Find An Expression For The Average Hbar_L And Nu_L For A Mixed BL, Where The Transition Occurs Somewhere

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Understanding heat transfer within a mixed boundary layer (BL) is crucial for many engineering applications, from thermal management in electronics to environmental modeling. When dealing with a mixed boundary layer, where both natural and forced convection influence the flow and heat transfer, deriving expressions for the average Nusselt number (Nu_L) and the average modified heat transfer coefficient ($\bar{H}_L$) becomes essential, especially when the transition point between these regimes occurs somewhere within the boundary layer. This article aims to provide a comprehensive guide on how to find these averages, accounting for the spatial variation of the boundary layer properties and the transition's location.

Understanding the Mixed Boundary Layer and Transition Phenomena

What Is a Mixed Boundary Layer?

A mixed boundary layer occurs when a fluid flow over a surface experiences combined effects of natural convection, driven by buoyancy, and forced convection, driven by an external velocity. This situation commonly arises in practical scenarios such as horizontal plates exposed to wind and temperature gradients or in internal flows where multiple heat transfer mechanisms coexist.

Transition Within the Boundary Layer

The boundary layer doesn't uniformly exhibit a single dominant convection mode throughout its extent. Instead, the transition point, where natural convection's influence overtakes forced convection or vice versa, can occur somewhere within the boundary layer. Identifying this point, often called the "transition point" or "crossover point," is key because the heat transfer characteristics differ significantly before and after it.

Mathematical Framework for Averaging Heat Transfer Coefficients

Fundamental Definitions

- Nusselt Number (Nu_L): A dimensionless number representing the ratio of convective to conductive heat transfer across a boundary layer of length L. It is defined as:

$$
Nu_L = \frac{h_L L}{k}
$$

where:
- h_L is the local heat transfer coefficient,
- L is the characteristic length,
- k is the thermal conductivity of the fluid.

- Modified Heat Transfer Coefficient ($\bar{H}_L$): Often used in convective heat transfer calculations, especially when considering complex boundary layers, defined similarly to h_L.

Spatial Variability in the Boundary Layer

In a mixed BL with a transition point at $x = x_t$, the properties such as Nu_x and H_x vary along the length x:

$$
Nu_x =
\begin{cases}
Nu_{forced}(x), & 0 \leq x \leq x_t \\
Nu_{natural}(x), & x_t < x \leq L
\end{cases}
$$

Similarly, for the heat transfer coefficient:

$$
H_x =
\begin{cases}
H_{forced}(x), & 0 \leq x \leq x_t \\
H_{natural}(x), & x_t < x \leq L
\end{cases}
$$

The goal is to find the average $\bar{H}_L$ and Nu_L over the entire length L, considering the transition point x_t exists somewhere within the boundary layer.

Deriving the Average Nusselt Number and $\bar{H}_L$

Formulating the Averages

The average Nusselt number over the length L:

$$
Nu_{avg} = \frac{1}{L} \int_0^L Nu_x \, dx
$$

Similarly, the average heat transfer coefficient:

$$
\bar{H}_L = \frac{1}{L} \int_0^L H_x \, dx
$$

Given the transition at $x = x_t$, these integrals split into two parts:

$$
Nu_{avg} = \frac{1}{L} \left(\int_0^{x_t} Nu_{forced}(x) \, dx + \int_{x_t}^{L} Nu_{natural}(x) \, dx \right)
$$

$$
\bar{H}_L = \frac{1}{L} \left(\int_0^{x_t} H_{forced}(x) \, dx + \int_{x_t}^{L} H_{natural}(x) \, dx \right)
$$

Note: To proceed, explicit expressions for $Nu_{forced}(x)$ and $Nu_{natural}(x)$, as well as $H_{forced}(x)$ and $H_{natural}(x)$, are required. These are typically derived from empirical correlations or similarity solutions.

Applying Empirical Correlations

Common correlations for forced and natural convection are:

- Forced convection (laminar or turbulent):

$$
Nu_{forced}(x) = C_f \, Re_x^{m} \, Pr^{n}
$$

where:
- Re_x is the local Reynolds number,
- Pr is the Prandtl number,
- C_f, m, n are empirical constants.

- Natural convection:

$$
Nu_{natural}(x) = C_n \, (Gr_x \, Pr)^p
$$

where:
- Gr_x is the local Grashof number,
- C_n, p are empirical constants.

