Find The Equation Of The Line That Is Perpendicular To The Given Lineand Passes Through The Given Point.

Find The Equation Of The Line That Is Perpendicular To The Given Lineand Passes Through The Given Point. Understanding how to find the equation of a line perpendicular to a given line and passing through a specific point is a fundamental skill in coordinate geometry. This process involves understanding the concepts of slopes, perpendicular lines, and point-slope forms. Whether you're preparing for a math exam, working on a geometry problem, or enhancing your algebra skills, mastering this topic is essential for solving a wide range of geometric problems efficiently.

Understanding the Basics: Lines, Slopes, and Perpendicularity

Before diving into the step-by-step process, it’s important to grasp some foundational concepts:

What Is a Line in Coordinate Geometry?

A line in the coordinate plane can be described mathematically by an equation that relates the x and y coordinates of any point lying on it. The most common form is the slope-intercept form:

$$ y = mx + b $$

Where:
- m is the slope of the line

- b is the y-intercept (the point where the line crosses the y-axis)

What Is the Slope of a Line?

The slope (m) measures the steepness of the line, calculated as:

$$m = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1}$$

This represents the ratio of the change in y to the change in x between two points on the line.

Perpendicular Lines and Their Slopes

Two lines are perpendicular if they intersect at a right angle (90 degrees). The key relationship between their slopes is:

$$m_1 \times m_2 = -1$$

This means that the slope of a line perpendicular to another is the negative reciprocal of the original line's slope. For example, if the original line has a slope of 2, the perpendicular line's slope will be $-\frac{1}{2}$.

Step-by-Step Guide to Find the Equation of the Perpendicular Line

The process involves several key steps, which are outlined below:

1. Identify the Slope of the Given Line

Begin by rewriting the given line in slope-intercept form or identifying its slope directly from the equation.

- If the line is in slope-intercept form: The slope is the coefficient of x.
- If the line is in standard form: Convert it to slope-intercept form or extract the slope directly.

Example:
Given line: $3x + 4y = 12$

Rearranged as:

$$4y = -3x + 12$$
$$y = -\frac{3}{4}x + 3$$

So, the slope $m_1 = -\frac{3}{4}$.

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2. Calculate the Slope of the Perpendicular Line

Using the negative reciprocal relationship:

$$m_2 = -\frac{1}{m_1}$$

Continuing the example:

$$m_2 = -\frac{1}{-\frac{3}{4}} = \frac{4}{3}$$

This is the slope of the line perpendicular to the original.

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3. Use the Point-Slope Form of a Line

Once you have the slope of the perpendicular line, use the point-slope form to find the line passing through the given point:

$$ y - y_1 = m (x - x_1) $$

Where:
- (x_1, y_1) is the given point
- m is the slope of the perpendicular line

Example:
Suppose the given point is $(2, 5)$.

Plugging into the point-slope form:

$$ y - 5 = \frac{4}{3} (x - 2) $$

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4. Simplify to Slope-Intercept or Standard Form

Expand and rearrange the equation to your preferred form.

Continuing the example:

$$ y - 5 = \frac{4}{3}x - \frac{8}{3} $$
$$ y = \frac{4}{3}x - \frac{8}{3} + 5 $$
$$ y = \frac{4}{3}x - \frac{8}{3} + \frac{15}{3} $$

$$ y = \frac{4}{3}x + \frac{7}{3} $$

This is the equation of the line perpendicular to the original line passing through $(2, 5)$.

Additional Tips and Common Variations

Handling Different Forms of the Given Line

- Standard form: $Ax + By = C$. Convert to slope-intercept form before proceeding.
- Vertical or horizontal lines: For vertical lines ($x = a$), the perpendicular line will be horizontal ($y = b$), and vice versa.

Special Cases

- When the slope of the original line is zero ($y = b$), the perpendicular line will have an undefined slope, i.e., it will be a vertical line.
- When the original line is vertical ($x = a$), the perpendicular line will be horizontal ($y = b$).

Example of Perpendicular to a Vertical Line

Given line: $x = 4$, passing through point $(2, 3)$.

- The perpendicular line will be horizontal: $y = 3$.

Practice Problems to Master Finding Perpendicular Line Equations

1. Find the equation of the line perpendicular to $y = 2x + 1$ passing through $(3, -2)$.
2. Determine the perpendicular line passing through $(0, 0)$ to the line $y = -\frac{1}{2}x + 4$.
3. Find the perpendicular line passing through $(5, 5)$ to the line $4x - 3y = 7$.

