Find The Parametric Equation Of The Line That Passes Through P(1, 0, 3) And Is Parallel To The Line With

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Understanding how to find the parametric equation of a line in three-dimensional space is fundamental in vector calculus and analytic geometry. Whether you're a student learning the basics or a professional tackling complex spatial problems, mastering this concept allows you to represent lines precisely and analyze their properties effectively. In particular, when given a point and a line parallel to the desired line, the task involves identifying the direction vector and applying the parametric form to find the equation of the line passing through the point.

In this comprehensive guide, we will explore the process of deriving the parametric equation of a line passing through a specific point, P(1, 0, 3), and parallel to another line. We will cover the foundational concepts, step-by-step procedures, and practical examples to ensure clarity and confidence in solving such problems.

Understanding Lines in Three-Dimensional Space

Before diving into the solution, it's crucial to understand how lines are represented in 3D space.

The Vector Equation of a Line

A line in three-dimensional space can be expressed using a vector equation:

$$\vec{r} = \vec{r_0} + t\vec{v}$$

where:
- $\vec{r}$ is the position vector of any point on the line,
- $\vec{r_0}$ is the position vector of a specific point on the line (known point),
- $\vec{v}$ is the direction vector of the line,
- t is a scalar parameter.

Example: If a line passes through point $P(x_1, y_1, z_1)$ with a direction vector $\vec{v} = \langle a, b, c \rangle$, then its vector equation is:

$$\vec{r} = \langle x_1, y_1, z_1 \rangle + t \langle a, b, c \rangle$$

Given Data and What is Required

Suppose we are given:
- A point $P(1, 0, 3)$,
- A line (say, Line L) with a known parametric or vector form, which we will refer to for its direction.

Objective: Find the parametric equations of a line passing through point $P(1, 0, 3)$ and parallel to Line L.

Step-by-Step Process to Find the Parametric Equation

Step 1: Understand the Line to Which the New Line Is Parallel

Since the new line is parallel to an existing line, it will share the same direction vector as that line.

Key idea: The direction vector of the new line is identical to the direction vector of the given line.

Step 2: Determine the Direction Vector of the Given Line

The problem statement says "parallel to the line with," which implies we need the line's direction vector, which might be given explicitly or deduced from its parametric or symmetric equations.

Example: Let's assume the given line (Line L) has the parametric equations:

$$
\begin{cases}
x = x_0 + a t \\
y = y_0 + b t \\
z = z_0 + c t
\end{cases}
$$

or its vector form:

\[
\vec{r} = \vec{r_0} + t \vec{v}
\]

where \(\vec{v} = \langle a, b, c \rangle\).

Note: If the direction vector of the original line is not given, but the line's equations are, you can extract the direction vector directly from the parametric equations.

Step 3: Write the Direction Vector

Suppose the given line L has the parametric equations:

\[
\begin{cases}
x = x_0 + a t \\
y = y_0 + b t \\
z = z_0 + c t
\end{cases}
\]

then:

\[
\vec{v} = \langle a, b, c \rangle
\]

Step 4: Write the Equation of the New Line

Using point \(P(1, 0, 3)\) and the direction vector \(\vec{v}\), the parametric equations of the new line are:

\[
\begin{cases}
x = 1 + a s \\
y = 0 + b s \\
z = 3 + c s
\end{cases}
\]

where \(s\) is a scalar parameter.

Note: The same direction vector $\langle a, b, c \rangle$ applies because the lines are parallel.

Practical Examples and Applications

Let's consider specific scenarios to illustrate the process more concretely.

Example 1: Line with Known Direction Vector

Suppose the line L has the parametric equations:

$$
\begin{cases}
x = 2 + 3 t \\
y = -1 + 4 t \\
z = 5 + 2 t
\end{cases}
$$

The direction vector:

$$
\vec{v} = \langle 3, 4, 2 \rangle
$$

Solution:

The line passing through $P(1, 0, 3)$ and parallel to this line has the parametric equations:

$$
\begin{cases}
x = 1 + 3 s \\
y = 0 + 4 s \\
z = 3 + 2 s
\end{cases}
$$

Example 2: Line Given in Symmetric Form

Suppose the original line is given as:

$$
\frac{x - 2}{3} = \frac{y + 1}{4} = \frac{z - 5}{2}
$$

Solution:

The direction vector is derived from denominators:

$$
\vec{v} = \langle 3, 4, 2 \rangle
$$

and a point on this line, for example, when the parameter equals zero:

$$
x = 2, \quad y = -1, \quad z = 5
$$

The new line passing through $P(1, 0, 3)$ with the same direction vector:

$$
\boxed{
\begin{cases}
x = 1 + 3 s \\
y = 0 + 4 s \\
z = 3 + 2 s
\end{cases}
}
$$

General Form of the Parametric Equation for the Line

The most general form for the parametric equations of the required line is:

$$
\boxed{
\begin{aligned}
x &= x_0 + a s \\
y &= y_0 + b s \\

z &= z_0 + c s
\end{aligned}
}
\]

where:
- (x_0, y_0, z_0) is the point $P(1, 0, 3)$,
- $\langle a, b, c \rangle$ is the direction vector of the original line.