The local Reynolds and Grashof numbers are functions of x:

$$
Re_x = \frac{U x}{\nu}
$$
$$
Gr_x = \frac{g \beta (T_s - T_\infty) x^3}{\nu^2}
$$

with U being the free-stream velocity, ν the kinematic viscosity, g gravity, β the thermal expansion coefficient, and ΔT the temperature difference.

Note: The exact forms depend on the flow regime and boundary conditions.

Calculating the Integrals for Averages

Integral Expressions

Assuming the correlations for Nu_x are known functions of x, the integrals become:

$$
\int_0^{x_t} Nu_{forced}(x) \, dx = \int_0^{x_t} C_f \, Re_x^{m} \, Pr^{n} \, dx
$$

$$
\int_{x_t}^{L} Nu_{natural}(x) \, dx = \int_{x_t}^{L} C_n \, (Gr_x \, Pr)^p \, dx
$$

Expressing Re_x and Gr_x explicitly in terms of x:

$$
Re_x = \frac{U x}{\nu}
$$
$$
Gr_x = \frac{g \beta \Delta T x^3}{\nu^2}
$$

Thus,

$$
Nu_{forced}(x) = C_f \left(\frac{U x}{\nu}\right)^m Pr^{n}
$$

\[

$$
Nu_{natural}(x) = C_n \left(\frac{g \beta \Delta T x^3}{\nu^2} \, Pr \right)^p
$$

The integrals then become:

$$
\int_0^{x_t} C_f \left(\frac{U}{\nu}\right)^m Pr^{n} x^{m} dx
$$

$$
\int_{x_t}^{L} C_n \left(\frac{g \beta \Delta T}{\nu^2} Pr \right)^p x^{3p} dx
$$

which are straightforward to evaluate:

$$
\int x^{q} dx = \frac{x^{q+1}}{q+1}
$$

Applying this:

$$
\int_0^{x_t} Nu_{forced}(x) dx = C_f \left(\frac{U}{\nu}\right)^m Pr^{n} \frac{x_t^{m+1}}{m+1}
$$

$$
\int_{x_t}^{L} Nu_{natural}(x) dx = C_n \left(\frac{g \beta \Delta T}{\nu^2} Pr \right)^p
\frac{L^{3p+1} - x_t^{3p+1}}{3p+1}
$$

The averages are then:

$$
Nu_{avg} = \frac{1}{L} \left[C_f \left(\frac{U}{\nu}\right)^m Pr^{n} \frac{x_t^{m+1}}{m+1} +
C_n \left(\frac{g \beta \Delta T}{\nu^2} Pr \right)^p \frac{L^{3p+1} - x_t^{3p+1}}{3p+1} \right]
$$

Similarly, $\bar{H}_L$ can be computed if corresponding correlations for H_x are available.

Considering the Transition Point x_t

Locating the Transition Point

The transition point x_t

Frequently Asked Questions

How can we derive an expression for the average Hbar_L in a mixed boundary layer where the transition occurs somewhere within the flow?

To derive the average Hbar_L, integrate the local Hbar_L profile across the boundary layer thickness, considering the point of transition. This involves modeling the velocity and turbulence profiles in both laminar and turbulent regions, then averaging over the entire boundary layer thickness.

What is the significance of the transition point in determining the average Nu_L in a mixed boundary layer?

The transition point marks where the flow changes from laminar to turbulent, significantly affecting the heat transfer characteristics. Accurately locating this point allows for proper modeling of the local Nusselt number (Nu_L) in each region, which in turn influences the calculation of the overall average Nu_L.

Can you provide a general expression for the average Nusselt number (Nu_L) in a mixed boundary layer with a transition occurring somewhere within?

Yes. The average Nu_L can be expressed as a weighted average:

$$\text{Average } Nu_L = (1 / \delta_{total}) \left[\int_0^{x_t} Nu_{L,laminar}\, dx + \int_{x_t}^{\delta_{total}} Nu_{L,turbulent}\, dx\right],$$

where x_t is the transition position, and $Nu_{L,laminar}$ and $Nu_{L,turbulent}$ are the local Nusselt numbers in laminar and turbulent regions, respectively.

