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Introduction

In the study of calculus, understanding the behavior of curves and their tangents is fundamental. One common problem involves determining the slope of the tangent line to a specific curve at a given point. This process involves differentiating the function and evaluating the derivative at the specified point. Here, we focus on the curve defined by the equation:

$$
Y = \frac{3}{\sqrt{x}}
$$

and aim to find the slope of the tangent line at the point where $x = A$, with $A > 0$. This article provides a detailed, step-by-step explanation of how to compute this slope, including the differentiation process, interpretation, and related concepts.

Understanding the Problem

The Given Curve

The function under consideration is:

$$
Y = \frac{3}{\sqrt{x}}
$$

which can also be written as:

$$
Y = 3x^{-\frac{1}{2}}
$$

The task involves:

- Differentiating this function with respect to x to find the derivative $\frac{dy}{dx}$, which represents the slope of the tangent at any point x.

- Evaluating this derivative at $x = A$, where $A > 0$, to find the specific slope M.

Significance of Finding the Slope

The slope of the tangent line at a point on a curve provides insight into the curve's instantaneous rate of change at that point. It tells us whether the function is increasing or decreasing at $x = A$, and how steep the tangent is.

Step-by-Step Solution

1. Differentiating the Function $Y = 3x^{-\frac{1}{2}}$

Since the function involves a power of x, we use the power rule for differentiation:

$$
\frac{d}{dx} [x^n] = n x^{n-1}
$$

Applying the differentiation to Y:

$$
\frac{dy}{dx} = 3 \times \frac{d}{dx} \left[x^{-\frac{1}{2}} \right]
$$

$$
\frac{dy}{dx} = 3 \times \left(-\frac{1}{2} \right) x^{-\frac{1}{2} - 1}
$$

$$
\frac{dy}{dx} = -\frac{3}{2} x^{-\frac{3}{2}}
$$

This derivative expression gives the slope of the tangent at any x.

2. Simplifying the Derivative

Rewrite $x^{-\frac{3}{2}}$ to a more manageable form:

$$
x^{-\frac{3}{2}} = \frac{1}{x^{\frac{3}{2}}}
$$

And $x^{\frac{3}{2}}$ can be expressed as:

$$
x^{\frac{3}{2}} = x^{1} \times x^{\frac{1}{2}} = x \sqrt{x}
$$

Therefore, the derivative becomes:

$$
\frac{dy}{dx} = -\frac{3}{2} \times \frac{1}{x \sqrt{x}} = -\frac{3}{2 x \sqrt{x}}
$$

---

3. Evaluating the Derivative at $x = A$

To find the slope M at the point where $x = A$, substitute A into the derivative:

$$
M = \left. \frac{dy}{dx} \right|_{x=A} = -\frac{3}{2 A \sqrt{A}}
$$

Since $A > 0$, the expression is well-defined, and the square root $\sqrt{A}$ is positive.

---

4. Interpreting the Result

Sign of the Slope

- The numerator is negative, and the denominator is positive (since $A > 0$ and $\sqrt{A} > 0$).

- Therefore, the slope M at $x = A$ is negative, indicating the curve is decreasing at that point.

Magnitude of the Slope

- The magnitude depends on the value of A.

- As A increases, the denominator increases, leading to a smaller magnitude of the slope (less steep).

Geometric Meaning

- The tangent line at $x = A$ has a negative slope, meaning it slopes downward from left to right.

- The steepness diminishes as A becomes larger.

---

5. Additional Considerations

Domain of the Function

- Since $\sqrt{x}$ appears in the denominator, x must be positive to avoid division by zero or complex numbers.

- Hence, the domain is $x > 0$.

Behavior of the Curve

- The function $Y = \frac{3}{\sqrt{x}}$ is a decreasing function for $x > 0$.

- As $x \to 0^+$, $Y \to +\infty$.

- As $x \to +\infty$, $Y \to 0^+$.

Applications of the Derivative

- Understanding the slope helps in sketching the graph.

- It is useful in optimization problems, where maximum or minimum values are sought.

---

6. Related Concepts and Formulas

Power Rule for Differentiation

$$
\frac{d}{dx} [x^n] = n x^{n-1}
$$

Chain Rule (if needed)

In more complex functions, the chain rule helps differentiate compositions of functions.

Derivative of $\frac{1}{\sqrt{x}}$

$$
\frac{d}{dx} \left(x^{-\frac{1}{2}} \right) = -\frac{1}{2} x^{-\frac{3}{2}}
$$

which aligns with our earlier calculation.

