Find The Charge On The Capacitor In An LRC-series Circuit At T = 0.03s When L = 0.05 H, R = 3, C = 0.008

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Understanding the behavior of an LRC series circuit is essential for students and professionals working in electronics and electrical engineering. Specifically, calculating the charge on the capacitor at a particular time involves analyzing the circuit's transient response, which is governed by differential equations that describe the interplay between inductance, resistance, and capacitance. This article provides a comprehensive guide to find the charge on the capacitor at T = 0.03 seconds for the given circuit parameters: inductance (L) = 0.05 H, resistance (R) = 3 Ω, and capacitance (C) = 0.008 F. We will explore the fundamental concepts, the mathematical formulation, step-by-step calculations, and practical insights to understand and solve this problem effectively.

Overview of LRC-Series Circuits

An LRC-series circuit comprises three fundamental components connected in series: an inductor (L), a resistor (R), and a capacitor (C). When connected to a voltage source, such as a battery or an AC supply, the circuit exhibits complex behaviors characterized by oscillations, damping, and transient phenomena.

Key Components of an LRC Circuit

- Inductor (L): Stores energy in a magnetic field when current flows through it.
- Resistor (R): Dissipates energy as heat, providing damping to oscillations.
- Capacitor (C): Stores energy in an electric field, influencing the circuit's frequency response.

Significance of the Circuit's Transient Response

The transient response describes how the circuit's current and voltage change over time after an initial disturbance (like switching on the circuit). It is particularly relevant for calculating the charge on the capacitor at specific moments, such as T = 0.03 seconds.

Mathematical Foundations of the Circuit

To find the charge on the capacitor at a specific time, we need to understand the mathematical modeling of the circuit's behavior.

Differential Equation Governing the Circuit

For a series LRC circuit with an initial charge Q_0 on the capacitor, the governing differential equation is:

$$
L \frac{d^2Q}{dt^2} + R \frac{dQ}{dt} + \frac{Q}{C} = 0
$$

where:

- $Q(t)$ is the charge on the capacitor at time t.
- $\frac{dQ}{dt}$ is the current $I(t)$.
- $\frac{d^2Q}{dt^2}$ relates to the rate of change of current.

Solution to the Differential Equation

The solution depends on the damping factor, which is determined by the roots of the characteristic equation:

$$
L r^2 + R r + \frac{1}{C} = 0
$$

The roots are:

$$
r_{1,2} = \frac{-R \pm \sqrt{R^2 - 4 L / C}}{2L}
$$

- Underdamped case: when $R^2 < 4L / C$, the roots are complex conjugates, leading to oscillatory behavior.
- Critically damped: when $R^2 = 4L / C$.
- Overdamped: when $R^2 > 4L / C$.

Given the parameters:

$$
L = 0.05\, H, \quad R = 3\, \Omega, \quad C = 0.008\, F
$$

Calculate the damping condition.

Calculating the Roots and Damping Condition

Step 1: Compute the Discriminant

$$
D = R^2 - 4 \frac{L}{C}
$$

Substituting values:

$$
D = (3)^2 - 4 \times 0.05 / 0.008
$$

Calculate step-by-step:

$$
4 \times 0.05 / 0.008 = 4 \times 6.25 = 25
$$

Thus:

$$
D = 9 - 25 = -16
$$

Since $D < 0$, the roots are complex conjugates, indicating an underdamped oscillatory response.

Step 2: Determine the Roots

$$
r_{1,2} = \frac{-R \pm j \sqrt{4L/C - R^2}}{2L}
$$

Calculate the imaginary part:

$$
\sqrt{4L / C - R^2} = \sqrt{25 - 9} = \sqrt{16} = 4
$$

Now, find the roots:

$$
r_{1,2} = \frac{-3 \pm j 4}{2 \times 0.05} = \frac{-3 \pm j 4}{0.1}
$$

Simplify:

$$
r_{1,2} = -30 \pm j 40
$$

The General Solution for Charge $Q(t)$

Given the underdamped nature, the general solution takes the form:

$$
Q(t) = e^{-\alpha t} \left(A \cos \omega_d t + B \sin \omega_d t \right)
$$

where:

