Find The Area Of The Region Enclosed Between Y = 4 Sin(2) And Y = 4 Cos(2) From X = 0 To X = 0.57. Hint:

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Understanding how to find the area enclosed between two curves is a fundamental skill in calculus. In this article, we will explore the process step-by-step to determine the area bounded by the functions $y = 4 \sin(2x)$ and $y = 4 \cos(2x)$ over the interval from $x = 0$ to $x = 0.57$. We will cover the necessary concepts, the steps involved in the calculation, and provide insights into why these methods work, making it easier for learners to grasp the topic.

Understanding the Problem: The Curves and the Interval

When asked to find the area enclosed between two curves, it’s essential to understand the functions involved and the specific interval over which the area is to be calculated.

The Given Functions

- $y = 4 \sin(2x)$
- $y = 4 \cos(2x)$

Both are trigonometric functions scaled by a factor of 4, with arguments involving $2x$. These functions are periodic and oscillate between -4 and 4.

The Interval of Interest

- From $x = 0$ to $x = 0.57$

This narrow interval suggests the area will be relatively small and close to the origin.

Key Concepts for Calculating the Enclosed Area

Before proceeding with the calculations, it’s important to review some core concepts.

1. Intersection Points of the Curves

- To find the region enclosed, identify where the two curves intersect.
- At intersection points, $y = 4 \sin(2x) = 4 \cos(2x)$.

2. The Integral Setup

- The area between two curves y_1 and y_2 over x from a to b is given by:

$$
\text{Area} = \int_a^b | y_1 - y_2 | \, dx
$$

- When the curves cross, the order of which is on top changes, so we need to split the integral at the intersection points and take the absolute value accordingly.

3. Symmetry and Behavior of the Functions

- The sine and cosine functions are phase-shifted versions of each other.
- Their points of intersection depend on their arguments.

Step-by-Step Approach to Find the Enclosed Area

Let’s now outline the steps involved in calculating the area.

Step 1: Find the Intersection Points

- Set $4 \sin(2x) = 4 \cos(2x)$, which simplifies to:

$$
\sin(2x) = \cos(2x)
$$

- Dividing both sides by $\cos(2x)$ (where $\cos(2x) \neq 0$):

$$
\tan(2x) = 1
$$

- Solve for $2x$:

$$
2x = \arctan(1) + n\pi
$$

\[

$$
2x = \frac{\pi}{4} + n\pi
$$

- For $x \in [0, 0.57]$, determine the relevant solutions:

$$
x = \frac{\pi}{8} + \frac{n\pi}{2}
$$

- Calculate x for $n=0$ and $n=1$:

- For $n=0$:

$$
x = \frac{\pi}{8} \approx 0.3927
$$

- For $n=1$:

$$
x = \frac{\pi}{8} + \frac{\pi}{2} \approx 0.3927 + 1.5708 = 1.9635
$$

Since $1.96 > 0.57$, only $x \approx 0.3927$ falls within our interval.

- Therefore, the curves intersect at approximately $x \approx 0.3927$.

Step 2: Set Up the Integral

- The interval from $x=0$ to $x=0.57$ is split into two parts:

1. From $x=0$ to $x \approx 0.3927$, determine which function is on top.
2. From $x \approx 0.3927$ to $x=0.57$, determine the top function again.

- Evaluate the functions at these points:

- At $x=0$:

$$
y_{1} = 4 \sin(0) = 0
$$

$$
y_{2} = 4 \cos(0) = 4
$$

So, $y=4 \cos(2x)$ is on top initially.

- At $x=0.2$:

$$
y_{1} = 4 \sin(0.4) \approx 4 \times 0.3894 \approx 1.5576
$$

$$
y_{2} = 4 \cos(0.4) \approx 4 \times 0.9211 \approx 3.6844
$$

So, $y=4 \cos(2x)$ remains on top.

- At $x=0.5$:

$$
y_{1} = 4 \sin(1) \approx 4 \times 0.8415 \approx 3.366
$$

$$
y_{2} = 4 \cos(1) \approx 4 \times 0.5403 \approx 2.1612
$$

Now, $y=4 \sin(2x)$ is on top after the intersection point.

- This confirms the change in the top function occurs at $x \approx 0.3927$.