---

7. Summary of the Calculation

Step	Description	Result
1	Rewrite Y as a power of x	$Y = 3x^{-\frac{1}{2}}$
2	Differentiate using power rule	$\frac{dy}{dx} = -\frac{3}{2} x^{-\frac{3}{2}}$
3	Express derivative in simplified form	$\frac{dy}{dx} = -\frac{3}{2 x \sqrt{x}}$
4	Evaluate at $x = A$	$M = -\frac{3}{2 A \sqrt{A}}$

---

8. Practical Example

Suppose $A = 4$, then:

$$
M = -\frac{3}{2 \times 4 \times \sqrt{4}} = -\frac{3}{2 \times 4 \times 2} = -\frac{3}{16}
$$

Thus, the slope of the tangent line at $x=4$ is $-\frac{3}{16}$.

---

9. Conclusion

Calculating the slope of the tangent line to a curve at a specific point is a core skill in calculus, with applications across physics, engineering, and mathematics. For the curve $Y = \frac{3}{\sqrt{x}}$, differentiating yields a clear expression for the slope at any positive x. Evaluating this derivative at $x = A$ provides the exact slope at that point, showing how the function's rate of change behaves relative to A. Understanding this process enhances the ability to analyze functions, interpret graphs, and solve real-world problems involving rates of change.

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10. Additional Resources

- Calculus Textbooks: For comprehensive explanations and exercises on derivatives and tangent lines.

- Online Calculus Tutorials: Interactive lessons on power rule, chain rule, and derivatives of rational functions.

- Graphing Calculators and Software: Tools like Desmos, GeoGebra, or Wolfram Alpha can help visualize the function and its tangent lines.

In summary, the slope M of the tangent to the curve $Y = \frac{3}{\sqrt{x}}$ at $x = A > 0$ is:

$$
\boxed{
M = -\frac{3}{2 A \sqrt{A}}
}
$$

This formula provides a direct way to compute the rate of change at any positive point along the curve, facilitating deeper analysis and understanding of the function's behavior.

Frequently Asked Questions

How do you find the slope of the tangent to the curve y = 3 / (sqrt(x)) at a point where x = A > 0?

To find the slope, differentiate y with respect to x to get dy/dx, then substitute x = A into the derivative.

What is the derivative of y = 3 / (sqrt(x)) with respect to x?

Rewriting y as $y = 3x^{(-1/2)}$, its derivative $dy/dx = - (3/2) x^{(-3/2)}$.

How do you evaluate the slope at x = A > 0 for y = 3 / (sqrt(x))?

Substitute x = A into dy/dx to get the slope $m = - (3/2) A^{(-3/2)}$.

What is the simplified expression for the slope of the tangent at x = A?

The slope is $m = - (3/2) (1 / A^{3/2})$ or equivalently $m = - (3/2) / A^{3/2}$.

Can you explain the significance of the slope in relation to the curve y = 3 / (sqrt(x))?

The slope indicates the rate of change of y with respect to x at a specific point, showing how steep the curve is there.

What is the importance of the condition A > 0 in

finding the tangent slope?

Since sqrt(x) is only real and defined for x > 0, A must be positive to compute the derivative and tangent slope accurately.

How would you write the final answer for the slope at x = A in a concise form?

The slope is m = - (3/2) / A^{3/2} at the point where x = A > 0.

Additional Resources

Find The Slope M Of The Tangent To The Curve Y = 3 / (√x) At The Point Where X = A > 0. (b) Find

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Introduction

Mathematics, especially calculus, offers a profound insight into the behavior of curves and their properties. Among these properties, the slope of a tangent line at a specific point on a curve is fundamental. It provides local information about the rate of change of the function at that point, which is essential in various fields such as physics, engineering, economics, and more.

In this article, we will delve into the problem of finding the slope \(M \) of the tangent line to the curve defined by \(y = \frac{3}{\sqrt{x}} \) at the point where \(x = A > 0 \). The task involves understanding the principles of differentiation, applying the chain rule, and interpreting the results in a meaningful way. We will also explore the theoretical background, detailed calculations, and broader implications of this problem, making it accessible to students, educators, and enthusiasts alike.

Understanding the Curve \(y = \frac{3}{\sqrt{x}} \)

The Function and Its Characteristics

The given function:

\[
y = \frac{3}{\sqrt{x}}
\]

can be rewritten for clarity as:

\[

$$
y = 3x^{-\frac{1}{2}}
$$

This form makes differentiation straightforward and reveals the nature of the function:

- Domain: Since $\sqrt{x}$ is defined for $x > 0$, the domain of y is $(0, \infty)$.
- Range: As $x \to 0^{+}$, $y \to \infty$; as $x \to \infty$, $y \to 0^{+}$.
- Behavior: The function is decreasing over its domain, tending to zero as $x \to \infty$, and increasing without bound as $x \to 0^{+}$.

Geometric Interpretation

Graphically, the curve resembles a hyperbola in the first quadrant, descending from infinity as $x \to 0^{+}$ to zero as $x \to \infty$. The curve is smooth and continuous, with no breaks or sharp points, which is typical of rational functions with fractional exponents.