What methods are typically used to determine the transition point within a mixed BL for calculating Hbar_L and Nu_L?

Common methods include empirical correlations based on Reynolds number thresholds, stability analysis, or turbulence models that predict transition location. For practical purposes, transition is often assumed at a critical Reynolds number, like $Re_x \approx 5\times10^5$ for flat-plate flows.

How does the presence of a mixed boundary layer influence the heat transfer predictions compared to purely laminar or turbulent boundary layers?

Mixed boundary layers exhibit a combination of laminar and turbulent heat transfer characteristics, leading to more complex profiles. This results in intermediate or transitional heat transfer rates, requiring models that account for both regimes to accurately predict Hbar_L and Nu_L.

Are there simplified formulas or correlations available for estimating the average Hbar_L and Nu_L in a mixed boundary layer with an internal transition point?

Yes, several empirical correlations exist that approximate the average Hbar_L and Nu_L by combining laminar and turbulent expressions weighted by the length or thickness of each region. These typically involve boundary layer parameters like Reynolds and Prandtl numbers and are validated against experimental data for specific flow configurations.

Additional Resources

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Introduction

In fluid dynamics, boundary layers (BLs) are critical regions where viscous effects are significant, especially near solid surfaces. When analyzing flow scenarios involving a mixed boundary layer—where a laminar boundary layer transitions into a turbulent boundary layer—the challenge lies in accurately capturing the average heat transfer and momentum transfer characteristics. Specifically, the quantities $\grave{H}_l$ (average heat transfer coefficient) and Nu_l (average Nusselt number) become more complex to evaluate when the transition occurs within the boundary layer at some point along the surface, rather than at a fixed location.

This review aims to provide a comprehensive derivation and discussion of the expressions for average $\grave{H}_l$ and $N\bar{u}_l$ in such a mixed boundary layer scenario, where the transition from laminar to turbulent flow occurs somewhere within the boundary layer, not necessarily at the end or beginning. We will explore the physical basis, mathematical formulations, and the assumptions involved, ensuring the derivations are rigorous and applicable across different flow regimes.

Physical Context and Significance

Why Is the Transition Location Important?

- The boundary layer is initially laminar near the leading edge of a surface, characterized by smooth, orderly flow with low momentum and heat transfer.
- As the flow progresses downstream, instabilities grow, and the boundary layer transitions to turbulence, significantly increasing mixing, momentum, and heat transfer rates.
- The location where this transition occurs, often denoted as x_t, influences the overall heat and momentum transfer characteristics of the surface.

Implications of a Transition Within the Boundary Layer

- When the transition occurs within the boundary layer (not at a boundary or at the end), the flow properties and transfer coefficients change along the surface.

- The boundary layer can be conceptually divided into two regions:
- Laminar region: from the leading edge to the transition point.
- Turbulent region: from the transition point to the trailing edge.

- The overall average heat transfer coefficient and Nusselt number must account for this spatial variation, requiring a weighted or composite approach.

Mathematical Framework

Fundamental Quantities

- $\grave{H}_l$ (Average heat transfer coefficient): Typically defined as the average of local heat transfer coefficients over the length of the boundary layer.
- $\bar{Nu}_l$ (Average Nusselt number): The non-dimensional measure of convective heat transfer, related to $\grave{H}_l$ through characteristic length scales.

Boundary Layer Thicknesses

- $\delta_l(x)$: Laminar boundary layer thickness at position x.
- $\delta_t(x)$: Turbulent boundary layer thickness at position x.
- x_t: Transition point along the surface where the boundary layer switches from laminar to turbulent.

Deriving Expressions for $\grave{H}_l$ and $\bar{Nu}_l$

Step 1: Defining the Spatial Domain

The overall boundary layer spans from the leading edge (x = 0) to the end of the surface (x = L):

- Laminar region: from x = 0 to $x = x_t$
- Turbulent region: from $x = x_t$ to x = L

The average quantities are then integrated over these regions, considering the distinct transfer characteristics:

\[
\boxed{
\text{Average Ḣ}_L = \frac{1}{L} \left(\int_0^{x_t} H_{lam}(x) \, dx + \int_{x_t}^L H_{turb}(x) \, dx \right)
}
\]

Similarly for the Nusselt number:

\[
\boxed{
\text{Nū}_L = \frac{1}{L} \left(\int_0^{x_t} Nu_{lam}(x) \, dx + \int_{x_t}^L Nu_{turb}(x) \, dx \right)

}
\]

Step 2: Expressing Local Heat Transfer Coefficients

The local heat transfer coefficients in laminar and turbulent regions are well-characterized by empirical correlations and similarity solutions.

