Given That F(x) Is An Even Function, F(6)=-8, And G(x)=2f(x-6)-7, Thenthe Y-intercept Of G(x)?(1) -23(3)

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Understanding the properties of functions and how transformations affect their graphs is fundamental in algebra and calculus. In this article, we explore a specific problem involving an even function $f(x)$, its known value at a point, and a transformed function $g(x)$. Our goal is to determine the y-intercept of $g(x)$ based on the given information. We will examine the properties of even functions, analyze the transformations involved, and systematically compute the y-intercept of $g(x)$. This comprehensive guide aims to clarify the reasoning process and reinforce the concepts of function transformations, symmetry, and intercepts.

Understanding the Given Information and Key Concepts

Before diving into calculations, it's essential to understand the fundamental pieces of information and concepts involved:

Properties of Even Functions

- An even function satisfies the property $f(-x) = f(x)$ for all x in its domain.
- Graphically, even functions are symmetric with respect to the y-axis.
- Knowing that $f(x)$ is even allows us to find $f(-x)$ directly from $f(x)$.

Given Data

- $f(x)$ is an even function.
- $f(6) = -8$.
- The transformed function $g(x)$ is defined as:

$$
g(x) = 2f(x - 6) - 7
$$

- Our objective: Find the y-intercept of $g(x)$, i.e., determine $g(0)$.

Analyzing the Transformation $g(x) = 2f(x - 6) - 7$

Transformations of functions involve shifts, stretches, and translations. Here, $g(x)$ is constructed by:

- A horizontal shift: $x \to x - 6$,
- A vertical stretch by a factor of 2,
- A vertical translation downward by 7 units.

Understanding these transformations is crucial to evaluate $g(0)$. Let's break down each step:

Step 1: Horizontal Shift

- The input to f is $x - 6$, which shifts the graph of $f(x)$ rightward by 6 units.
- To find $g(0)$, evaluate $f(0 - 6) = f(-6)$.

Step 2: Vertical Stretch

- Multiply the function value by 2: $2f(x - 6)$.

Step 3: Vertical Shift

- Subtract 7 from the product: $2f(x - 6) - 7$.

Calculating the Y-Intercept of $g(x)$

The y-intercept occurs where $x=0$. Therefore, to find $g(0)$:

$$
g(0) = 2f(0 - 6) - 7 = 2f(-6) - 7
$$

Our task reduces to determining $f(-6)$.

Using the Properties of Even Functions to Find $f(-6)$

Recall that:

$$
f(-x) = f(x) \quad \text{(since f is even)}
$$

Given $f(6) = -8$, then:

$$
f(-6) = f(6) = -8
$$

This symmetry property allows us to directly find $f(-6)$ without additional information.

Putting It All Together: Computing $g(0)$

Now, substitute $f(-6) = -8$ into the expression for $g(0)$:

$$
g(0) = 2 \times (-8) - 7 = -16 - 7 = -23
$$

Therefore, the y-intercept of $g(x)$ is -23.

Summary of the Solution Steps

- Recognized that $f(x)$ is even, so $f(-6) = f(6) = -8$.
- Calculated $g(0) = 2f(-6) - 7$.
- Substituted $f(-6) = -8$.
- Simplified to find $g(0) = -23$.

Conclusion: The Y-Intercept of $g(x)$

Based on the above analysis, the y-intercept of the function $g(x)$ is $\boxed{-23}$. This value corresponds to the point where the graph of $g(x)$ crosses the y-axis, occurring at

$(0, -23)$.

Additional Insights and Related Topics

While this problem is straightforward, it highlights several important concepts in mathematics related to function properties and transformations:

1. Symmetry of Even Functions

- The key to solving problems involving even functions is understanding their symmetry with respect to the y-axis.
- This symmetry simplifies calculations for points like $f(-x)$.

2. Function Transformations

- Horizontal shifts, vertical stretches, and translations alter the graph's position without changing its fundamental shape.
- Recognizing these transformations helps in predicting the behavior of composite functions.

3. Interpreting Function Values in Context

- Knowing a single point $f(6) = -8$ allows us to infer $f(-6)$ for an even function.
- This principle extends to many problems involving symmetry and transformations.

