Find The Constant C Such That The Function = F(x) = {cx? 0 < X < 4 Otherwise 0 B- Compute P(1 <

Find The Constant C Such That The Function = F(x) = {cx? 0 < X < 4 Otherwise 0 B- Compute P(1 < X < 2)

In probability theory and statistics, defining a probability density function (pdf) correctly requires ensuring that the total probability over the entire sample space equals 1. When given a piecewise function like F(x) = {cx for 0 < X < 4, otherwise 0}, the primary goal is to find the constant C that makes this function a valid pdf. Once the constant is determined, we can then compute the probability P(1 < X < 2), which involves integrating the pdf over the specified interval.

Understanding the Function and Its Domain

The function in question is defined as:

- F(x) = cx for 0 < X < 4
- F(x) = 0 otherwise

This indicates that the density function is non-zero only within the interval (0, 4). Outside this interval, the function is zero, meaning the probability of the random variable X falling outside (0, 4) is zero.

Our task is two-fold: first, determine the value of C such that the total probability across (0, 4) is 1, and second, compute the probability that X falls between 1 and 2.

Part 1: Finding the Constant C

Step 1: Recall the Properties of a Probability Density Function

For a function to be a valid pdf, it must satisfy:

1. **Non-negativity:** F(x) ≥ 0 for all x
2. **Normalization:** The total probability over the entire domain must be 1:

Mathematically, this is expressed as:

∫ from -∞ to ∞ of F(x) dx = 1

Given the function's support is from 0 to 4, the integral simplifies to:

$\int_0^4 cx\, dx = 1$

Step 2: Set Up the Integral Equation

Calculate the integral:

$$\int_0^4 cx\, dx = c \int_0^4 x\, dx$$

The integral of x over [0, 4] is:

$$\int_0^4 x\, dx = [x^2 / 2]_0^4 = (4^2 / 2) - (0^2 / 2) = (16 / 2) - 0 = 8$$

Therefore:

$$c \quad 8 = 1$$

Step 3: Solve for C

Dividing both sides by 8 gives:

$$c = 1 / 8$$

Thus, the constant C that makes F(x) a valid pdf is **1/8**.

Part 2: Computing P(1 < X < 2)

Step 1: Express the Probability as an Integral

The probability that X falls between 1 and 2 is given by:

$$P(1 < X < 2) = \int_1^2 F(x)\, dx$$

Recall that within the support (0, 4), the pdf is $F(x) = (1/8)\, x$.

Since both 1 and 2 are within (0, 4), we can proceed with the integral:

Step 2: Set Up the Integral with the Determined C

$$P(1 < X < 2) = \int_1^2 (1/8)\, x\, dx$$

Step 3: Calculate the Integral

$$\int_1^2 x\,dx = [x^2 / 2]_1^2 = (2^2 / 2) - (1^2 / 2) = (4 / 2) - (1 / 2) = 2 - 0.5 = 1.5$$

$$P(1 < X < 2) = (1/8) \quad 1.5 = 1.5 / 8 = 0.1875$$

Therefore, the probability that X is between 1 and 2 is **0.1875** or 18.75%.

Summary of Key Results

- The constant C that makes the function a valid probability density function is **1/8**.
- The probability that X falls between 1 and 2 is **0.1875**.

Understanding how to find the constant C involves integrating the function over its support and setting the integral equal to 1. After determining C, calculating probabilities over specific intervals involves integrating the pdf over those intervals. This approach is fundamental in probability theory and essential for accurate statistical analysis.

Additional Considerations and Applications

Implications of the Constant C

- The value of C ensures the total probability across the support (0, 4) sums to 1, satisfying the normalization condition.
- This approach can be extended to other functions and supports by integrating accordingly and solving for the unknown constant.

Applications in Real-World Scenarios

- **Modeling Continuous Data:** Defining suitable pdfs for data such as measurements, durations, or other continuous variables.
- **Probability Computations:** Calculating the likelihood of an event occurring within a specified range, as in the example of $P(1 < X < 2)$.
- **Statistical Inference:** Estimating parameters based on observed data, where such functions are often used as probability models.

