x = -\frac{9}{24}x
$$

Sum:

$$
-\frac{64}{24}x - \frac{9}{24}x = -\frac{73}{24}x
$$

Thus,

$$
-\frac{73}{24}x = -3 - y_0 - \frac{8}{3}x_0
$$

Now, solving for x:

$$
x = \frac{24}{73} \left(3 + y_0 + \frac{8}{3}x_0 \right)
$$

The intersection point depends on the choice of (x_0, y_0). Alternatively, if we specify a point through which $G(x)$ passes, say (x_p, y_p), then:

$$
\boxed{
G(x): y = y_p - \frac{8}{3}(x - x_p)
}
$$

Case 2: $G(x)$ Intersects the Original Line at a Known Point

Suppose the problem specifies that $G(x)$ passes through a particular point, say (x_0, y_0). For illustration, assume $(x_0, y_0) = (0, y_0)$, where y_0 is to be determined or is arbitrary.

The line $G(x)$ with slope $-8/3$ passing through (x_0, y_0) is:

$$
\boxed{
G(x): y = y_0 - \frac{8}{3}(x - x_0)
}
$$

This general form allows us to write an explicit equation once the point is known.

Special Cases and Additional Constraints

In many practical problems, additional constraints are specified:

- Intersects a particular point: For example, $G(x)$ passes through (x_1, y_1).

- Is tangent or intersects a line at a specific point: The intersection point (x_i, y_i) must satisfy both equations.

- Passes through the origin: $(0,0)$.

In absence of such, the family of solutions is characterized by the general form:

$$
G(x): y = y_0 - \frac{8}{3}(x - x_0)
$$

where (x_0, y_0) is any point on the line.

Summary of the Solution

- The original line's slope is $\frac{3}{8}$.

- The slope of the perpendicular line $G(x)$ is $-\frac{8}{3}$.

- The general form of $G(x)$, passing through a point (x_0, y_0), is:

$$
\boxed{
G(x): y = y_0 - \frac{8}{3}(x - x_0)
}
$$

- To find a specific equation, additional information about the intersection point or the point $G(x)$ passes through is required.

Broader Implications and Applications

Geometric Significance

Understanding perpendicular lines is fundamental in coordinate geometry, facilitating the analysis of angles, distances, and intersections. The negative reciprocal relationship between slopes simplifies the process of identifying perpendicularity, which is vital in various fields such as computer graphics, navigation, and structural engineering.

Practical Applications

- Design and Construction: Ensuring perpendicularity between structural components.

- Physics: Analyzing forces and motion along perpendicular axes.

- Data Analysis: Orthogonal regression lines.

Conclusion

The process of finding a linear function \(

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