# Find An Equation Of The Tangent Line To The Curve At Each Given Point.x=t2-4, Y=t2 - 2t T2-2tat (0, 0)at

**Find An Equation Of The Tangent Line To The Curve At Each Given Point.x=t^2-4, Y=t^2 - 2t T2-2tat (0, 0)at**

Understanding how to find the tangent line to a curve at a specific point is a fundamental skill in calculus, with applications spanning physics, engineering, and mathematics. In this guide, we will explore the process step-by-step to determine the equations of tangent lines for a parametric curve defined by the given functions. Specifically, we'll analyze the curve defined by the parametric equations:

- $x = t^2 - 4$
- $y = t^2 - 2t$

and find the equations of the tangent lines at various points, including the point $(0,0)$. By the end of this comprehensive tutorial, you'll understand how to approach such problems systematically and apply the principles to similar curves.

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## Understanding the Problem: Parametric Curves and Tangent Lines

### What Is a Parametric Curve?

A parametric curve is one where the coordinates of points on the curve are defined in terms of a parameter, often denoted as $t$. For the given equations:

$$
x = t^2 - 4
$$
$$
y = t^2 - 2t
$$

each value of $t$ corresponds to a specific point $(x, y)$ on the curve.

## Why Find the Tangent Line?

The tangent line at a specific point on a curve indicates the instantaneous direction of the curve at that point. It is crucial in understanding the behavior of the curve, such as identifying maxima, minima, and points of inflection.

## Key Concepts Needed

- Derivative of the parametric equations: To find the slope of the tangent line, we need $\frac{dy}{dx}$, which can be obtained via derivatives of $x(t)$ and $y(t)$.
- Point on the curve: The particular point at which we want the tangent line, often corresponding to a specific value of $t$.
- Equation of the tangent line: Using the point-slope form: $y - y_0 = m (x - x_0)$, where $m$ is the slope at the point.

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# Step 1: Find $\frac{dy}{dt}$ and $\frac{dx}{dt}$

To determine the slope of the tangent line at any point, we first compute the derivatives of the parametric equations with respect to $t$.

## Calculate $\frac{dx}{dt}$

Given $x = t^2 - 4$:

$$
\frac{dx}{dt} = 2t
$$

## Calculate $\frac{dy}{dt}$

Given $y = t^2 - 2t$:

$$
\frac{dy}{dt} = 2t - 2
$$

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# Step 2: Find $\frac{dy}{dx}$ in terms of $t$

Since both $x$ and $y$ are functions of $t$, the derivative $\frac{dy}{dx}$ can be found using the chain rule:

$$
\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{2t - 2}{2t} \quad \text{(provided } 2t \neq 0)
$$

Simplify:

$$
\frac{dy}{dx} = \frac{2(t - 1)}{2t} = \frac{t - 1}{t}
$$

This formula gives the slope of the tangent line at any point corresponding to $t$.

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# Step 3: Find the specific points and their tangent slopes

To find the tangent line at a particular point, we need:

- The corresponding value of $t$
- The point coordinates $(x, y)$
- The slope $m = \frac{dy}{dx}$

Let's analyze at the point $(0, 0)$ first, then at other points of interest.

## Finding the parameter $t$ for $(0, 0)$

Set $x = 0$ and $y = 0$:

$$
x = t^2 - 4 = 0 \Rightarrow t^2 = 4 \Rightarrow t = \pm 2
$$

Check $y$ at these $t$:

- For $t = 2$:

$$
y = 2^2 - 2 \times 2 = 4 - 4 = 0
$$

\]

- For \(t = -2\):

\[
y = (-2)^2 - 2 \times (-2) = 4 + 4 = 8
\]

Since \(y = 0\) at \(t=2\), the point \((0, 0)\) corresponds to \(t=2\).