- $\alpha = \frac{R}{2L} = 30$ (damping factor)
- $\omega_0 = \frac{1}{\sqrt{LC}}$ (natural frequency)
- $\omega_d = \sqrt{\omega_0^2 - \alpha^2}$ (damped angular frequency)

Step 1: Calculate ω_0

$$
\omega_0 = \frac{1}{\sqrt{L C}} = \frac{1}{\sqrt{0.05 \times 0.008}} = \frac{1}{\sqrt{0.0004}} = \frac{1}{0.02} = 50\, \text{rad/sec}
$$

Step 2: Calculate ω_d

$$
\omega_d = \sqrt{\omega_0^2 - \alpha^2} = \sqrt{50^2 - 30^2} = \sqrt{2500 - 900} = \sqrt{1600} = 40\, \text{rad/sec}
$$

Determining Constants A and B

To find A and B, initial conditions are necessary. Typically, for a circuit initially uncharged and at rest:

- $Q(0) = Q_0$ (initial charge on the capacitor)
- $I(0) = \frac{dQ}{dt}\big|_{t=0} = 0$ (initial current is zero)

Step 1: Assume initial charge Q_0

If not specified, consider that the capacitor is initially charged to some known value, or zero. For this problem, since no initial charge is provided, assume $Q_0 = 0$.

Step 2: Apply initial conditions

- At $t=0$:

\[

Q(0) = e^{0} (A \times 1 + B \times 0) = A
\]

Thus:

\[
A = Q_0
\]

- The derivative:

\[
\frac{dQ}{dt} = e^{-\alpha t} \left[-\alpha A \cos \omega_d t - A \omega_d \sin \omega_d t + \alpha B \sin \omega_d t + B \omega_d \cos \omega_d t \right]
\]

At \(t=0 \):

\[
\frac{dQ}{dt}\big|_{t=0} = -\alpha A + \omega_d B
\]

Given \(I(0) = 0 \):

\[
0 = -\alpha A + \omega_d B
\]

Solve for \(B \):

\[
B = \frac{\alpha A}{\omega_d}
\]

Since \(A = Q_0 \), if initial charge is zero (\(Q_0 = 0 \)), then:

\[
A = 0 \Rightarrow B = 0
\]

Alternatively, if initial charge \(Q_0 \) is known, these constants can be calculated accordingly.

Calculating the Charge at \(T = 0.03\, \text{s} \)

Assuming initial charge \(Q_0 = 0 \), the simplified solution becomes:

\[

Q(t) = 0
\]

indicating no charge if initially uncharged and no external voltage is applied. But in a realistic scenario, suppose the capacitor was initially charged to \(Q_0 \). For this case, let's assume an initial charge \(Q_0 \).

Step 1: Example initial charge \(Q_0 \)

Suppose the capacitor was initially charged to 1 μC:

\[
Q_0 = 1 \times 10^{-6}\, C
\]

Step 2: Constants \(A \) and \(B \)

\[
A = Q

Frequently Asked Questions

What is the initial charge on the capacitor in an LRC series circuit at t = 0.03 s with given parameters?

The initial charge on the capacitor can be found using the formula $Q(t) = Q_0 e^{(-Rt/2L)} \cos(\omega_d t)$, but additional information about initial voltage or charge is needed. If the circuit starts uncharged, the charge at $t = 0.03$ s depends on the initial conditions and excitation source.

How do you calculate the damped angular frequency (ω_d) in an LRC circuit with L = 0.05 H, R = 3 Ω, and C = 0.008 F?

The damped angular frequency is given by $\omega_d = \sqrt{1/LC - (R/2L)^2}$. Substituting values: $\omega_d = \sqrt{1/(0.050.008) - (3/20.05)^2}$.

What is the formula to find the charge on the capacitor at a specific time in an underdamped LRC circuit?

The charge is given by $Q(t) = Q_0 e^{(-Rt/2L)} \cos(\omega_d t + \varphi)$, where φ is the phase constant determined by initial conditions.

How does the resistance R affect the charge on the capacitor at t = 0.03 s?

Higher R values increase damping, leading to faster decay of oscillations and a reduced charge amplitude at any given time, including $t = 0.03$ s.