Step 3: Write the Integral Expression

- The total enclosed area:

$$
\text{Area} = \int_0^{0.3927} [4 \cos(2x) - 4 \sin(2x)] \, dx + \int_{0.3927}^{0.57} [4 \sin(2x) - 4 \cos(2x)] \, dx
$$

- Simplify the integrals:

$$
\text{Area} = 4 \int_0^{0.3927} [\cos(2x) - \sin(2x)] \, dx + 4 \int_{0.3927}^{0.57} [\sin(2x) - \cos(2x)] \, dx
$$

Step 4: Calculate the Integrals

- Focus on the indefinite integral:

$$
\int \cos(2x) \, dx = \frac{1}{2} \sin(2x) + C
$$

\[

\int \sin(2x) \, dx = -\frac{1}{2} \cos(2x) + C
\]

- Applying these:

- First integral:

\[
4 \left[\frac{1}{2} \sin(2x) + \frac{1}{2} \cos(2x) \right] = 2 \sin(2x) + 2 \cos(2x)
\]

- Second integral:

\[
4 \left[-\frac{1}{2} \cos(2x) - \frac{1}{2} \sin(2x) \right] = -2 \cos(2x) - 2 \sin(2x)
\]

- Evaluate these expressions at the bounds:

- At \(x=0.3927 \):

\[
2 \sin(2 \times 0.3927) + 2 \cos(2 \times 0.3927)
\]

Compute:

\[
2 \sin(0.7854) + 2 \cos(0.7854)
\]

\[
2 \times 0.7071 + 2 \times 0.7071 = 1.4142 + 1.4142 = 2.8284
\]

- At \(x=0 \):

\[
2 \sin(0) + 2 \cos(0) = 0 + 2 = 2
\]

- The

Frequently Asked Questions

What is the main goal when finding the area of the region

enclosed between Y = 4 sin(2x) and Y = 4 cos(2x) from x = 0 to x = 0.57?

The main goal is to calculate the total area between the two curves over the specified interval, which involves integrating the difference of the functions where one is above the other.

How do you determine which function is on top between Y = 4 sin(2x) and Y = 4 cos(2x) within the interval?

You compare the values of 4 sin(2x) and 4 cos(2x) at points within the interval or analyze their intersection points to identify which function is greater at each sub-interval.

What is the significance of the hint provided in the problem regarding finding the area?

The hint likely suggests analyzing the points where the two functions intersect or points where their difference changes sign, which is essential for setting up the correct integral bounds for the area calculation.

How can we find the points where Y = 4 sin(2x) and Y = 4 cos(2x) intersect within the interval?

Set 4 sin(2x) = 4 cos(2x) and solve for x, which simplifies to sin(2x) = cos(2x); solving this gives the intersection points, typically at $x = \pi/8 + n\pi/2$.

What is the integral setup to find the enclosed area between the two curves?

The area is given by the integral of the absolute difference of the functions over the interval: Area = ∫ from a to b |4 sin(2x) - 4 cos(2x)| dx, split at points where the functions cross if necessary.

Why is it important to consider the points of intersection when calculating the area?

Because the relative position of the curves may change at intersection points, requiring splitting the integral into sub-intervals where one function is consistently on top to accurately compute the area.

What techniques can be used to evaluate the integral of |4 sin(2x) - 4 cos(2x)| dx over the interval?

You can evaluate the integral by first determining which function is on top in each sub-interval, removing the absolute value accordingly, and then integrating directly or using substitution techniques.

How does understanding the periodicity of sine and cosine functions help in solving this problem?

Since sine and cosine are periodic with a period of π, understanding their behavior helps predict where they intersect and how to split the integral for accurate area calculation within the given interval.

What is the final step after setting up the integrals for the different regions to find the total enclosed area?

The final step is to evaluate the integrals over each sub-interval and sum their absolute values to obtain the total area enclosed between the two curves.

Additional Resources

Find The Area Of The Region Enclosed Between Y = 4 Sin(2x) And Y = 4 Cos(2x) From x = 0 To x = 0.57 is a classic problem in calculus that combines understanding of functions, intersections, and integration techniques to determine the enclosed area between two curves over a specified interval. This type of problem is fundamental in analyzing the geometric relationships of trigonometric functions, and solving it provides insight into the practical application of calculus concepts such as definite integrals, function intersections, and area calculations.

Understanding the Problem

Before diving into calculations, it’s essential to interpret what the problem is asking:

- Given functions:
$y = 4 \sin(2x)$ and $y = 4 \cos(2x)$.

- Interval:
From $x = 0$ to $x = 0.57$.

- Goal:
Find the area of the region enclosed between these two curves within the specified interval.

Step 1: Visualizing the Functions and Their Intersection Points

Graphical Insight

Visualizing the two functions helps understand the behavior of the curves:

- Both are sinusoidal functions with amplitude 4.
- The functions oscillate between -4 and 4.
- Their period is $\frac{\pi}{2}$ because the argument is $2x$, and the period of $\sin(kx)$ or

$\cos(kx)$ is $\frac{2\pi}{k}$. Here, $k = 2$, so the period is π.

Intersection Points

To find where the curves intersect within the interval, solve:

$$
4 \sin(2x) = 4 \cos(2x)
$$

Dividing both sides by 4:

$$
\sin(2x) = \cos(2x)
$$

Recall that:

$$
\sin(\theta) = \cos(\theta) \implies \tan(\theta) = 1
$$

So:

$$
2x = \frac{\pi}{4} + n\pi
$$

for integers n.