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# Step 4: Calculate the slope at \(t=2\)

Using \(\frac{dy}{dx} = \frac{t - 1}{t}\):

\[
m = \frac{2 - 1}{2} = \frac{1}{2}
\]

The point on the curve at \(t=2\):

\[
x = 2^2 - 4 = 4 - 4 = 0
\]
\[
y = 4 - 4 = 0
\]

Thus, the tangent line at \((0, 0)\) has a slope of \(\frac{1}{2}\).

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# Step 5: Write the equation of the tangent line at \((0, 0)\)

Using point-slope form:

\[
y - y_0 = m (x - x_0)
\]

Substitute \(x_0 = 0\), \(y_0= 0\), \(m= \frac{1}{2}\):

\[
y = \frac{1}{2} x

\]

Equation of the tangent line at $(0, 0)$:

$$
\boxed{ y = \frac{1}{2} x }
$$

---

# Additional Points and Their Tangent Lines

To deepen your understanding, let's analyze other points on the curve and find their tangent lines.

## Example 1: Point at $t = 1$

- Find $x$ and $y$:

$$
x = 1^2 - 4 = -3
$$
$$
y = 1^2 - 2 \times 1 = 1 - 2 = -1
$$

- Compute the slope:

$$
m = \frac{1 - 1}{1} = 0
$$

- Equation of the tangent line:

$$
y - (-1) = 0 \times (x - (-3))
$$
$$
y + 1 = 0
$$
$$
\boxed{ y = -1 }
$$

Thus, at $(-3, -1)$, the tangent line is horizontal.

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## Example 2: Point at $t = -1$

- Find $x$ and $y$:

$$
x = (-1)^2 - 4 = 1 - 4 = -3
$$
$$
y = 1 - 2 \times (-1) = 1 + 2 = 3
$$

- Compute the slope:

$$
m = \frac{-1 - 1}{-1} = \frac{-2}{-1} = 2
$$

- Equation of the tangent line:

$$
y - 3 = 2 (x + 3)
$$
$$
y - 3 = 2x + 6
$$
$$
y = 2x + 9
$$

Equation of the tangent line at $(-3, 3)$:

$$
\boxed{ y = 2x + 9 }
$$

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## Summary of the Process for Finding Tangent Lines to Parametric Curves

Here's a concise roadmap to tackle similar problems:

1. Identify the parametric equations $x(t)$ and $y(t)$.

2. Compute $\frac{dx}{dt}$ and $\frac{dy}{dt}$.

3. Find $\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}}$, ensuring $\frac{dx}{dt} \neq 0$.
4. Determine the value(s) of $t$ corresponding to the point(s) of interest.
5. Calculate the coordinates $(x, y)$ at these $t$-values.
6. Compute the slope $m = \frac{dy}{dx}$ at each $t$.
7. Write the equation of the tangent line

## Frequently Asked Questions

### What is the equation of the curve given by $x = t^2 - 4$ and $y = t^2 - 2t$?

The parametric equations define a curve where $x = t^2 - 4$ and $y = t^2 - 2t$.

### How do I find the slope of the tangent line to the curve at a given point?

Calculate dy/dt and dx/dt, then find dy/dx = (dy/dt) / (dx/dt) to get the slope at the point.

### What is the derivative dy/dt for the given parametric equations?

dy/dt = 2t - 2.

### What is the derivative dx/dt for the given parametric equations?

dx/dt = 2t.

### How do I find the slope of the tangent line at the point

## (0,0)?

Solve for t where x=0 and y=0. Then compute dy/dx at that t value using dy/dt and dx/dt.

## At which value of t does the curve pass through the point (0,0)?

Set x = 0: $t^2 - 4 = 0 \Rightarrow t^2 = 4 \Rightarrow t = \pm 2$. Check y: for t=2, y= 4 - 4=0; for t=-2, y=4 + 4=8. So at t=2, the point is (0, 0).

## What is the slope of the tangent line at the point (0,0)?

At t=2, dy/dt=2(2)-2=2, dx/dt=2(2)=4, so slope m = (dy/dt)/(dx/dt)= 2/4= 1/2.

