

# From A Population With A Variance Of 529, A Sample Of 289 Items Is Selected. What Is The Margin Of Error

## From A Population With A Variance Of 529, A Sample Of 289 Items Is Selected. What Is The Margin Of Error

Understanding margin of error is essential in statistics, especially when interpreting the reliability of sample data in relation to the entire population. In this article, we will explore how to determine the margin of error given a population variance of 529 and a sample size of 289 items. We will delve into the foundational concepts, step-by-step calculation procedures, and practical implications to help you grasp this important statistical measure.

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## Introduction to Margin of Error in Statistics

The margin of error (MOE) quantifies the amount of random sampling error in survey results or statistical estimates. It provides a range within which the true population parameter (e.g., mean, proportion) is expected to fall, with a specified confidence level. The lower the margin of error, the more precise the estimate.

Key points:

- It reflects the uncertainty inherent in using a sample to estimate a population parameter.
- Commonly expressed as a percentage or a decimal.
- Influenced by sample size, variability in the population, and the confidence level chosen.

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## Understanding Population Variance and Its Role

Population variance is a measure of the spread or dispersion of data points in a population. It indicates how much individual data points differ from the population mean.

Given Data:

- Population Variance ( $\sigma^2$ ) = 529
- Sample Size ( $n$ ) = 289

Implications:

- The population standard deviation ( $\sigma$ ) =  $\sqrt{\sigma^2} = \sqrt{529} = 23$

Knowing the population variance allows us to determine the standard deviation, which is

critical in calculating the margin of error, especially when the population standard deviation is known or estimated.

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## Calculating the Margin of Error: Step-by-Step Guide

To compute the margin of error, especially for the mean, the following formula is typically used when the population standard deviation is known:

$$\text{Margin of Error (MOE)} = z (\sigma / \sqrt{n})$$

Where:

- $z$  = z-score corresponding to the desired confidence level (e.g., 1.96 for 95% confidence)
- $\sigma$  = population standard deviation
- $n$  = sample size

Step 1: Determine the Confidence Level and Corresponding z-Score

Common confidence levels and their z-scores:

| Confidence Level | z-Score |
|------------------|---------|
| 90%              | 1.645   |
| 95%              | 1.96    |
| 99%              | 2.576   |

For most practical purposes, a 95% confidence level is standard unless specified otherwise.

Step 2: Calculate the Standard Error

$$\text{Standard Error (SE)} = \sigma / \sqrt{n}$$

Using the given data:

- $\sigma = 23$
- $n = 289$

$$SE = 23 / \sqrt{289} = 23 / 17 = 1.3529 \text{ (approximate)}$$

Step 3: Compute the Margin of Error

Assuming a 95% confidence level:

$$MOE = z SE = 1.96 \times 1.3529 \approx 2.65$$

Result: The margin of error is approximately  $\pm 2.65$  units for the population mean at 95% confidence.

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## Understanding the Significance of the Margin of Error

The calculated margin of error provides vital insight into the precision of the sample estimate:

- A lower MOE indicates higher precision.
- The MOE depends on the variability in the population ( $\sigma$ ) and the sample size ( $n$ ).
- Increasing the sample size reduces the MOE, leading to more accurate estimates.

In our scenario, a margin of error of approximately  $\pm 2.65$  suggests that the true population mean is likely within 2.65 units of the sample mean, with 95% confidence.

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## Factors Affecting the Margin of Error

Several factors influence the magnitude of the margin of error:

Population Variance (or Standard Deviation)

- Higher variability increases the MOE.
- Knowing the population variance allows for precise calculations.

Sample Size

- Larger samples reduce the MOE.
- Doubling the sample size decreases the MOE by approximately the square root of 2.

Confidence Level

- Higher confidence levels (e.g., 99%) lead to larger z-scores, increasing the MOE.
- Lower confidence levels (e.g., 90%) result in smaller MOEs.

Practical tips:

- To improve estimate precision, increase the sample size.
- Balance the cost of data collection with the desired accuracy.

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## Practical Example: Applying the Concepts

Suppose you conduct a survey within a population where the variance is known to be 529. You randomly select 289 items and compute a sample mean of some measurement. To report the estimate with 95% confidence, you calculate the margin of error as approximately  $\pm 2.65$  units.

Implications:

- If the sample mean is, for example, 50, then the population mean is likely between 47.35 and 52.65.
- This range helps policymakers or researchers understand the reliability of the estimate.

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## Limitations and Assumptions in Margin of Error Calculations

While the calculations seem straightforward, certain assumptions underpin the process:

- Normality Assumption: The sampling distribution of the mean is approximately normal, especially for large  $n$ , due to the Central Limit Theorem.
- Known Population Variance: In practice, population variance is often unknown; in such cases, the sample variance is used along with the t-distribution.
- Random Sampling: The sample must be randomly selected to ensure representativeness.

