Frequency Modulated (FM) Signal XFM (t) = 10cos (102mt + 10. Sin (4710t)) Is Given. (a) Find The Carrier

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Understanding Frequency Modulated (FM) Signals

Frequency modulation (FM) is a widely used method of encoding information in a carrier wave by varying its instantaneous frequency. Unlike amplitude modulation (AM), where the amplitude of the carrier wave is varied, FM modifies the frequency according to the information signal. This technique offers advantages such as improved noise immunity and better signal quality, making it predominant in radio broadcasting, audio transmissions, and data communication systems.

In this article, we analyze an FM signal given by the mathematical expression:

$X_{FM}(t) = 10 \cos(102\pi t + 10 \sin(4710t))$

Our goal is to determine the carrier frequency component embedded within this FM signal. We will explore the concepts involved, methods to extract the carrier frequency, and related calculations to understand the signal comprehensively.

Decoding the Given FM Signal

Mathematical Expression Breakdown

The given FM signal is:

$X_{FM}(t) = 10 \cos(102\pi t + 10 \sin(4710t))$

Let’s analyze each component:

- Amplitude: 10 (constant, indicating the maximum amplitude of the wave)
- Carrier angular frequency: 102π radians/sec
- Frequency deviation: 10 (indicating the maximum shift from the carrier frequency)
- Modulating signal: $\sin(4710t)$

This structure indicates that the instantaneous phase of the carrier wave is modulated by a sine wave of frequency 4710 rad/sec. The modulation index can be derived from the parameters provided, which determines the extent of frequency deviation.

Understanding the Components

- Carrier Signal: The term $\cos(102\pi t)$ represents the carrier wave.
- Modulation Term: $10 \sin(4710t)$ is the phase deviation term, which varies the phase of the carrier, and thereby the frequency, in FM modulation.

Identifying the Carrier Frequency

Carrier Frequency in FM Signals

In FM signals, the carrier frequency (f_c) is the frequency of the unmodulated carrier wave. It is essential because it forms the basis of the transmitted signal, around which deviations occur due to modulation.

The general form of the carrier wave is:

$$\cos(2\pi f_c t)$$

In the given expression, the argument of the cosine function is:

$$102\pi t + 10 \sin(4710t)$$

Compare this with the standard form:

$$\cos(2\pi f_c t + \beta \sin(2\pi f_m t))$$

where:

- f_c = carrier frequency
- f_m = modulating frequency
- β = modulation index

From the expression, the coefficient of t inside the cosine function relates directly to the carrier frequency.

Extracting the Carrier Frequency

The argument of the cosine function is:

$$102\pi t + 10 \sin(4710t)$$

- The carrier angular frequency (ω_c) is the coefficient of t in the cosine argument:

$$\omega_c = 102\pi \text{ radians/sec}$$

- To find the carrier frequency (f_c) in Hz:

$$f_c = \omega_c / (2\pi)$$

Calculating:

$f_c = (102\pi) / (2\pi) = 102 / 2 = 51$ Hz

Therefore, the carrier frequency is 51 Hz.

Understanding Modulation Index and Bandwidth

Modulation Index (β)

The modulation index in FM is given by:

$\beta = \Delta f / f_m$

where:

- Δf = peak frequency deviation (in Hz)
- f_m = modulating frequency (in Hz)

In the given expression:

- The phase deviation term is $10 \sin(4710t)$

- The maximum phase deviation, $\Delta\theta_{max}$, is 10 radians

- The instantaneous frequency deviation:

$\Delta f = (1 / 2\pi)\ d(\text{phase})/dt$

Since phase deviation is $10 \sin(4710t)$, its derivative:

$d/dt\ [10 \sin(4710t)] = 10\ 4710 \cos(4710t)$

The maximum of this derivative:

Maximum $d(\text{phase})/dt = 10\ 4710 = 47{,}100$ radians/sec

Converting to Hz:

$\Delta f = (1 / 2\pi)$ maximum frequency deviation in radians/sec

But in FM, the peak frequency deviation Δf is given directly by the maximum variation in the instantaneous frequency, which is:

$\Delta f = (1 / 2\pi)$ maximum angular frequency deviation $= 47{,}100 / (2\pi) \approx 7{,}500$ Hz

Similarly, the modulating frequency:

$f_m = 4710 / (2\pi) \approx 750$ Hz

Thus, the modulation index:

$\beta = \Delta f / f_m \approx 7500 / 750 \approx 10$

This indicates a significant modulation index, implying extensive frequency deviation.