Conclusion: Mastering the Art of Perpendicular Line Equations

Understanding how to find the equation of a line perpendicular to a given line and passing through a specific point is crucial in geometry and algebra. The key steps involve identifying the original line's slope, calculating the negative reciprocal for the perpendicular slope, and then using the point-slope form to derive the complete equation. With practice, these skills become second nature, enabling you to solve complex geometric problems efficiently and accurately.

Whether you're working with real-world data, tackling exam questions, or developing a deeper understanding of geometric relationships, mastering this technique opens the door to a wide array of mathematical applications. Remember to carefully handle different line forms, special cases, and verify your solutions for correctness.

If you're looking to improve your skills further, consider exploring related topics such as the distance formula, the equation of a line passing through two points, and the concept of parallel lines. Each of these concepts reinforces your understanding of the fundamental principles of coordinate geometry and enhances your problem-solving toolkit.

Frequently Asked Questions

How do I find the equation of a line perpendicular to a given line that passes through a specific point?

First, determine the slope of the given line. Then, find the negative reciprocal of that slope for the perpendicular line. Finally, use the point-slope form with this new slope and the given point to find the equation.

What is the formula for the slope of a perpendicular line if the original line's slope is known?

If the original line has a slope m, the perpendicular line's slope is -1/m, provided m is not zero.

How do I handle a horizontal or vertical line when finding a perpendicular line passing through a point?

For a horizontal line (slope 0), the perpendicular line is vertical with an undefined slope and passes through the given point's x-coordinate. For a vertical line (undefined slope), the perpendicular line is horizontal passing through the given point's y-coordinate.

Can you give an example of finding the perpendicular line passing through a point?

Sure. Given the line y = 2x + 3 and point (4, 1): The slope of the given line is 2. The perpendicular slope is -1/2. Using point-slope form: y - 1 = -1/2(x - 4). Simplifying gives the equation of the perpendicular line.

What is the point-slope form of the line equation used in these

problems?

The point-slope form is $y - y_1 = m(x - x_1)$, where (x_1, y_1) is the point the line passes through and m is its slope.

How do I verify that the line I found is perpendicular to the original line?

Calculate the slopes of both lines. If their slopes are negative reciprocals, the lines are perpendicular.

What if the given line is in standard form, like Ax + By + C = 0? How do I find its slope?

Rewrite the line in slope-intercept form $y = mx + b$ to identify the slope m. Then find the negative reciprocal for the perpendicular line.

Are there any special cases I should be aware of when finding perpendicular lines?

Yes. If the given line is vertical ($x = a$), the perpendicular line is horizontal ($y = k$). If the given line is horizontal ($y = b$), the perpendicular line is vertical ($x = h$).

Is there a quick way to write the equation of the perpendicular line once I have the slope and point?

Yes. Use the point-slope form $y - y_1 = m(x - x_1)$, then simplify to slope-intercept form $y = mx + b$ for the final equation.

Why is it important to find the negative reciprocal when determining

perpendicular lines?

Because perpendicular lines in the plane have slopes that are negative reciprocals, ensuring the lines intersect at a right angle.

Additional Resources

Perpendicular Lines: Mastering the Art of Finding the Equation of a Line Passing Through a Point and Perpendicular to a Given Line

In the world of analytic geometry, understanding how to find the equation of a line that is perpendicular to a given line and passes through a specific point is an essential skill. Whether you're a student tackling coursework, a teacher preparing lessons, or a professional working on geometric modeling, mastering this process opens doors to solving complex problems efficiently and accurately. This article delves deeply into the concept, methodology, and practical applications of this problem, presenting it as a comprehensive guide for learners and experts alike.

Understanding the Foundations: The Geometric and Algebraic Principles

Before diving into formulas and calculations, it's crucial to understand the foundational concepts that underpin the problem.

The Concept of Perpendicular Lines

Perpendicular lines are two lines that intersect at a right angle (90 degrees). Their relationship is significant because it influences the slopes of the lines involved:

- If one line has slope m_1, then the line perpendicular to it will have a slope m_2 such that:

$$
m_1 \times m_2 = -1
$$

- This means $m_2 = -\frac{1}{m_1}$ (assuming $m_1 \neq 0$).

Key Point: The negative reciprocal of the slope of the original line gives the slope of the perpendicular line.

