Find The Directional Derivative Of The Function AtPin The Direction Ofv.f(x,y)=x3y3,P(8,5),v=22(i+j)Find

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Understanding how to compute the directional derivative of a function provides valuable insight into how the function changes at a particular point in a specific direction. In this article, we will explore the step-by-step process to find the directional derivative of the function $f(x, y) = x^3 y^3$ at the point $P(8, 5)$, in the direction of the vector $\mathbf{v} = 2 2 (\mathbf{i} + \mathbf{j})$. We'll break down the concepts involved, perform necessary calculations, and clarify each step to ensure a comprehensive understanding.

Understanding the Concept of the Directional Derivative

What Is a Directional Derivative?

The directional derivative measures the rate at which a function $f(x, y)$ changes at a point (x_0, y_0) in the direction of a given vector $\mathbf{v}$. Unlike partial derivatives which measure change along coordinate axes, the directional derivative assesses change in an arbitrary direction.

Mathematically, the directional derivative of f at (x_0, y_0) in the direction of a unit vector $\mathbf{u} = (u_x, u_y)$ is defined as:

$$
D_{\mathbf{u}}f(x_0, y_0) = \lim_{h \to 0} \frac{f(x_0 + h u_x, y_0 + h u_y) - f(x_0, y_0)}{h}
$$

Alternatively, it can be computed using the gradient ∇f:

$$
D_{\mathbf{u}}f(x_0, y_0) = \nabla f(x_0, y_0) \cdot \mathbf{u}
$$

where $\nabla f(x, y) = \left(\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}\right)$.

Step-by-Step Approach to Find the Directional Derivative

1. Find the Gradient $\nabla f(x, y)$

The gradient vector contains the partial derivatives of f:

$$
\nabla f(x, y) = \left(\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}\right)
$$

Given $f(x, y) = x^3 y^3$, compute:

$$
\frac{\partial f}{\partial x} = 3x^2 y^3
$$

$$
\frac{\partial f}{\partial y} = 3x^3 y^2
$$

2. Evaluate the Gradient at the Given Point $P(8, 5)$

Substitute $x=8$, $y=5$:

$$
\frac{\partial f}{\partial x} \bigg|_{(8, 5)} = 3 \times 8^2 \times 5^3
$$

$$
\frac{\partial f}{\partial y} \bigg|_{(8, 5)} = 3 \times 8^3 \times 5^2
$$

Calculate each:
- $8^2 = 64$
- $5^3 = 125$
- $8^3 = 512$
- $5^2 = 25$

Thus:

$$
\frac{\partial f}{\partial x} \bigg|_{(8, 5)} = 3 \times 64 \times 125 = 3 \times 8000 = 24,000
$$

$$
\frac{\partial f}{\partial y} \bigg|_{(8, 5)} = 3 \times 512 \times 25 = 3 \times 12,800 = 38,400
$$

So, the gradient vector at $P(8, 5)$ is:

$$
\nabla f(8, 5) = (24,000, 38,400)
$$

---

3. Normalize the Direction Vector $\mathbf{v}$

Given $\mathbf{v} = 2 2 (\mathbf{i} + \mathbf{j})$. First, interpret this:

- The vector $\mathbf{v}$ is scaled by 2, and in the direction of $\mathbf{i} + \mathbf{j}$.

So:
$$
\mathbf{v} = 2 \times (\mathbf{i} + \mathbf{j}) = 2(1, 1) = (2, 2)
$$

Next, normalize $\mathbf{v}$:
$$
|\mathbf{v}| = \sqrt{2^2 + 2^2} = \sqrt{4 + 4} = \sqrt{8} = 2 \sqrt{2}
$$

The unit vector $\mathbf{u}$ in the direction of $\mathbf{v}$ is:
$$
\mathbf{u} = \frac{\mathbf{v}}{|\mathbf{v}|} = \left(\frac{2}{2 \sqrt{2}}, \frac{2}{2 \sqrt{2}}\right) = \left(\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right)
$$

---

4. Compute the Directional Derivative

Using the gradient and the unit vector:
$$
D_{\mathbf{u}}f(8, 5) = \nabla f(8, 5) \cdot \mathbf{u}
$$
$$
= (24,000, 38,400) \cdot \left(\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right)
$$
$$
= 24,000 \times \frac{1}{\sqrt{2}} + 38,400 \times \frac{1}{\sqrt{2}} = \frac{24,000 + 38,400}{\sqrt{2}}
$$

$$
= \frac{62,400}{\sqrt{2}} = 62,400 \times \frac{\sqrt{2}}{2} = 62,400 \times \frac{\sqrt{2}}{2}
$$

$$
= 31,200 \sqrt{2}
$$

Final Result and Interpretation

The directional derivative of the function $f(x, y) = x^3 y^3$ at the point $P(8, 5)$ in the direction of the vector $\mathbf{v} = 2 \mathbf{i} + 2 \mathbf{j}$ is:

$$
\boxed{
D_{\mathbf{u}}f(8, 5) = 31,200 \sqrt{2}
}
$$

This value indicates the rate at which the function increases (or decreases) at the point $P(8, 5)$ when moving in the specified direction. A positive value signifies an increase, while a negative value (not in this case) would indicate a decrease.