Conclusion

Determining the constant C in a piecewise probability density function is a foundational task in probability theory. By integrating the function over its support and equating the result to 1, we ensure the function is a valid pdf. Once C is established, computing probabilities over particular intervals becomes straightforward through integration. In our example, we found C = 1/8, and the probability that X falls between 1 and 2 is 0.1875. These techniques are fundamental for statisticians and data scientists working with continuous probability models.

Frequently Asked Questions

How do you determine the constant C in the function F(x) = cx for 0 < x < 4 such that F(x) is a valid probability density function?

To find C, you integrate F(x) over its domain and set the integral equal to 1: $\int_0^4$ cx dx = 1. Solving this gives C = 1/8.

What is the purpose of finding the constant C in the given function?

The constant C ensures that the function F(x) qualifies as a probability density function by making the total probability over its domain equal to 1.

How do you compute P(1 < X < 4) given the density function F(x) = cx?

First, find C, then compute P(1 < X < 4) by integrating F(x) from 1 to 4: P = $\int_1^4$ cx dx.

What is the value of the constant C for the given probability density function?

C = 1/8, obtained by solving $\int_0^4$ cx dx = 1, which yields C = 1/8.

How do you calculate P(1 < X < 4) once C is known?

Calculate P(1 < X < 4) = $\int_1^4$ (C x) dx = C ((4)^2/2 - (1)^2/2) = C (8 - 0.5) = C 7.5.

What is the significance of the interval 0 < x < 4 in the context of this problem?

The interval 0 < x < 4 defines the domain where the probability density function is non-zero, indicating the range of possible values for the random variable X.

Additional Resources

Find The Constant C Such That The Function = F(x) = {cx for 0 < x < 4, Otherwise 0} and Compute P(1 < X).

In the realm of probability theory, one often encounters the challenge of defining probability density functions (pdfs) that model real-world phenomena. A common task is to determine an unknown constant within a pdf to ensure it adheres to the fundamental property that the total probability over its domain equals 1. Furthermore, once the density function is properly defined, calculating probabilities for specific events becomes straightforward. This article delves into the process of finding the constant C that makes $F(x)$ a valid probability density function, and subsequently, computing the probability $P(1 < X)$.

Understanding the Problem

The function under consideration is defined as:

$$
F(x) =
\begin{cases}
cx, & 0 < x < 4 \\
0, & \text{otherwise}
\end{cases}
$$

Our goals are two-fold:

1. Determine the constant C such that $F(x)$ qualifies as a probability density function.
2. Compute the probability $P(1 < X < \text{some value})$, which involves integrating the density over the specified interval.

Before proceeding, it's important to clarify what a probability density function (pdf) entails. For a function $f(x)$ to be a valid pdf over a domain D:

- $f(x) \geq 0$, for all $x \in D$
- The total integral over the domain equals 1:

$$
\int_D f(x) \, dx = 1
$$

In our case, the domain is $(0, 4)$. So, the first step is to find C such that:

$$
\int_{0}^{4} c x \, dx = 1
$$

Step 1: Finding the Constant C

Setting Up the Integral

Given the density function $f(x) = c x$, the total probability must be 1:

$$
\int_{0}^{4} c x \, dx = 1
$$

Since c is constant, it can be factored out:

$$
c \int_{0}^{4} x \, dx = 1
$$

Calculating the Integral

The integral of x over $[0, 4]$ is:

$$
\int_{0}^{4} x \, dx = \left[\frac{1}{2} x^2 \right]_0^{4} = \frac{1}{2} \times 4^2 - 0 = \frac{1}{2} \times 16 = 8
$$

Solving for C

Plugging back into the equation:

$$
c \times 8 = 1 \implies c = \frac{1}{8}
$$

Result: The constant C is $\frac{1}{8}$, making the probability density function:

$$
f(x) =
\begin{cases}
\frac{x}{8}, & 0 < x < 4 \\
0, & \text{otherwise}
\end{cases}
$$

Step 2: Computing $P(1 < X < \text{some value})$

Now that the density function is established, let's compute the probability $P(1 < X < a)$, where a is a specific value within the domain $(0, 4)$. For the purpose of this discussion, assume we're asked to find $P(1 < X < 3)$.