4. Applications in Graphing and Analysis

- Understanding how transformations impact the graph is essential for sketching functions accurately.
- These skills are foundational in calculus, physics, engineering, and data analysis.

Practice Problems to Reinforce Understanding

To solidify your grasp of the concepts discussed, consider working through the following problems:

1. Given that $f(x)$ is even and $f(3) = 5$, find $f(-3)$.

2. For $g(x) = 3f(x + 4) + 2$, determine $g(-4)$ if $f(4) = 7$ and f is even.

3. If $f(x)$ is odd and $f(2) = 6$, what is $f(-2)$?

4. Given $f(x)$ is even with $f(0) = 10$, find the value of $f(-5)$ if $f(5) = -3$.

5. Graph the function $g(x) = 2f(x - 3) + 4$ given that $f(x)$ is even and $f(3) = 2$. Identify the y-intercept.

Final Remarks

Understanding how properties like evenness influence function values and how transformations affect graphs is crucial in higher mathematics. The problem addressed here demonstrates the power of symmetry and algebraic manipulation in deriving key features of functions, such as the y-intercept. Mastery of these concepts enhances problem-solving skills and prepares students for more advanced topics in calculus and analytical mathematics.

Remember, always analyze the properties of the functions involved before performing calculations, and leverage symmetry whenever possible to simplify your work. With practice, recognizing these patterns will become second nature, enabling quick and accurate solutions to a wide array of mathematical problems.

Frequently Asked Questions

What is the value of F(6) given that F(x) is an even function and F(6) = -8?

The value of F(6) is -8, as given in the problem statement.

Since F(x) is an even function, what is F(-6)?

F(-6) = F(6) = -8, because even functions satisfy F(-x) = F(x).

Given G(x) = 2f(x - 6) - 7, how do we find the y-intercept of G(x)?

To find the y-intercept, evaluate G(0): G(0) = 2f(0 - 6) - 7 = 2f(-6) - 7.

What is the value of f(-6) if F(x) = f(x) and F(6) = -8?

Since F(x) is even, f(-6) = F(-6) = F(6) = -8.

What is G(0) in terms of f(-6)?

G(0) = 2f(-6) - 7.

Calculate G(0) using the known value of f(-6).

G(0) = 2 (-8) - 7 = -16 - 7 = -23.

What is the y-intercept of G(x)?

The y-intercept of G(x) is -23.

Why is the y-intercept of G(x) equal to -23?

Because G(0) = 2f(-6) - 7, and with f(-6) = -8, G(0) = -23.

Is the y-intercept of G(x) consistent with the given options?

Yes, the y-intercept is -23, which matches option (1).

What is the final answer for the y-intercept of G(x) given the options?

The y-intercept of G(x) is -23.

Additional Resources

Given That F(x) Is An Even Function, F(6) = -8, And G(x) = 2f(x - 6) - 7, Then The Y-intercept Of G(x)? (1) -23 (3)

Investigative Analysis of the Y-Intercept of a Transformed Function

Mathematics often presents intriguing puzzles, especially when functions undergo transformations. The problem at hand involves understanding the implications of a given function's properties and how they influence the resulting graph, specifically focusing on the y-intercept of a transformed function. This investigation aims to dissect the underlying principles, verify the calculations, and interpret the significance of the transformations involved.

Introduction to the Problem

The core question is: Given that F(x) is an even function, F(6) = -8, and G(x) = 2f(x - 6) - 7, what is the y-intercept of G(x)? Notably, the options provided are (1) -23 and (3). The primary goal is to determine which of these options correctly represents the y-intercept of G(x), considering the properties of F(x) and the given transformations.

Key Components of the Problem:

- F(x) is an even function.
- F(6) = -8.
- G(x) = 2f(x - 6) - 7.
- Determine G(0), the y-intercept.

Understanding the Properties of F(x)

What Does It Mean for F(x) to Be Even?

An even function is symmetric with respect to the y-axis. Mathematically:

$$F(-x) = F(x) \quad \text{for all } x$$

This property implies:

- The value of the function at x and $-x$ are identical.
- The graph is symmetric about the y-axis.