Additional Insights and Applications

Why Is the Directional Derivative Important?

The directional derivative helps in understanding the behavior of multivariable functions, especially in optimization problems, gradient-based algorithms, and in fields like physics and engineering where the rate of change in specific directions is critical.

Applications Include:

- Finding the maximum rate of increase of a function at a point (the magnitude of the gradient).

- Determining the direction in which a function increases most rapidly.
- Designing gradients for optimization algorithms like gradient descent.
- Modeling physical phenomena such as heat transfer or fluid flow where directional rates matter.

Summary

In this article, we've outlined the process to compute the directional derivative of a function at a specific point in a given direction. From calculating the gradient, normalizing the direction vector, to applying the dot product, each step is crucial. Mastering these steps enhances your understanding of multivariable calculus and its applications across various scientific disciplines.

In conclusion, the key steps to find the directional derivative are:

1. Compute the gradient \(\nabla f\).
2. Evaluate the gradient at the point of interest.
3. Normalize the direction vector to obtain a unit vector.
4. Take the dot product of the gradient and the unit vector.

By applying these principles, you can analyze how functions change in any specified direction, empowering you to solve complex problems involving multivariable functions efficiently.

Frequently Asked Questions

What is the general formula for the directional derivative of a function at a point in a given direction?

The directional derivative of a function f at point P in the direction of a unit vector v is given by $D_v f(P) = \nabla f(P) \cdot \hat{v}$, where $\nabla f(P)$ is the gradient of f at P and $\hat{v}$ is the unit vector in the direction of v.

How do you compute the gradient of the function $f(x, y) = x^3 y^3$?

The gradient $\nabla f(x, y)$ is found by taking partial derivatives: $\partial f/\partial x = 3x^2 y^3$ and $\partial f/\partial y = 3x^3 y^2$, so $\nabla f(x, y) = (3x^2 y^3, 3x^3 y^2)$.

What is the process to find the directional derivative at point P(8,5) in the direction of vector v = 22(i + j)?

First, evaluate the gradient at P: $\nabla f(8,5)$. Then, normalize the given vector v to a unit vector $\hat{v}$. Finally, compute the dot product of $\nabla f(8,5)$ and $\hat{v}$ to find the directional derivative.

How do you normalize the vector v = 22(i + j) to get the unit vector $\hat{v}$?

Calculate the magnitude of v: $|v| = \sqrt{(22)^2 + (22)^2} = \sqrt{(484 + 484)} = \sqrt{968} \approx 31.11$. Then, $\hat{v} = (1/|v|)\, v = (22/31.11, 22/31.11)$.

What are the steps to compute the directional derivative of f at P(8,5) in the given direction?

Step 1: Compute $\nabla f(8,5)$. Step 2: Normalize v to get $\hat{v}$. Step 3: Calculate the dot product $\nabla f(8,5) \cdot \hat{v}$. The result is the directional derivative at P in that direction.

What is the significance of the directional derivative in multivariable calculus?

The directional derivative measures the rate of change of a function in a specific direction at a given point, indicating how the function increases or decreases along that direction.

Can the directional derivative be zero, and what does that imply about the function at that point in the given direction?

Yes, the directional derivative can be zero if the function does not change in that direction at the point, implying a potential local maximum, minimum, or saddle point in that direction.

Additional Resources

Directional Derivative

Understanding how a function changes as we move in different directions is fundamental in multivariable calculus, especially when analyzing the behavior of surfaces, optimizing functions, or modeling real-world phenomena. The directional derivative provides a precise measure of this rate of change at a specific point in a specified direction. In this article, we will explore how to compute the directional derivative of the function $f(x, y) = x^3 y^3$ at the point $P(8, 5)$ in the direction of the vector $\mathbf{v} = 2 \mathbf{i} + 2 \mathbf{j}$.

Understanding the Concept of Directional Derivative

The directional derivative of a function $f(x, y)$ at a point $P = (x_0, y_0)$ in the direction of a unit vector $\mathbf{u} = \langle u_x, u_y \rangle$ quantifies how fast the function changes as we move from P in that specific direction. It is a generalization of partial derivatives, which consider only the change along coordinate axes.

Mathematically, the directional derivative $D_{\mathbf{u}} f (P)$ is defined as:

$$
D_{\mathbf{u}} f (P) = \lim_{h \to 0} \frac{f(x_0 + h u_x, y_0 + h u_y) - f(x_0, y_0)}{h}
$$

Alternatively, it can be computed using the gradient of f:

$$
D_{\mathbf{u}} f (P) = \nabla f (P) \cdot \mathbf{u}
$$

where:

- $\nabla f (P)$ is the gradient vector of f at P — comprising partial derivatives.
- $\mathbf{u}$ is the unit vector in the specified direction.