Within the interval $x \in [0, 0.57]$, the relevant solutions are:

$$
2x = \frac{\pi}{4} \Rightarrow x = \frac{\pi}{8} \approx 0.3927
$$

and the next one:

$$
2x = \frac{\pi}{4} + \pi = \frac{\pi}{4} + \frac{4\pi}{4} = \frac{5\pi}{4}
$$

which corresponds to:

$$
x = \frac{5\pi}{8} \approx 1.9635
$$

but since $1.9635 > 0.57$, it is outside our interval.

Therefore, the only intersection point within the interval is at approximately $x = 0.3927$.

Step 2: Determining Which Function Is Above the Other

To find the enclosed area, identify which function is on top between the interval endpoints and the intersection point.

- At $x=0$:

$$
y_{sin} = 4 \sin(0) = 0
$$
$$
y_{cos} = 4 \cos(0) = 4
$$

Since $4 > 0$, the cosine function starts above the sine function at $x=0$.

- At $x=0.57$:

Calculate:

$$
y_{sin} \approx 4 \sin(2 \times 0.57) = 4 \sin(1.14)
$$
$$
\sin(1.14) \approx 0.9093
$$
$$
y_{sin} \approx 4 \times 0.9093 \approx 3.637
$$

$$
y_{cos} \approx 4 \cos(1.14)
$$
$$
\cos(1.14) \approx 0.4161
$$
$$
y_{cos} \approx 4 \times 0.4161 \approx 1.664
$$

At $x=0.57$, y_{sin} is still above y_{cos}, indicating that between 0 and 0.3927, the cosine curve is on top, and after that, the sine curve becomes on top.

Step 3: Setting Up the Integral for Enclosed Area

The total area enclosed between the two functions over $x \in [0, 0.57]$ can be computed by splitting the interval at the intersection point:

$$
\text{Area} = \int_{0}^{x_{intersect}} \left[\text{top function} - \text{bottom function} \right] dx + \int_{x_{intersect}}^{0.57} \left[\text{top function} - \text{bottom function} \right] dx
$$

Where:

- $x_{intersect} \approx 0.3927$

Between:

- $0 \leq x \leq 0.3927$: $y=4 \cos(2x)$ is on top.
- $0.3927 \leq x \leq 0.57$: $y=4 \sin(2x)$ is on top.

Thus:

$$
\boxed{
\text{Area} = \int_0^{0.3927} \left[4 \cos(2x) - 4 \sin(2x) \right] dx + \int_{0.3927}^{0.57} \left[4 \sin(2x) - 4 \cos(2x) \right] dx
}
$$

Step 4: Computing the Integrals

Simplify the integrals:

$$
\text{Area} = 4 \int_0^{0.3927} [\cos(2x) - \sin(2x)] dx + 4 \int_{0.3927}^{0.57} [\sin(2x) - \cos(2x)] dx
$$

Let's focus on the indefinite integral:

$$
I = \int (\cos(2x) - \sin(2x)) dx
$$

Recall:

$$
\int \cos(2x) dx = \frac{1}{2} \sin(2x)
$$
$$
\int \sin(2x) dx = -\frac{1}{2} \cos(2x)
$$

Therefore:

\[
I = \frac{1}{2} \sin(2x) + \frac{1}{2} \cos(2x) + C
\]

Similarly, the second integral is:

\[
J = \int (\sin(2x) - \cos(2x)) dx = -\frac{1}{2} \cos(2x) - \frac{1}{2} \sin(2x) + C
\]

Step 5: Evaluating the Definite Integrals

First integral:

\[
4 \left[\frac{1}{2} \sin(2x) + \frac{1}{2} \cos(2x) \right]_{0}^{0.3927}
\]

\[
= 2 \left[\sin(2x) + \cos(2x) \right]_{0}^{0.3927}
\]

Calculate at the bounds:

- At \(x = 0.3927 \):

\[
2x \approx 0.7854 \, \text{radians} \ (\text{which is } \frac{\pi}{4})
\]

\[
\sin(0.7854) \approx 0.7071
\]
\[
\cos(0.7854) \approx 0.7071
\]

Sum:

\[
0.7071 + 0.7071 = 1.4142
\]

- At \(x=0 \):

\[
2 \times 0 = 0
\]
\[

\sin(0) = 0
\]
\[
\cos(0) = 1
\]
Sum:

\[
0 + 1 = 1
\]

So the first integral:

\[
= 2 \times (1.4142 - 1) = 2 \times 0.4142 = 0.8284
\]

Second integral:

\[
4 \left[-\frac{1}{2} \cos(2x) - \frac{1}{2} \sin(2x) \right]_{0.3927}^{0.57}
\]

\[
= -2 \left[\cos(2x) + \sin(2x) \right]_{0.3927}^{0.57}
\]

Calculate at \(x=0.57 \):

\[
2 \times 0.57 = 1.14
\]
\[
\cos(1.14) \approx 0.4161
\]
\[
\sin(1.14

Find The Area Of The Region Enclosed Between Y 4 Sin 2 And Y 4 Cos 2 From X 0 To X 0 57 Hint

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