Practical Applications and Significance

Why Is Determining the Carrier Frequency Important?

Knowing the carrier frequency is vital for:

- Tuning radio receivers to the correct frequency band
- Filtering signals to remove unwanted frequencies
- Analyzing bandwidth requirements for transmission
- Designing communication systems with optimal performance

Other Uses of FM Signal Analysis

Analyzing FM signals like the one provided can help in:

- Signal demodulation: Extracting the original modulating information
- Bandwidth calculation: Using Carson's rule
- Spectral analysis: Understanding signal behavior in the frequency domain
- Noise suppression: FM's inherent noise immunity

Summary and Conclusion

In this comprehensive analysis, we've examined the FM signal:

$X_{FM}(t) = 10 \cos (102\pi t + 10 \sin (4710t))$

and determined that the carrier frequency embedded within the signal is 51 Hz. This conclusion was drawn from the coefficient of t inside the cosine function, which directly relates to the carrier's angular frequency.

Understanding the carrier frequency helps in multiple facets of communication engineering, including system design, signal processing, and radio broadcasting. The modulation index, bandwidth, and spectral characteristics further provide insights into the behavior of the FM signal.

Key Takeaways:

- The carrier angular frequency (ω_c) is 102π radians/sec.
- The carrier frequency (f_c) is 51 Hz.
- The modulation index (β) is approximately 10, indicating substantial frequency deviation.
- The modulating signal frequency (f_m) is approximately 750 Hz.

By mastering the analysis of such FM signals, engineers and technicians can optimize communication systems for clarity, efficiency, and robustness, ensuring reliable transmission of information across various platforms.

Meta description: Discover how to determine the carrier frequency in a Frequency Modulated (FM) signal, using the example $X_{FM}(t) = 10\cos(102\pi t + 10 \sin(4710t))$. Learn about FM signal components, calculations, and practical applications in communication systems.

Frequently Asked Questions

What is the general form of a frequency modulated (FM) signal?

An FM signal is generally expressed as $XFM(t) = A \cos[\omega_c t + \beta \sin(\omega_m t)]$, where A is amplitude, ω_c is the carrier angular frequency, β is the modulation index, and ω_m is the message (modulating) angular frequency.

How do you identify the carrier frequency in an FM signal given as $XFM(t) = 10 \cos(102\pi t + 10 \sin(4710 t))$?

The carrier frequency is determined by the coefficient multiplying t inside the cosine function's first term. Here, it is 102π, so the carrier angular frequency $\omega_c = 102\pi$ rad/sec, and the carrier frequency $f_c = \omega_c / (2\pi) = 102\pi / (2\pi) = 51$ Hz.

What is the significance of the term 10 sin (4710 t) in the FM signal?

The term $10 \sin(4710 t)$ represents the modulating signal, which influences the instantaneous frequency of the carrier. Its amplitude (10) relates to the modulation index, and 4710 rad/sec is the message signal's angular frequency.

How do you calculate the modulation index (β) for the given FM signal?

The modulation index β is given by the ratio of the peak frequency deviation to the message signal frequency. In this case, β = frequency deviation / message frequency, which can be derived from the amplitude of the modulating signal and the message frequency 4710 rad/sec.

What is the message signal frequency in Hz for the given FM signal?

The message signal angular frequency is 4710 rad/sec. To convert to Hz, divide by 2π: $4710 / (2\pi) \approx 750$ Hz.

How can you verify the carrier frequency in the provided FM signal?

By inspecting the argument of the cosine function, the carrier angular frequency ω_c is 102π rad/sec, leading to a carrier frequency $f_c = 102\pi / (2\pi) = 51$ Hz. This confirms the carrier frequency is 51 Hz.

What are the typical steps to analyze an FM signal like XFM(t) = 10 cos (102π t + 10 sin (4710 t))?