This involves integrating the density function over the interval $(1, 3)$:

$$
P(1 < X < 3) = \int_{1}^{3} f(x) \, dx = \int_{1}^{3} \frac{x}{8} \, dx
$$

Step 3: Performing the Integration

Set up the integral:

$$
\int_{1}^{3} \frac{x}{8} \, dx
$$

Factor out the constant:

$$
\frac{1}{8} \int_{1}^{3} x \, dx
$$

Compute the integral:

$$
\int_{1}^{3} x \, dx = \left[\frac{1}{2} x^2 \right]_1^{3} = \frac{1}{2} (3^2 - 1^2) = \frac{1}{2} (9 - 1) = \frac{1}{2} \times 8 = 4
$$

Final probability:

$$
P(1 < X < 3) = \frac{1}{8} \times 4 = \frac{4}{8} = \frac{1}{2}
$$

Result: The probability that X falls between 1 and 3 is $\frac{1}{2}$, or 50%.

Broader Implications and Related Calculations

While the specific interval $(1, 3)$ was used for illustrative purposes, the process generalizes to any interval within the domain $(0, 4)$:

$$
P(a < X < b) = \int_{a}^{b} \frac{x}{8} \, dx
$$

which simplifies to:

$$
\frac{1}{8} \times \left[\frac{1}{2} x^2 \right]_a^{b} = \frac{1}{16} (b^2 - a^2)
$$

This formula allows for quick calculations of probabilities over any subinterval within the domain.

Visualizing the Density Function

Understanding the shape of the density function can provide valuable intuition. The function $f(x) = \frac{x}{8}$ over $(0, 4)$ is a linear function starting from 0 at $x=0$ and increasing to a maximum at $x=4$:

- At $x=0$, $f(0) = 0$
- At $x=4$, $f(4) = \frac{4}{8} = \frac{1}{2}$

The graph is a straight line ascending from zero to 0.5, forming a triangle when combined with the x-axis over $(0, 4)$. The total area under this line over the domain is 1, confirming it as a valid pdf.

Practical Applications and Significance

The ability to determine the constant C in a density function is fundamental in probability and statistics, especially when modeling real-world data where the functional form is suggested but parameters are unknown. Once the density is correctly normalized, it becomes a powerful tool to compute probabilities, expectations, variances, and other statistical measures.

For example, suppose an engineer models the time until a machine failure with such a density. Knowing the probability that failure occurs within a certain time frame allows for better maintenance scheduling. Similarly, in finance, modeling asset returns with specific density functions enables risk assessment.

Summary of Key Steps

- Identify the domain over which the density is defined.
- Integrate the unnormalized function over the domain to find the total probability.
- Solve for the constant C by setting the integral equal to 1.
- Use the normalized density function to compute specific probabilities via integration over desired intervals.

Final Thoughts

Determining the constant in a probability density function is a foundational skill in statistics, enabling meaningful interpretation of probabilistic models. The process exemplified here—integrating, solving for the constant, and computing probabilities—is applicable across a wide range of problems involving continuous random variables. Whether analyzing physical phenomena, financial data, or engineering systems, mastering these calculations ensures accurate and insightful results.

In conclusion, for the given function $F(x) = cx$ over $(0, 4)$, the constant C that makes it a valid pdf is $\frac{1}{8}$. Using this, the probability that X falls within specific intervals can be precisely calculated, as demonstrated for $P(1 < X < 3)$, which equals $\frac{1}{2}$. These steps exemplify the essential procedures in probability theory that underpin much of statistical analysis and decision-making.

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