Additional Considerations and Special Cases

When the Direction Vector is Zero

If the original line's direction vector is $\vec{0}$, which means the line is actually a point, then the line passing through P and parallel to it is just a point or undefined in the usual sense.

Lines in Symmetric Form

The parametric equations can sometimes be converted into symmetric form:

$$
\frac{x - x_0}{a} = \frac{y - y_0}{b} = \frac{z - z_0}{c}
$$

which is useful for visualization or solving intersections.

Handling Cases with Unknown Direction Vectors

If the direction vector isn't directly given but the line's equations are, always extract the direction vector from the parametric or symmetric forms.

Summary and Key Takeaways

- To find the parametric equation of a line passing through a point $P(x_1, y_1, z_1)$ and parallel to a given line, identify the direction vector of the given line.

- The direction vector is key; lines are parallel if and only if their direction vectors are scalar multiples.
- The parametric form uses the point and the direction vector:

$$
x = x_1 + a s, \quad y = y_1 + b s, \quad z = z_1 + c s
$$

- Always verify the direction vector from the given line's equations before proceeding.

Final Remarks

Mastering the process of deriving parametric equations for lines in 3D space is essential for advanced studies in geometry, physics, engineering, and computer graphics. By understanding how to extract the direction vector and apply the point-direction form, you can efficiently solve various problems involving lines, including intersections, distances, and projections.

Remember, practice with different forms and scenarios enhances your understanding and problem-solving skills. Whether working with given parametric equations, symmetric forms, or geometric descriptions, the core principle remains consistent: identify the direction vector and use the point to construct the parametric

Frequently Asked Questions

How do I find the parametric equations of a line passing through point P(1, 0, 3) and parallel to a given line?

First, identify the direction vector of the given line. Then, use that vector as the direction vector for the new line passing through P(1, 0, 3). The parametric equations are formed by adding the scaled direction vector components to the point coordinates.

What information do I need to write the parametric equation of a line parallel to a given line?

You need a point through which the new line passes (here, P(1, 0, 3)) and the direction vector of the given line. The direction vector determines the line's orientation and ensures the new line is parallel.

Can you provide an example of finding the parametric equations of a line passing through P(1, 0, 3) and parallel to line with direction vector v?

Yes. If the given line has direction vector $v = (a, b, c)$, then the parametric equations are: $x = 1 + a t$, $y = 0 + b t$, $z = 3 + c t$, where t is a parameter.

If the line to which the new line is parallel has a parametric form, how do I determine the parametric equations of the new line?

Identify the direction vector from the existing line's parametric equations. Use this vector and the point P(1, 0, 3) to write the new line's equations as P plus t times the direction vector.

What are common mistakes when deriving the parametric equations of a line parallel to another?

Common mistakes include misidentifying the direction vector, using incorrect point coordinates, or mixing up the components. Always ensure the direction vector matches the parallel line and the point is correctly substituted.

Is it necessary to know the equation of the original line to find the parametric equation of a parallel line passing through a specific point?

No, it's not necessary if you already know the direction vector of the original line. The key is to use the same direction vector and the given point to write the parametric equations of the parallel line.

Additional Resources

Find The Parametric Equation Of The Line That Passes Through P(1, 0, 3) And Is Parallel To The Line With

In the realm of three-dimensional analytic geometry, the task of determining the parametric equations of a line based on given conditions is fundamental. The problem statement, “Find the parametric equation of the line that passes through P(1, 0, 3) and is parallel to the line with,” introduces a classic scenario where geometric intuition and algebraic precision converge. This article endeavors to thoroughly dissect this problem, elucidate the theoretical underpinnings, and detail methodological approaches to arrive at the solution.

Understanding the Problem Statement

Before diving into the solution, it's essential to clarify the problem's components:

- Given Point P(1, 0, 3): This is a specific point in three-dimensional space through which the required line must pass.