First, identify the carrier frequency from the first term, determine the message signal parameters (amplitude and frequency), compute the modulation index, and understand how the message influences the instantaneous frequency of the carrier.

Why is understanding the carrier frequency important in FM signal analysis?

The carrier frequency determines the primary spectral component of the FM signal, which is essential for modulation, demodulation, filtering, and bandwidth calculations in communication systems.

What is the significance of the amplitude '10' in the FM signal expression?

In the given expression, 10 is the amplitude of the carrier signal, which affects the signal's power but does not directly influence the modulation index. The modulation index depends on the message signal's amplitude and frequency.

Additional Resources

Frequency Modulated (FM) Signal XFM(t) = 10 cos (102πt + 10 sin (4710πt)) is a classic example of a frequency modulation (FM) signal, widely studied in communication systems and signal processing. Understanding and analyzing such signals is fundamental for engineers and students working in wireless communications, radar, audio transmission, and related fields. In this guide, we will delve into the process of identifying the carrier frequency of this FM signal, breaking down the concepts, mathematical derivations, and practical steps involved.

Understanding the FM Signal Structure

Before we analyze the given signal, it’s essential to understand the general form of an FM signal.

What is Frequency Modulation?

Frequency modulation involves varying the instantaneous frequency of a carrier wave in accordance with the modulating signal. The general form of an FM wave can be expressed as:

$$XFM(t) = A \cos (2\pi f_c t + \beta \sin (2\pi f_m t))$$

Where:
- A is the amplitude,
- f_c is the carrier frequency,
- β is the frequency deviation constant (modulation index),
- f_m is the modulating frequency,
- The term $\beta \sin(2\pi f_m t)$ introduces the frequency deviation.

In the provided signal, the instantaneous phase is explicitly given, which helps in identifying the carrier frequency and the characteristics of the modulation.

Step-by-Step Breakdown of the Given Signal

The given FM signal is:

$X_{FM}(t) = 10 \cos(102\pi t + 10 \sin(4710\pi t))$

Let's dissect its components:

1. Amplitude (A)
- The amplitude of the signal is 10 volts (or units).

2. Instantaneous Phase
- The phase of the cosine function is $\psi(t) = 102\pi t + 10 \sin(4710\pi t)$.

This phase consists of two parts:
- A linear term: $102\pi t$
- A sinusoidal term: $10 \sin(4710\pi t)$

3. Carrier and Modulating Components
- The linear term $102\pi t$ corresponds to the carrier.
- The sinusoidal term $10 \sin(4710\pi t)$ represents the frequency deviation due to the modulating signal.

Identifying the Carrier Frequency

The core objective here is to find the carrier frequency of the given FM signal.

How is the Carrier Frequency Defined?

In a typical FM signal:

- The carrier frequency (f_c) is the frequency of the unmodulated carrier wave.
- It appears as the coefficient multiplying t in the phase term, scaled appropriately.

Extracting the Carrier Frequency from the Phase

Given the phase:

$\psi(t) = 102\pi t + 10 \sin(4710\pi t)$

- The term $102\pi t$ corresponds to the carrier component.
- Since phase $\psi(t)$ is in radians, the instantaneous frequency $f_i(t)$ is:

$f_i(t) = (1/2\pi)\, d\psi(t)/dt$

Let's differentiate:

$d\psi(t)/dt = 102\pi + 10 \times d/dt\, [\sin(4710\pi t)]$

Recall that:

$d/dt\ [\sin(\omega t)] = \omega \cos(\omega t)$

where ω is the angular frequency.

So,

$d\varphi(t)/dt = 102\pi + 10 \times 4710\pi \cos(4710\pi t)$

The instantaneous frequency becomes:

$f_i(t) = (1/2\pi)\ [102\pi + 47100\pi \cos(4710\pi t)]$

Simplify:

$f_i(t) = (1/2\pi) \times 102\pi + (1/2\pi) \times 47100\pi \cos(4710\pi t)$

which simplifies further to:

$f_i(t) = (102\pi) / (2\pi) + (47100\pi) / (2\pi) \times \cos(4710\pi t)$

$f_i(t) = 51\ Hz + 23{,}550\ Hz \times \cos(4710\pi t)$

Interpretation:

- The instantaneous frequency oscillates around 51 Hz with a maximum deviation of 23,550 Hz.
- The carrier frequency is the center frequency of the FM signal, which is the average of the instantaneous frequency.