What is the significance of the time t = 0.03 seconds in analyzing the capacitor's charge?

This specific time allows us to evaluate the capacitor's charge during the transient response of the circuit, showing how oscillations decay over time.

Can the charge on the capacitor be calculated without initial conditions? If not, what additional data is needed?

No, initial conditions such as initial charge or voltage are necessary to compute the charge at a specific time. Without them, only the general solution can be provided.

How do you determine the maximum charge on the capacitor in an LRC circuit?

The maximum charge occurs at the initial moment if the circuit is initially charged. For an uncharged circuit, the maximum charge depends on the energy supplied by the source and circuit damping.

What role does the inductance L play in the charge dynamics of the capacitor at t = 0.03 s?

Inductance L influences the oscillation frequency and the rate at which current and charge change; higher L results in lower frequency and slower charge variation.

How do you interpret the behavior of the capacitor's charge at t = 0.03 s in a damped LRC circuit?

At t = 0.03 s, the charge on the capacitor reflects the damping effect, showing reduced amplitude compared to initial charge, with oscillations gradually decreasing over time.

Additional Resources

Find The Charge On The Capacitor In An LRC-Series Circuit At T = 0.03s When L = 0.05 H, R = 3 Ω, C = 0.008 F

Introduction

In electrical engineering and circuit analysis, understanding the behavior of RLC series circuits is fundamental. These circuits, composed of a resistor (R), inductor (L), and capacitor (C) connected in series, exhibit complex transient and steady-state responses when subjected to initial conditions and external stimuli like voltage sources. One of the key parameters of interest in such circuits is the charge on the capacitor at a specific instant, which depends on the initial conditions, circuit parameters, and the type of excitation.

This detailed review delves into the process of determining the charge on the capacitor at a specified time — specifically, at T = 0.03 seconds — for a given RLC series circuit with known component values. We will explore the underlying theory, derive the mathematical expressions, analyze the transient response, and apply the appropriate calculations step-by-step.

Understanding the RLC Series Circuit

Basic Configuration

An RLC series circuit consists of:

- Resistor (R): Provides resistance, dissipates energy as heat.
- Inductor (L): Stores energy in magnetic fields, opposes changes in current.
- Capacitor (C): Stores energy in electric fields, opposes voltage changes.

Typically, the circuit may be connected to a voltage source, but for transient analysis, it is often considered with initial energy stored in the inductor or capacitor, or with a step input.

Governing Differential Equation

Applying Kirchhoff’s Voltage Law (KVL) around the loop:

$$ V_R + V_L + V_C = 0 $$

Expressed as:

$$ R i(t) + L \frac{di(t)}{dt} + \frac{1}{C} \int i(t) dt = 0 $$

Differentiating to eliminate the integral:

$$ L \frac{d^2 q(t)}{dt^2} + R \frac{dq(t)}{dt} + \frac{q(t)}{C} = 0 $$

Where:

- $q(t)$: Charge on the capacitor at time t
- $i(t) = \frac{dq(t)}{dt}$: Current in the circuit

This second-order linear differential equation characterizes the circuit dynamics.

Circuit Parameters and Initial Conditions

Given:

- $L = 0.05\, \text{H}$
- $R = 3\, \Omega$
- $C = 0.008\, \text{F}$

The key to solving for $q(t)$ at a specific time is knowing the initial conditions:

- The initial charge on the capacitor, $q(0)$
- The initial current in the circuit, $i(0)$

Note: If the problem specifies initial energy stored or initial current/charge, that info should be incorporated. If not specified, common assumptions include zero initial energy or initial charge.

Step 1: Calculating the Damping Factor and Natural Frequency

Before solving the differential equation, determine the circuit's damping characteristic:

- Resonant or natural frequency:

$$
\omega_0 = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{0.05 \times 0.008}}
$$

Calculate:

$$
\omega_0 = \frac{1}{\sqrt{0.0004}} = \frac{1}{0.02} = 50\, \text{rad/sec}
$$

- Damping coefficient:

$$
\alpha = \frac{R}{2L} = \frac{3}{2 \times 0.05} = \frac{3}{0.1} = 30\, \text{rad/sec}
$$

- Damped natural frequency:

$$
\omega_d = \sqrt{\omega_0^2 - \alpha^2} = \sqrt{50^2 - 30^2} = \sqrt{2500 - 900} = \sqrt{1600} = 40\, \text{rad/sec}
$$

Since $\alpha < \omega_0$, the circuit is underdamped, leading to oscillatory transient behavior.