## What is the equation of the tangent line at the point (0,0)?

Using point-slope form: y - 0 = (1/2)(x - 0), so y = (1/2)x.

## How do I generalize the tangent line equation at any point t on the curve?

Calculate dy/dx at t, then use the point (x(t), y(t)) in y - y(t) = m(x - x(t)) to find the tangent line equation.

## What is the final step to find the tangent line at the point (0,0)?

Substitute t=2 into x and y to confirm the point, compute dy/dx at t=2, and then write the tangent line equation using point-slope form.

# Additional Resources

Finding the Equation of the Tangent Line to a Parametric Curve at a Given Point

Understanding the behavior of curves and their tangents is fundamental in calculus, especially when analyzing the instantaneous rate of change at specific points. When a curve is defined parametrically, finding the tangent line involves a nuanced process that combines derivatives with parameterization. This comprehensive guide delves into the step-by-step methodology for determining the equation of the tangent line to a parametric curve at a specified point, using the example functions provided:

- $x = t^2 - 4$
- $y = t^2 - 2t$

Given specific points such as (0,0) or others, we'll explore how to find the corresponding tangent lines, interpret the results, and understand their significance in the broader context of calculus and geometry.

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## Understanding Parametric Curves and Their Significance

Parametric equations describe a curve by expressing the coordinates of points on the curve as functions of one or more parameters—in this case, the parameter $t$. Unlike explicit functions $y = f(x)$, parametric equations can represent more complex curves, including circles, ellipses, and other non-function graphs.

Advantages of Using Parametric Equations:

- They can describe a wider variety of curves, especially those that are not functions in the traditional sense.
- They facilitate the analysis of motion along a path, where $t$ might represent time.
- They allow for easier computation of derivatives when dealing with complex shapes.

Given Functions:

- $x(t) = t^2 - 4$
- $y(t) = t^2 - 2t$

Our goal is to find the tangent line at points corresponding to specific values of $t$, particularly at $t = 0$, which yields the point (0, 0).

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## Step-by-Step Methodology for Finding the Tangent Line

The process of finding the tangent line to a parametric curve at a point involves several key steps:

1. Identify the Point on the Curve

- Determine the parameter value $t = t_0$ corresponding to the point of interest.
- Calculate the coordinates $(x(t_0), y(t_0))$.

2. Compute the Derivatives $dx/dt$ and $dy/dt$

- Derivatives with respect to $t$ give the instantaneous rate of change of $x$ and $y$ along the curve.
- These derivatives are essential for finding the slope of the tangent line.

3. Find the Slope of the Tangent Line ($dy/dx$)

- Use the chain rule for parametric derivatives:

$$
\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}}
$$

- Evaluate $dy/dt$ and $dx/dt$ at $t = t_0$.

4. Write the Equation of the Tangent Line

- Use the point-slope form:

$$
y - y_0 = m (x - x_0)
$$

where:

- $(x_0, y_0)$ is the point on the curve.
- $m = dy/dx$ at $t = t_0$.

---

# Applying the Method to the Given Functions

Let's apply this methodology step-by-step to the specific functions provided:

- $x(t) = t^2 - 4$
- $y(t) = t^2 - 2t$

and particularly focus on the point $(0, 0)$.

Step 1: Find \( t \) corresponding to \( (0, 0) \)

Set \( x(t) = 0 \):

\[
t^2 - 4 = 0 \implies t^2 = 4 \implies t = \pm 2
\]

Verify which \( t \) gives \( y(t) = 0 \):

- For \( t = 2 \):

\[
y(2) = (2)^2 - 2 \times 2 = 4 - 4 = 0
\]

- For \( t = -2 \):

\[
y(-2) = (-2)^2 - 2 \times (-2) = 4 + 4 = 8
\]

Since \( y = 0 \) occurs only at \( t=2 \), the point \( (0, 0) \) on the curve corresponds to \( t=2 \).