Derivation of the Slope M at $x = A$

Step 1: Find the Derivative $\frac{dy}{dx}$

The slope of the tangent line at any point on the curve is given by the derivative of y with respect to x:

$$
\frac{dy}{dx} = \frac{d}{dx} \left(3x^{-\frac{1}{2}} \right)
$$

Applying the power rule:

$$
\frac{dy}{dx} = 3 \times \left(-\frac{1}{2} \right) x^{-\frac{1}{2} - 1} = -\frac{3}{2} x^{-\frac{3}{2}}
$$

Step 2: Simplify the Derivative

Expressing the derivative in radical form:

$$
\frac{dy}{dx} = -\frac{3}{2} x^{-\frac{3}{2}} = -\frac{3}{2} \times \frac{1}{x^{\frac{3}{2}}} = -\frac{3}{2} \times \frac{1}{x \sqrt{x}}
$$

Thus,

$$
\boxed{
\frac{dy}{dx} = -\frac{3}{2x \sqrt{x}}
}
$$

This derivative provides the slope of the tangent to the curve at any point $x > 0$.

Step 3: Find the Slope at $x = A$

Substitute $x = A$:

$$
\boxed{
M = \frac{dy}{dx}\bigg|_{x=A} = -\frac{3}{2A \sqrt{A}}
}
$$

Since $A > 0$, the denominator is positive, and the slope M is negative, confirming the decreasing nature of the function.

Step 4: Expressing the Tangent Line Equation (Optional)

The tangent line at $x = A$ passes through the point:

$$
\left(A, y(A) \right) = \left(A, \frac{3}{\sqrt{A}} \right)
$$

The equation of the tangent line:

$$
y - y(A) = M (x - A)
$$

or explicitly:

$$
y - \frac{3}{\sqrt{A}} = -\frac{3}{2A \sqrt{A}} (x - A)
$$

This line provides a local linear approximation of the curve near $x = A$.

Broader Implications and Applications

Significance of the Slope M

The slope M reflects how rapidly the function $y = \frac{3}{\sqrt{x}}$ decreases as x increases. As $A \to 0^{+}$, the denominator $2A \sqrt{A}$ approaches zero, causing $|M|$ to tend toward infinity, indicating an extremely steep tangent near the origin. Conversely, as $A \to \infty$, $M \to 0$, reflecting a gentler slope.

Practical Examples

- Physics: If y models a quantity like velocity or intensity that diminishes over a variable x, understanding the slope provides insights into the rate at which the quantity decreases.
- Economics: In cost or demand functions similar to this form, the derivative indicates marginal changes and sensitivities.
- Engineering: Structural analysis often involves slopes of curves representing stress-strain relationships or other physical properties.

Limitations and Considerations

- The derivative is valid only where the function is differentiable, which is for $x > 0$ in this case.
- As $x \to 0^{+}$, the slope tends to $-\infty$, indicating an asymptote and a vertical tangent.
- The negative sign of M confirms the decreasing trend of the curve.

Analytical and Conceptual Insights

Power Rule Application

The derivation exemplifies how the power rule simplifies the differentiation of functions involving fractional exponents. Recognizing the function in exponential form:

$$
y = 3 x^{-\frac{1}{2}}
$$

is crucial for applying basic calculus techniques.

Significance of the Result

The explicit expression:

$$
M = -\frac{3}{2A \sqrt{A}}
$$

encapsulates the local behavior of the curve at any positive A. It highlights the inverse relationship between A and the slope's magnitude—larger A corresponds to less steep (closer to horizontal) tangents, while smaller A yields steeper, more vertical tangents.

Extension to General Cases

While this analysis focuses on a specific function, the methodology extends naturally to other rational or radical functions. The derivative's form provides a template for analyzing similar curves, emphasizing the importance of the chain rule and exponential differentiation.

Conclusion

The process of finding the slope M of the tangent to the curve $y = \frac{3}{\sqrt{x}}$ at the point where $x = A > 0$ illustrates core calculus principles. By differentiating the function, applying the power rule, and substituting the specific x-value, we obtain a precise measure of the curve's local inclination. The final expression:

$$
\boxed{
M = -\frac{3}{2A \sqrt{A}}
}
$$

not only quantifies the slope but also provides deeper understanding into the geometric and analytical behavior of the function. This exercise underscores the importance of derivatives in analyzing the infinitesimal changes and local properties of mathematical models, which are integral to scientific and engineering disciplines.

References and Further Reading

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Final Remarks

Understanding how to calculate the slope of a tangent line at any point on a curve is a foundational skill in calculus. It combines algebraic manipulation, differentiation rules, and geometric interpretation, forming a bridge between abstract mathematics and real-world applications. Whether analyzing physical phenomena or optimizing functions in economics, mastering these concepts enhances problem-solving capabilities and deepens mathematical insight.

Find The Slope M Of The Tangent To The Curve Y 3 Sqrt X At The Point Where X A Gt 0 B Find

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