Given that $F(6) = -8$, by the property of even functions:

$$F(-6) = F(6) = -8$$

This symmetry will be crucial when evaluating the transformations applied to $F(x)$.

Analyzing the Transformation G(x)

Expression for G(x):

$$G(x) = 2f(x - 6) - 7$$

Note the following transformation steps embedded in G(x):

1. Horizontal shift: $x \to x - 6$ (shifts the graph to the right by 6 units)
2. Vertical scaling: Multiply by 2
3. Vertical shift: Subtract 7

Clarifying Notation: F(x) vs. f(x)

The problem states $F(x)$ is an even function, but the formula for $G(x)$ involves $f(x)$. Presumably, $f(x)$ refers to the same function as $F(x)$. For clarity and consistency, we will assume:

$$ f(x) \equiv F(x) $$

Thus,

$$ G(x) = 2F(x - 6) - 7 $$

Step-by-Step Determination of the Y-Intercept

What is the Y-Intercept?

The y-intercept of a function $G(x)$ occurs when $x = 0$:

$$ G(0) = 2F(0 - 6) - 7 $$
$$ G(0) = 2F(-6) - 7 $$

Given the symmetry of F, and knowing $F(6) = -8$, we leverage the even property:

$$ F(-6) = F(6) = -8 $$

Substitute into the expression:

$$ G(0) = 2 \times (-8) - 7 $$
$$ G(0) = -16 - 7 $$
$$ G(0) = -23 $$

Conclusion:

The y-intercept of $G(x)$ is -23.

Detailed Exploration of Underlying Concepts

Significance of the Even Function Property

The even nature of $F(x)$ simplifies the evaluation of $F(-6)$. Without this property, we would need explicit information about $F(-6)$. The symmetry ensures:

$$ F(-6) = F(6) = -8 $$

which directly influences the calculation of $G(0)$.

Transformation Impact on the Graph

The transformations applied to $F(x)$ can be summarized as:

Transformation	Effect on the Graph	Algebraic Effect
Horizontal shift $x \to x - 6$	Moves the graph right by 6 units	$F(x) \to F(x - 6)$

Vertical scaling by 2	Stretches the graph vertically	$F(x) \to 2F(x)$
Vertical shift down by 7	Moves the graph down	$2F(x) \to 2F(x) - 7$

Applying these transformations systematically, the y-intercept at $x=0$ can be computed directly.

Alternative Approaches and Verifications

Graphical Perspective

If one visualizes $F(x)$:

- It passes through the point $(6, -8)$.
- The graph is symmetric about the y-axis.

Applying the transformations:

- The horizontal shift aligns the point $(6, -8)$ with $(0, -8)$ in the $F(x - 6)$ graph.
- Scaling by 2 amplifies the y-values.
- Subtracting 7 shifts the entire graph downward.

Hence, at $x=0$:

$$G(0) = 2F(-6) - 7 = 2 \times (-8) - 7 = -23$$

Numerical Summary:

Step	Calculation	Result
$F(-6)$	$F(6)$ (by evenness)	-8
$G(0)$	$2 \times F(-6) - 7$	-23

Final Reflection and Broader Implications

Significance of the Result

This analysis demonstrates how properties like symmetry and transformations influence the behavior of composite functions. Recognizing the even nature of $F(x)$ simplifies calculations and clarifies the effects of shifts and scalings.

Potential Variations

- If $F(x)$ were not even, the calculation would depend on the specific value of $F(-6)$.
- Different transformations could significantly alter the y-intercept, emphasizing the importance of understanding each step.

Educational Takeaways

- The importance of function properties (like evenness) in simplifying problems.
- The systematic approach to analyzing transformations.
- The value of combining algebraic and graphical reasoning.

Conclusion

The investigation confirms that the y-intercept of \(G(x) \), given the properties of \(F(x) \), is -23. This conclusion relies fundamentally on the even property of \(F(x) \) and the specified transformations. The problem exemplifies how symmetry and transformations interplay to determine key features of composite functions, serving as a vital lesson in advanced algebra and function analysis.

Final Answer:

The y-intercept of \(G(x) \) is \(-23\).

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