The Carrier Frequency: Final Calculation

The key insight is that the carrier frequency (f_c) is the frequency around which the FM is centered.

From the above:

$f_c = 51\ Hz$

This is the carrier frequency of the FM signal.

Additional Insights and Clarifications

Why is the Carrier Frequency 51 Hz?

- The linear term in phase ($102\pi t$) corresponds to a base frequency:

$f_c = (\text{coefficient of } t \text{ in phase}) / (2\pi)$

- Since the phase coefficient is 102π, dividing by 2π gives:

$f_c = 102\pi / 2\pi = 51\ Hz$

This confirms that the carrier frequency is 51 Hz.

Modulation Index and Frequency Deviation

In FM, the modulation index (β) is given by:

$\beta = \Delta f / f_m$

Where:
- Δf is the maximum frequency deviation,
- f_m is the modulating frequency.

From the phase:

$10 \sin(4710\pi t)$

- The amplitude 10 in the phase corresponds to the maximum phase deviation.

The maximum frequency deviation Δf is:

$\Delta f = (d/dt \text{ of phase deviation})/2\pi$

Since the phase deviation is $10 \sin(4710\pi t)$, its derivative:

$d/dt\,[10 \sin(4710\pi t)] = 10 \times 4710\pi \cos(4710\pi t)$

Maximum value:

Maximum $d/dt = 10 \times 4710\pi = 47100\pi$

Dividing by 2π:

$\Delta f = 47100\pi / 2\pi = 23{,}550$ Hz

The modulating frequency f_m is:

$f_m = (4710\pi) / (2\pi) = 2355$ Hz

Thus, the modulation index:

$\beta = \Delta f / f_m = 23{,}550 / 2{,}355 = 10$

which matches the amplitude of the sinusoidal phase deviation, confirming the consistency of the analysis.

Summary of Key Findings

Parameter	Value	Explanation
Carrier frequency (f_c)	51 Hz	Derived from phase coefficient ($102\pi/2\pi$)
Modulating frequency (f_m)	2,355 Hz	From derivative of phase sinusoid ($4710\pi/2\pi$)
Frequency deviation (Δf)	23,550 Hz	Max derivative of phase deviation divided by 2π
Modulation index (β)	10	Ratio of Δf to f_m

Practical Considerations and Applications

Understanding how to extract the carrier frequency from an FM signal is crucial in various applications:

- Receiver Design: Accurate carrier frequency estimation ensures proper demodulation.
- Spectral Analysis: Knowing the carrier helps in spectral shaping and filtering.
- Communication System Optimization: Adjusting the carrier and modulation parameters for optimal bandwidth utilization.

Summary and Final Remarks

In this comprehensive guide, we analyzed the given FM signal:

XFM(t) = 10 cos (102π t + 10 sin (4710π t))

to determine its carrier frequency. By examining the phase term and differentiating accordingly, we identified that:

- The carrier frequency (f_c) is 51 Hz.
- The modulation involves a sinusoid at approximately 2,355 Hz with a frequency deviation of about 23,550 Hz and a modulation index of 10.

This method of extracting the carrier frequency from the phase expression is fundamental in signal processing. It highlights the importance of understanding phase relationships and derivatives in analyzing complex modulation schemes. Whether for designing communication systems or interpreting signals in the field, mastering these principles is essential for accurate signal analysis and system performance.

References and Further Reading

- Proakis, J. G., & Salehi, M. (2008). Digital Communications. McGraw-Hill.
- Haykin, S. (2001). Communication Systems. Wiley.
- Lyons, R. G. (2010). Understanding Digital Signal Processing. Pearson.
- Online resources on FM modulation, phase analysis, and spectral analysis techniques.

By mastering the analysis of signals like XFM(t), engineers and students can develop a deeper understanding of the intricacies of frequency modulation and its vital role in modern communications.

Frequency Modulated Fm Signal Xfm T 10cos 102mt 10 Sin 4710t Is Given A Find The Carrier

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