The Importance of the Point Through Which the Line Passes

Given a specific point (x_0, y_0), our goal is to find the line that:

- Passes through (x_0, y_0)
- Is perpendicular to the given line

This point acts as a fixed anchor, anchoring the new line in space and guiding the calculation.

Step-by-Step Approach to Deriving the Equation of the

Perpendicular Line

The process involves several systematic steps, ensuring clarity and precision at each stage.

Step 1: Identify the Equation of the Given Line

The first step is to express the given line in a form suitable for slope calculation. Common forms include:

- Slope-Intercept Form: \(y = mx + b \)
- Standard Form: \(Ax + By + C = 0 \)

Example:

Suppose the given line is:

\[
2x + 3y - 6 = 0
\]

Rearranged into slope-intercept form:

\[
3y = -2x + 6 \Rightarrow y = -\frac{2}{3}x + 2
\]

The slope of this line is:

\[

m_1 = -\frac{2}{3}

\]

Step 2: Determine the Slope of the Perpendicular Line

Using the negative reciprocal relationship:

\[

m_2 = -\frac{1}{m_1}

\]

For our example:

\[

m_2 = -\frac{1}{-\frac{2}{3}} = \frac{3}{2}

\]

Note:

- If the original line is vertical (\(x = c \)), its slope is undefined, and the perpendicular line will be horizontal (\(y = k \)).
- If the original line is horizontal (\(y = k \)), then the perpendicular line will be vertical (\(x = c \)).

Step 3: Use the Point-Slope Form to Find the Equation

Once the slope m_2 is known, and the point (x_0, y_0) is given, we apply the point-slope form:

$$
y - y_0 = m_2 (x - x_0)
$$

Continuing the example:

Suppose the point given is $(4, 1)$. The equation becomes:

$$
y - 1 = \frac{3}{2} (x - 4)
$$

Expanding:

$$
y - 1 = \frac{3}{2}x - 6
$$

Adding 1 to both sides:

$$
y = \frac{3}{2}x - 5
$$

This is the equation of the line passing through $(4, 1)$ and perpendicular to the original line.

Step 4: Convert to Desired Form (Optional)

Depending on the context, you might want to express the line in slope-intercept form, standard form, or point-slope form.

- Slope-Intercept Form: Already obtained as $y = \frac{3}{2}x - 5$.
- Standard Form: Multiply through by 2 to clear fractions:

$$
2y = 3x - 10 \Rightarrow 3x - 2y = 10
$$

Special Cases and Additional Considerations

While the above steps cover most scenarios, some special cases require additional attention.

Vertical and Horizontal Lines

- Original line is vertical: $x = c$

The perpendicular line is horizontal: $y = y_0$

- Original line is horizontal: $y = k$

The perpendicular line is vertical: $x = x_0$

Example:

Given $x = 3$, and a point $(2, 5)$:

- The perpendicular line is $y = 5$.

Given $y = 4$, and point $(1, 2)$:

- The perpendicular line is $x = 1$.

Handling Lines with Zero or Infinite Slopes

- Zero slope (horizontal line): $y = k$
- Infinite slope (vertical line): $x = c$

In such cases, the perpendicular line is straightforward to find, often involving switching between these forms.

Practical Applications and Real-World Examples

Understanding how to find these lines is not just an academic exercise; it has tangible applications:

- Engineering Design: Ensuring components are perpendicular to a given line or surface.

- Navigation and Mapping: Calculating paths perpendicular to roads or boundaries.
- Computer Graphics: Rendering perpendicular lines for shading, modeling, or collision detection.
- Robotics: Planning movement paths orthogonal to obstacles or paths.

Summary of the Procedure: A Quick Reference

Step	Action	Key Point
1	Find the slope of the given line	Convert to slope-intercept or standard form
2	Compute the perpendicular slope $m_2 = -1/m_1$	Handle special cases (vertical/horizontal lines)
3	Use the point-slope form with the given point	$y - y_0 = m_2 (x - x_0)$
4	Simplify to preferred form	Slope-intercept or standard form

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Conclusion: Mastery Through Practice

The ability to find the equation of a line perpendicular to a given line and passing through a specific point is a fundamental skill that combines algebraic manipulation, geometric insight, and problem-solving acumen. By understanding the underlying principles—slope relationships, line forms, and special cases—you can approach such problems with confidence and precision.

Practice by applying these steps to various lines and points, including challenging scenarios involving vertical or horizontal lines. With consistent effort, this process will become an intuitive part of your mathematical toolkit, empowering you to tackle complex geometric problems and real-world applications with ease and expertise.

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