Step-by-Step Calculation of the Directional Derivative

To compute the directional derivative, we need to follow several key steps:

1. Calculate the gradient $\nabla f(x, y)$.
2. Evaluate the gradient at the point $P(8, 5)$.
3. Normalize the given direction vector $\mathbf{v}$ to obtain a unit vector $\mathbf{u}$.
4. Compute the dot product of the gradient and the unit vector to find the directional derivative.

Let's examine each step carefully.

Step 1: Find the Gradient \(\nabla f(x, y) \)

Given \(f(x, y) = x^3 y^3 \), we compute the partial derivatives:

- Partial derivative with respect to x:

\[
\frac{\partial f}{\partial x} = 3 x^2 y^3
\]

- Partial derivative with respect to y:

\[
\frac{\partial f}{\partial y} = 3 y^2 x^3
\]

Thus, the gradient vector is:

\[
\nabla f(x, y) = \left\langle 3 x^2 y^3, \quad 3 y^2 x^3 \right\rangle
\]

Step 2: Evaluate the Gradient at \(P(8, 5) \)

Substitute \(x=8 \) and \(y=5 \):

- For the x-component:

\[
3 \times (8)^2 \times (5)^3 = 3 \times 64 \times 125
\]

Calculate:

\[
64 \times 125 = 8000

$$
3 \times 8000 = 24,000
$$

- For the y-component:

$$
3 \times (5)^2 \times (8)^3 = 3 \times 25 \times 512
$$

Calculate:

$$
25 \times 512 = 12800
$$

$$
3 \times 12800 = 38400
$$

Therefore,

$$
\nabla f(8, 5) = \langle 24,000, 38,400 \rangle
$$

Step 3: Normalize the Direction Vector $\mathbf{v} = 2 \mathbf{i} + 2 \mathbf{j}$

Given:

$$
\mathbf{v} = \langle 2, 2 \rangle
$$

Calculate its magnitude:

$$
|\mathbf{v}| = \sqrt{2^2 + 2^2} = \sqrt{4 + 4} = \sqrt{8} = 2 \sqrt{2}
$$

\]

Normalize \(\mathbf{v} \) to obtain the unit vector \(\mathbf{u} \):

\[
\mathbf{u} = \frac{\mathbf{v}}{|\mathbf{v}|} = \left\langle \frac{2}{2\sqrt{2}}, \frac{2}{2\sqrt{2}} \right\rangle = \left\langle \frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}} \right\rangle
\]

Step 4: Compute the Directional Derivative

Using the formula:

\[
D_{\mathbf{u}} f (P) = \nabla f (P) \cdot \mathbf{u}
\]

Calculate the dot product:

\[
(24,000) \times \frac{1}{\sqrt{2}} + (38,400) \times \frac{1}{\sqrt{2}} = \frac{1}{\sqrt{2}} (24,000 + 38,400)
\]

Sum inside the parentheses:

\[
24,000 + 38,400 = 62,400
\]

Thus, the directional derivative:

\[
D_{\mathbf{u}} f (8, 5) = \frac{62,400}{\sqrt{2}} = 62,400 \times \frac{\sqrt{2}}{2} = 62,400 \times \frac{\sqrt{2}}{2}
\]

Simplify:

\[
= 62,400 \times \frac{\sqrt{2}}{2} = 31,200 \times \sqrt{2}

\]

Final Result and Interpretation

The directional derivative of the function \(f(x, y) = x^3 y^3 \) at the point \((8,5) \) in the direction of \(\mathbf{v} = 2 \mathbf{i} + 2 \mathbf{j} \) is:

\[
\boxed{
D_{\mathbf{u}} f (8, 5) = 31,200 \times \sqrt{2}
}
\]

Numerically, since \(\sqrt{2} \approx 1.4142 \):

\[
D_{\mathbf{u}} f (8, 5) \approx 31,200 \times 1.4142 \approx 44,155.7
\]

This value indicates the rate of change of the function \(f \) at the specified point in the given direction. A positive value signifies that moving in this direction increases the function's value.

Key Takeaways

- The gradient vector points in the direction of the steepest ascent of the function.
- Normalizing the direction vector ensures the derivative measures the rate of change per unit movement.
- The directional derivative combines the gradient and the direction to provide a precise rate of change in any specified direction.
- Understanding these concepts enhances our ability to analyze multivariable functions in fields like physics, engineering, and data science.

Additional Tips for Calculation

- Always verify the units of your vectors and ensure the direction vector is normalized before computing the dot product.
- When dealing with complex functions, breaking down the derivatives and evaluation steps simplifies the process.
- Remember the geometric interpretation: the directional derivative is the projection of the gradient onto the desired direction.

In conclusion, calculating the directional derivative involves understanding the gradient, normalization of the direction vector, and applying the dot product. Mastery of these steps enables a comprehensive analysis of how functions behave under various movements, an essential tool in advanced calculus and applied mathematics.

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