Laminar Region

- For a flat plate with constant free-stream velocity U, the local Nusselt number (for constant wall temperature) is given by:

$$
Nu_{lam}(x) = C_{lam} \, Re_x^{m}
$$

where:

- $Re_x = \frac{U x}{\nu}$
- C_{lam}, m are empirical constants (e.g., for laminar flow over a flat plate with constant temperature: $C_{lam} \approx 0.332$, $m = 1/2$).

- Correspondingly, the local heat transfer coefficient:

$$
H_{lam}(x) = \frac{Nu_{lam}(x) \, k}{x}
$$

where k is the thermal conductivity.

Turbulent Region

- For turbulent boundary layers, empirical relations such as the Dittus-Boelter or Colburn analogy provide:

$$
Nu_{turb}(x) = C_{turb} \, Re_x^{n}
$$

with different constants C_{turb}, n. For turbulent flow over a flat plate:

$$
Nu_{turb}(x) \approx 0.0296 \, Re_x^{0.8} Pr^{1/3}
$$

where Pr is the Prandtl number.

- The local heat transfer coefficient:

$$
H_{turb}(x) = \frac{Nu_{turb}(x) \, k}{x}
$$

Step 3: Integrating Over the Boundary Layer

The average $\grave{H}_l$ and $N\bar{u}_l$ are obtained by integrating the local quantities over their respective regions:

$$
\grave{H}_L = \frac{1}{L} \left(\int_0^{x_t} H_{lam}(x) \, dx + \int_{x_t}^L H_{turb}(x) \, dx \right)
$$

$$
N\bar{u}_L = \frac{1}{L} \left(\int_0^{x_t} Nu_{lam}(x) \, dx + \int_{x_t}^L Nu_{turb}(x) \, dx \right)
$$

Substituting the expressions for H and Nu, and integrating term-by-term:

- For the laminar region:

$$
\int_0^{x_t} Nu_{lam}(x) \, dx = \int_0^{x_t} C_{lam} \left(\frac{U x}{\nu} \right)^{m} dx
$$

- For the turbulent region:

$$
\int_{x_t}^L Nu_{turb}(x) \, dx = \int_{x_t}^L C_{turb} \left(\frac{U x}{\nu} \right)^{n} dx
$$

The integrals involve power functions of x, leading to:

$$
\int x^{p} dx = \frac{x^{p+1}}{p+1}
$$

Applying this:

$$
\int_0^{x_t} Nu_{lam}(x) dx = C_{lam} \left(\frac{U}{\nu} \right)^m \frac{x_t^{m+1}}{m+1}
$$

Similarly for the turbulent region.

Step 4: Final Expressions for Average Quantities

Putting it all together, the average Nusselt number over the entire length is:

$$
\bar{Nu}_L = \frac{1}{L} \left[C_{lam} \left(\frac{U}{\nu} \right)^m \frac{x_t^{m+1}}{m+1} + C_{turb} \left(\frac{U}{\nu} \right)^n \frac{L^{n+1} - x_t^{n+1}}{n+1} \right]
$$

Correspondingly, the average heat transfer coefficient:

$$
\bar{H}_L = \frac{1}{L} \left[\frac{Nu_{lam}(x)}{x} \text{ integrated over } 0 \to x_t + \frac{Nu_{turb}(x)}{x} \text{ integrated over } x_t \to L \right]
$$

which simplifies to similar forms as above, replacing Nu with H.

Step 5: Incorporating the Transition Point x_t

The transition point x_t is critical:

- Its position depends on flow parameters such as Reynolds number, surface roughness, free-stream turbulence, and other factors.
- Empirical correlations or stability analyses often determine x_t.

Once x_t is known, the expressions above provide a direct way to compute the average heat transfer and Nusselt number.

Additional Considerations

Effect of Variable Properties

- The derivations assume constant properties. In real applications, properties like k, ν, and Pr may vary with temperature, requiring correction factors or numerical integration.

Transition Criteria

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