- Line Parallelism: The desired line must be parallel to another line, whose equation or direction vector is provided (though it appears incomplete in the prompt). Typically, such problems specify the line's equation or at least its direction vector, which is crucial for defining the parallel line.

For the purpose of this detailed discussion, we will assume that the line to which the new line must be parallel is given by its parametric form or its direction vector. If not explicitly provided, the general method applies once the direction vector is known.

General Approach to Finding the Parametric Equation of a Line

To construct the parametric equations of the line in question, the key components are:

1. A point through which the line passes — in this case, P(1, 0, 3).

2. A direction vector — which determines the orientation of the line in space.

Given these, the parametric equations are formulated as:

- $x = x_0 + a t$
- $y = y_0 + b t$
- $z = z_0 + c t$

where:

- (x_0, y_0, z_0) is the point P through which the line passes.
- (a, b, c) is the direction vector.
- t is the parameter.

The primary challenge lies in accurately identifying or constructing the direction vector of the line that is parallel to the given line.

Interpreting the Parallelism Condition

Assumption of the Given Line's Equation

Suppose the original line (to which the new line must be parallel) is given in the form:

L: $r = r_0 + \lambda v$

where:

- r_0 is a point on the line (e.g., P_0),
- v is the direction vector of the line,
- λ is a scalar parameter.

In many problems, the line is specified either through its symmetric equations, parametric equations, or a point and a direction vector.

Example:

Suppose the line is given as:

L: $x = 2 + 3\lambda$, $y = 4 - \lambda$, $z = 1 + 2\lambda$

Then, the direction vector v = (3, -1, 2).

The line passing through P(1, 0, 3) and parallel to this line must have a direction vector parallel to v.

Determining the Direction Vector

The direction vector of the new line is parallel to the original, meaning:

- It is a scalar multiple of v, i.e.,

(a, b, c) = k (3, -1, 2)

for some scalar $k \neq 0$.

Choosing $k = 1$ for simplicity, we have:

$(a, b, c) = (3, -1, 2)$

Constructing the Parametric Equations

Given the point P(1, 0, 3) and the direction vector $v = (3, -1, 2)$, the parametric equations are:

- $x = 1 + 3t$
- $y = 0 - t$
- $z = 3 + 2t$

where $t \in \mathbb{R}$.

This set of equations fully describes the line passing through P(1, 0, 3) and parallel to the original line.

Special Cases and Additional Considerations

Line Given in Symmetric Form

If the original line is provided in symmetric equations, such as:

$(x - x_1)/a = (y - y_1)/b = (z - z_1)/c$

then the direction vector is simply (a, b, c). The process then involves:

- Identifying the direction vector directly.
- Using the point P to write the parametric equations.

Line Given by Two Points

If the original line is given by two points, say, $Q_1(x_1, y_1, z_1)$ and $Q_2(x_2, y_2, z_2)$, then:

- The direction vector $v = Q_2 - Q_1 = (x_2 - x_1, y_2 - y_1, z_2 - z_1)$.
- The parametric equations are then built using the point P and this vector.

Handling Multiple or Unknown Directions

In cases where the original line's direction is not explicitly given, additional information is necessary to define the problem fully. Without the direction vector, the problem cannot be solved definitively.

Verification and Visualization

Once the parametric equations are established, it's prudent to verify:

- The point P lies on the line (by substituting $t=0$).
- The line's direction vector is consistent with the original line (if known).
- The line's geometric relation to the original line (parallelism) is maintained.

Graphical tools or vector analysis can be employed to visualize the line, confirming the geometric intuition aligns with algebraic results.

Conclusion

In summary, the process of finding the parametric equation of a line passing through a point and parallel to a given line hinges critically on identifying the line's direction vector. The general steps involve:

1. Extracting or determining the direction vector of the existing line.
2. Using the point through which the new line passes.
3. Formulating the parametric equations based on these components.

Example Final Solution:

Suppose the original line is given by:

$x = 2 + 3\lambda$, $y = 4 - \lambda$, $z = 1 + 2\lambda$

The parametric equations for the line passing through P(1, 0, 3) and parallel to this line are:

- $x = 1 + 3t$
- $y = 0 - t$
- $z = 3 + 2t$

where $t \in \mathbb{R}$.

This approach exemplifies the fundamental principles of vector geometry and parametric representations, illustrating a robust method applicable across diverse problems in three-dimensional analytic geometry.

In essence, the key to solving such problems lies in understanding the geometric constraints, accurately extracting the direction vector, and applying the parametric form systematically. With these tools, one can navigate the complexities of three-dimensional space to derive precise and meaningful equations of lines under various conditions.

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