Note: If the population variance is unknown, the margin of error is calculated using the sample standard deviation and the t-distribution, which affects the z-score used.

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## Conclusion: Summary of Key Points

- The population variance of 529 translates to a population standard deviation of 23.
- With a sample size of 289, the standard error is approximately 1.35.
- Using a 95% confidence level, the margin of error is approximately  $\pm 2.65$ .
- The margin of error indicates the range within which the true population mean likely falls.
- Increasing the sample size reduces the margin of error, improving estimate precision.
- Confidence levels influence the z-score and, consequently, the margin of error.
- Accurate margin of error estimation relies on assumptions like normality and randomness.

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# Final Thoughts and Practical Applications

Understanding and calculating the margin of error is fundamental in fields like survey research, quality control, and data analysis. It allows researchers and analysts to quantify the uncertainty associated with their estimates and make informed decisions.

In your statistical endeavors, always consider the population variability, sample size, and confidence level to accurately interpret your results. When working with known population variance, leveraging this information can lead to precise and meaningful margins of error, ultimately enhancing the credibility and reliability of your findings.

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Keywords for SEO Optimization:

- Margin of error calculation
- Population variance and standard deviation
- Sample size importance
- Confidence level in statistics
- How to calculate margin of error
- Statistical estimation techniques
- Confidence intervals in research
- Sampling error reduction
- Population mean estimation

Meta Description:

Learn how to calculate the margin of error for a population with a variance of 529 and a sample size of 289 items. This comprehensive guide explains the concepts, formulas, and practical implications to improve your statistical analysis.

## Frequently Asked Questions

### **What is the formula to calculate the margin of error for a population with a known variance?**

The margin of error (E) is calculated as  $E = z (\sigma / \sqrt{n})$ , where z is the z-score for the desired confidence level,  $\sigma$  is the population standard deviation (square root of variance), and n is the sample size.

### **Given a population variance of 529 and a sample size of 289, how do you determine the margin of error at a 95% confidence level?**

First, find the population standard deviation:  $\sigma = \sqrt{529} = 23$ . Then, for a 95% confidence level,  $z \approx 1.96$ . Calculate  $E = 1.96 (23 / \sqrt{289}) = 1.96 (23 / 17) \approx 1.96 \cdot 1.3529 \approx 2.65$ .

## **Why is it important to know the population variance when calculating the margin of error?**

Knowing the population variance allows for an accurate calculation of the population standard deviation, which is essential for determining the margin of error, especially when the population standard deviation is known or estimated from variance.

## **How does increasing the sample size affect the margin of error in this scenario?**

Increasing the sample size reduces the margin of error because the standard error ( $\sigma / \sqrt{n}$ ) decreases as  $n$  increases, leading to more precise estimates.

## **Can the margin of error be directly calculated without the population standard deviation? If so, how?**

Yes, if the population standard deviation is unknown, the margin of error can be estimated using the sample standard deviation and the t-distribution instead of the z-distribution, especially for smaller samples.

## **Additional Resources**

Margin of Error: Unlocking the Confidence in Your Data Analysis

Understanding the intricacies of statistical measures is essential in making informed decisions across various fields—from market research and social sciences to quality control and public policy. Among these measures, the margin of error (MoE) plays a pivotal role in determining the reliability of survey results, sample estimations, and data-driven conclusions. Today, we dive deep into an intriguing problem: From a population with a variance of 529, a sample of 289 items is selected. What is the margin of error?—a question that encapsulates core principles of inferential statistics and showcases the importance of precision in data interpretation.

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## **Understanding the Fundamentals: Variance, Sample Size, and Margin of Error**

Before tackling the specific problem, it's crucial to understand the foundational concepts involved.

## Variance: The Measure of Data Dispersion

Variance quantifies the spread or dispersion within a dataset. It measures how much individual data points differ from the mean:

$$\sigma^2 = \frac{1}{N} \sum_{i=1}^N (X_i - \mu)^2$$

where:

- $\sigma^2$  is the population variance,
- $X_i$  are individual data points,
- $\mu$  is the population mean,
- $N$  is the total number of data points.

In our context, the population variance is given as 529, which indicates a certain degree of variability in the entire population.

## Sample Size: The Number of Observations

Sample size ( $n$ ) impacts the accuracy and reliability of statistical estimates. Larger samples tend to produce estimates closer to the true population parameters, reducing sampling error.

In our scenario:

- $n = 289$

This is a substantial sample size, which generally leads to more precise estimates.

## Margin of Error: Confidence and Precision

The margin of error quantifies the amount of random sampling error associated with a survey or estimate. It indicates the range within which the true population parameter is expected to lie, with a specified level of confidence (commonly 95%).