Step 2: Compute derivatives \( dx/dt \) and \( dy/dt \)

\[
\frac{dx}{dt} = 2t
\]
\[
\frac{dy}{dt} = 2t - 2
\]

Evaluate at \( t=2 \):

\[
dx/dt|_{t=2} = 2 \times 2 = 4
\]
\[
dy/dt|_{t=2} = 2 \times 2 - 2 = 4 - 2 = 2
\]

Step 3: Calculate \( dy/dx \) at \( t=2 \)

\[
\frac{dy}{dx} = \frac{dy/dt}{dx/dt} = \frac{2}{4} = \frac{1}{2}
\]

This slope indicates the tangent line's steepness at the point \( (0, 0) \).

Step 4: Write the Equation of the Tangent Line

Using the point $(x_0, y_0) = (0, 0)$ and slope $m = \frac{1}{2}$:

$$
y - 0 = \frac{1}{2} (x - 0)
$$
$$
y = \frac{1}{2} x
$$

Final Equation:

$$
\boxed{
\textbf{The tangent line at } (0, 0) \text{ is } y = \frac{1}{2} x
}
$$

---

# Analyzing Other Points and Different Values of $t$

While the above demonstrates the process at $t=2$, similar steps can be used for other points on the curve or for different $t$ values.

Example: Find the tangent line at $t=-2$, which yields $(x, y) = (0, 8)$.

- Derivatives at $t=-2$:

$$
dx/dt = 2 \times (-2) = -4
$$

$$
dy/dt = 2 \times (-2) - 2 = -4 - 2 = -6
$$

- Slope:

$$
dy/dx = \frac{-6}{-4} = \frac{3}{2}
$$

- Equation of tangent line at $(0, 8)$:

$$
y - 8 = \frac{3}{2} (x - 0) \implies y = \frac{3}{2} x + 8

\]

This illustrates that different points on the curve can have varying tangent slopes, reflecting the curve's local behavior.

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# Deeper Insights: The Role of Derivatives and Geometry

Understanding the derivative $dy/dx$ in the context of parametric curves is crucial for grasping the geometric nature of the curve.

Geometric Interpretation:

- The derivative $dy/dx$ at a point provides the slope of the tangent line, indicating the curve’s direction at that point.
- When $dy/dx$ is positive, the curve is increasing locally; when negative, decreasing.
- A zero $dy/dx$ signifies a horizontal tangent; an undefined $dy/dx$ indicates a vertical tangent.

Why the Derivative is the Key:

In parametric forms, the derivatives $dx/dt$ and $dy/dt$ encode how the position changes with respect to the parameter. The ratio $dy/dt$ over $dx/dt$ effectively measures how $y$ changes relative to $x$ as $t$ varies, which is exactly the slope of the tangent in the Cartesian plane.

Limitations and Special Cases:

- When $dx/dt = 0$, the slope $dy/dx$ becomes undefined, indicating a potential vertical tangent.
- In such cases, the tangent line is vertical, and its equation is simply $x = x_0$, where $x_0$ is the $x$-coordinate at that point.

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# Extensions and Applications of Tangent Line Calculations in Parametric Curves

Understanding tangent lines extends beyond simple geometric curiosity. It has numerous applications:

- Physics: Analyzing the velocity and acceleration of moving objects along a path.
- Engineering: Designing curves with specific tangent properties for roads, tracks, or aerodynamic surfaces.
- Mathematics: Studying curvature, inflection points, and the shape of graphs.
- Computer Graphics: Rendering smooth curves and understanding their tangent directions for shading and motion.

Calculating the Tangent Line at Any Parameter \( t \

## Find An Equation Of The Tangent Line To The Curve At Each Given Point X T2 4 Y T2 2t T2 2tat 0 0 At

Find An Equation Of The Tangent Line To The Curve At Each Given Point X T2 4 Y T2 2t T2 2tat 0 0 At


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