Step 2: General Solution for Charge $q(t)$

The homogeneous differential equation:

$$
L \frac{d^2 q}{dt^2} + R \frac{dq}{dt} + \frac{q}{C} = 0
$$

has a general solution:

$$
q(t) = e^{-\alpha t} \left(A \cos \omega_d t + B \sin \omega_d t \right)
$$

Where:

- A and B are constants determined by initial conditions.

Step 3: Applying Initial Conditions

Suppose the initial charge on the capacitor is $q(0) = q_0$, and initial current $i(0) = i_0$.

From the general solution:

$$
q(0) = A
$$

And the current:

$$
i(t) = \frac{dq}{dt} = e^{-\alpha t} \left[-\alpha A \cos \omega_d t - A \omega_d \sin \omega_d t + B \left(-\alpha \sin \omega_d t + \omega_d \cos \omega_d t \right) \right]
$$

At $t=0$:

$$
i(0) = \frac{dq}{dt} \bigg|_{t=0} = -\alpha A + \omega_d B
$$

Thus, with known initial charge q_0 and initial current i_0, we can solve for A and B:

$$
A = q_0
$$

$$
B = \frac{i_0 + \alpha q_0}{\omega_d}
$$

Step 4: Calculating $q(t)$ at $T = 0.03\, \text{s}$

With all constants determined, the charge at $t=0.03\, \text{s}$ is:

$$
q(0.03) = e^{-\alpha \times 0.03} \left[A \cos \omega_d \times 0.03 + B \sin \omega_d \times 0.03 \right]
$$

Given the component values:

$$
\alpha = 30\, \text{rad/sec}
$$

$$
\omega_d = 40\, \text{rad/sec}
$$

And assuming initial conditions:

- q_0: initial charge on the capacitor
- i_0: initial current

Note: If specific initial conditions are not provided, common assumptions are:

- Zero initial charge: $q_0 = 0$
- Zero initial current: $i_0 = 0$

In the absence of initial energy, the charge remains zero unless an external source is involved.

Step 5: Numerical Calculation (Assuming Zero Initial Conditions)

Suppose the initial conditions are:

- $q_0 = 0$
- $i_0 = 0$

Then:

$$
A = 0
$$

$$
B = \frac{0 + 30 \times 0}{40} = 0
$$

Hence:

$$
q(0.03) = e^{-30 \times 0.03} \times (0 + 0) = 0
$$

Interpretation: The charge remains zero if initially uncharged and with no energy supplied.

Inclusion of External Voltage or Initial Energy

If the circuit was energized with a voltage source or stored energy initially, the initial conditions would change. For example, if the capacitor was initially charged to Q_0:

$$
q(0) = Q_0
$$

and if the initial current $i(0)$ is known, we can plug in values to get precise A and B.

Additional Considerations

Effect of External Input

- If the circuit was energized with a step voltage source V_s, the particular solution must be added to the homogeneous solution.
- Steady-state charge on the capacitor in DC conditions is $Q_{ss} = C V_s$.

Energy Dissipation

In an underdamped circuit, energy oscillates between magnetic and electric fields, gradually dissipating due to resistance R, leading to exponential decay.

Summary of Key Steps to Find $q(t)$:

1. Calculate ω_0, α, and ω_d.
2. Write the general solution for $q(t)$.
3. Apply initial conditions to find constants A and B.
4. Evaluate $q(t)$ at the specific time $T=0.03\, \text{s}$.

Conclusion

Determining the charge on the capacitor at a specific instant in an RLC series circuit requires a thorough understanding of the circuit’s transient response. Using the differential equations approach, combined with initial conditions and known component values, allows precise calculation. The underdamped nature of the circuit

Find The Charge On The Capacitor In An Lrc Series Circuit At

T 0 03s When L 0 05 H R 3 C 0 008

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