Mathematically, for a population mean:

$$\text{MoE} = z \times \frac{\sigma}{\sqrt{n}}$$

where:

- $z$  is the z-score corresponding to the desired confidence level,
- $\sigma$  is the population standard deviation,
- $n$  is the sample size.

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# Step-by-Step Calculation of the Margin of Error

Having established fundamental concepts, let's now methodically calculate the margin of error for our problem.

## Step 1: Extract Known Data

| Parameter                          | Value | Explanation                       |
|------------------------------------|-------|-----------------------------------|
| Population variance ( $\sigma^2$ ) | 529   | Variance of the entire population |
| Sample size ( $n$ )                | 289   | Number of items sampled           |

From the variance, the standard deviation ( $\sigma$ ) can be derived:

$$\sigma = \sqrt{\sigma^2} = \sqrt{529} = 23$$

This value signifies the typical deviation of individual data points from the mean in the population.

## Step 2: Determine the Confidence Level and Corresponding $z^*$

The margin of error depends on the confidence level, which indicates how certain we are that the true parameter lies within our interval. Common choices include:

- 90% confidence:  $z^* \approx 1.645$
- 95% confidence:  $z^* \approx 1.96$
- 99% confidence:  $z^* \approx 2.576$

Without an explicit confidence level specified, the most typical choice is 95%, which balances precision and certainty.

## Step 3: Calculate the Standard Error (SE)

The standard error measures the variability of the sample mean estimate:

$$SE = \frac{\sigma}{\sqrt{n}} = \frac{23}{\sqrt{289}} = \frac{23}{17} \approx 1.3529$$

A smaller SE indicates that the sample mean more closely approximates the true population mean.

## Step 4: Compute the Margin of Error

Applying the formula:

$$\text{MoE} = z^* \times SE$$

for a 95% confidence level ( $z^* \approx 1.96$ ):

$$\text{MoE} = 1.96 \times 1.3529 \approx 2.65$$

Thus, the margin of error is approximately 2.65 units.

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## Interpreting the Results: What Does the Margin of Error Tell Us?

The calculated margin of error, approximately 2.65, signifies that with 95% confidence, the true population mean lies within  $\pm 2.65$  units of the sample mean.

This interpretation is crucial when presenting survey results or making data-driven decisions, as it contextualizes the accuracy of the estimate. For instance, if the sample mean is 50, then the true population mean is likely between 47.35 and 52.65, with 95% confidence.

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## Factors Influencing the Margin of Error

Understanding what affects the margin of error helps in designing better studies:

### 1. Sample Size

- Larger samples reduce the MoE.
- Since  $\text{MoE} \propto 1/\sqrt{n}$ , quadrupling the sample size halves the margin of error.

### 2. Variability in the Population

- Higher variance (or standard deviation) increases the MoE.
- Efforts to reduce variability, such as stratified sampling, can improve precision.

### 3. Confidence Level

- A higher confidence level (e.g., 99%) increases the  $(z^{\wedge})$  value, thus enlarging the MoE.
- The choice depends on the desired certainty versus precision trade-off.

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## Practical Implications and Applications

In real-world scenarios, the margin of error influences how data is communicated and interpreted:

- Market Research: Understanding customer preferences with a known MoE helps in strategic planning.
- Public Policy: Ensuring survey results accurately reflect population attitudes guides policy decisions.
- Quality Control: Precise estimates of defect rates inform manufacturing processes.

In our specific case, knowing the margin of error allows stakeholders to assess the reliability of their sample estimates, plan future studies with appropriate sample sizes, and communicate findings transparently.

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## Extensions and Considerations

While the above calculation assumes the population standard deviation is known and the data is approximately normally distributed, real-world data may require additional considerations:

- Unknown Population Variance: When  $(\sigma)$  is unknown, the t-distribution is used, especially with smaller samples.
- Non-Normal Distributions: For skewed or non-normal data, alternative methods or larger sample sizes may be necessary.
- Finite Population Correction (FPC): If sampling from a small finite population, an FPC factor adjusts the MoE accordingly:

$$\text{FPC} = \sqrt{\frac{N - n}{N - 1}}$$

where  $(N)$  is the population size.

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# Conclusion: The Power of Precise Estimation

Calculating the margin of error is a fundamental step in inferential statistics, providing a quantified measure of the uncertainty inherent in sample estimates. In our scenario—sampling 289 items from a population with a variance of 529—the margin of error at a 95% confidence level is approximately 2.65 units.

This figure empowers data analysts, researchers, and decision-makers to interpret their findings with confidence, understanding the bounds of their estimations. It emphasizes the importance of sample size, population variability, and confidence levels in designing studies that yield reliable, actionable insights. Whether in academic research, business analytics, or public health, mastering the concept of the margin of error ensures that data-driven decisions are both informed and credible.

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