Given The Functions H(x) = 9x +1 And F(x)=(x/5)^2 , On A Standard Coordinate Graph Of Y = H(f(x)), Which

Given The Functions H(x) = 9x +1 And F(x)=(x/5)^2 , On A Standard Coordinate Graph Of Y = H(f(x)), Which presents an intriguing exploration into the composition of functions and their graphical representations. Understanding how these functions interact and transform the coordinate plane is fundamental in algebra and calculus, enabling students and mathematicians alike to analyze complex relationships visually. This article will delve into the properties of these functions, how to graph the combined function Y = H(f(x)), and interpret the resulting graph's features.

Understanding the Given Functions

Before examining the composite function Y = H(f(x)), it is essential to understand each component individually.

Function H(x) = 9x + 1

This is a linear function characterized by:

- Slope (m): 9, indicating a steep increase.
- Y-intercept (b): 1, where the line crosses the y-axis.

Graphically, H(x) is a straight line passing through (0,1), rising rapidly due to its slope of 9.

Function $F(x) = (x/5)^2$

This is a quadratic function, specifically a parabola opening upwards, with:

- Vertex at (0,0): Since the squared term is zero at $x=0$.
- Shape: Wider than standard $y = x^2$ because of the division by 5, which stretches the graph along the x-axis.

The function produces non-negative outputs for all real x, with the minimum at $x=0$.

Constructing the Composite Function $Y = H(f(x))$

The composite function involves applying $F(x)$ first, then $H(x)$ to the result:

- Step 1: Evaluate $f(x) = (x/5)^2$.
- Step 2: Plug $f(x)$ into $H(x)$: $Y = H(f(x)) = 9 f(x) + 1$.

Expressed explicitly:

$$ Y = 9 \times \left(\frac{x}{5}\right)^2 + 1 $$

which simplifies to:

$$ Y = 9 \times \frac{x^2}{25} + 1 = \frac{9x^2}{25} + 1 $$

This is a quadratic function with a vertical stretch and a shift.

Graphing Y = H(f(x))

Understanding the graph involves analyzing the shape, key features, and transformations.

Shape and General Form

The resulting function:

$$ Y = \frac{9x^2}{25} + 1 $$

is a parabola opening upwards, with:

- Vertex at (0,1): Since the quadratic term is zero at $x=0$, $y=1$.
- Axis of symmetry: $x=0$.
- Minimum point: At the vertex, $y=1$.

This parabola is wider than the standard $y = x^2$ because the coefficient $\frac{9}{25}$ is less than 1, indicating a less steep parabola compared to $y = x^2$.

Key Features of the Graph

- Vertex: Located at (0,1).
- Y-intercept: When $x=0$, $y=1$.
- X-intercepts: The points where $y=0$. Solve for x:

$$
0 = \frac{9x^2}{25} + 1 \Rightarrow \frac{9x^2}{25} = -1
$$

Since $\frac{9x^2}{25}$ is non-negative, it cannot equal -1, indicating there are no x-intercepts. The parabola does not cross the x-axis.

- Domain: All real numbers $\mathbb{R}$, because quadratic functions are defined everywhere.
- Range: $[1, \infty)$, because the minimum y-value is 1 at the vertex.

Transformations and Interpretations

Analyzing the composition reveals several transformations from the basic quadratic and linear functions.

Scaling and Stretching

- The quadratic is multiplied by $\frac{9}{25}$, which is less than 1, resulting in a wider parabola compared to $y = x^2$.
- This indicates a vertical compression factor of $\frac{9}{25}$.

Vertical Shift

- The +1 shifts the entire parabola upward by 1 unit.

Implications for the Graph

- The vertex at (0,1) is higher than the origin.
- The parabola opens upward and is less steep than $y = x^2$, which would have a coefficient of 1.

Applications and Significance of the Graph

Understanding the graph of Y = H(f(x)) has practical applications in various fields such as physics, engineering, and economics, where modeling relationships involving transformations of functions is essential.

Analyzing Function Behavior

- The composite function shows how applying a quadratic transformation followed by a linear transformation affects the shape and position of the graph.
- Recognizing these transformations helps in predicting the behavior of more complex functions.

Interpreting Real-World Data

- For example, if f(x) models a physical quantity such as energy or displacement, then applying H(x) can represent a linear response or scaling factor.
- The shape of the parabola indicates the nature of the relationship, such as minimum or maximum points, symmetry, and end behavior.

Advanced Insights: Derivatives and Critical Points

For those interested in calculus, analyzing the derivative of Y = $\frac{9x^2}{25} + 1$ provides insights into the slope and rate of change.

Calculating the Derivative

$$
Y' = \frac{d}{dx} \left(\frac{9x^2}{25} + 1\right) = \frac{18x}{25}
$$

- The derivative is zero at x=0, confirming the vertex is a critical point.

- For $x<0$, the derivative is negative, indicating decreasing function.
- For $x>0$, the derivative is positive, indicating increasing function.

Interpretation

- The parabola is symmetric, with the minimum point at the vertex.
- The rate of change increases linearly with x.

Conclusion: Visualizing and Understanding the Composite Function

Graphing the function Y = H(f(x)) where H and F are given functions reveals how quadratic and linear functions combine to produce a wider parabola shifted upward. Recognizing the transformations involved—scaling, shifting, and symmetry—is crucial for interpreting the graph's features and behavior. Such analysis is invaluable in both theoretical mathematics and practical applications, enabling precise modeling of real-world phenomena. With a solid understanding of the individual functions and their composition, students and professionals can better analyze complex relationships and make informed predictions based on the graphical representations.

Keywords: composite functions, graph transformations, quadratic functions, linear functions, parabola, coordinate graph, function analysis, mathematical modeling, algebra, calculus

Frequently Asked Questions

What is the composite function y = H(f(x)) when H(x) = 9x + 1 and F(x) = (x/5)^2?

The composite function $y = H(f(x))$ becomes $y = 9 ((x/5)^2) + 1$, which simplifies to $y = 9 (x^2 / 25) + 1 = (9/25) x^2 + 1$.

What is the shape of the graph of y = H(f(x))?

The graph of $y = H(f(x))$ is a parabola opening upwards since it simplifies to a quadratic function in x.

How does the transformation H(x) = 9x + 1 affect the graph of F(x) = (x/5)^2?

Applying H to F stretches the graph vertically by a factor of 9 and shifts it upward by 1 unit.

What is the domain and range of y = H(f(x))?

The domain is all real numbers, and the range is $y \geq 1$, since the quadratic term is always non-negative and multiplied by 9, then shifted upward by 1.

At what x-value does y = H(f(x)) equal 10?

Set $(9/25) x^2 + 1 = 10$, then $(9/25) x^2 = 9$, so $x^2 = 25$, giving $x = \pm 5$.

What are the vertex coordinates of the graph y = H(f(x))?

The vertex occurs at $x = 0$, $y = (9/25)(0)^2 + 1 = 1$, so the vertex is at (0, 1).

How does changing the coefficient in H(x) = 9x + 1 affect the parabola's steepness?

Increasing the coefficient makes the parabola steeper; decreasing it makes it wider and less steep.

Is the graph of y=H(f(x)) symmetric? If so, about which axis?

Yes, since the quadratic is even in x, the parabola is symmetric about the y-axis.

Additional Resources

Mathematical Transformation Analysis: Exploring the Graph of Y = H(f(x)) with H(x) = 9x + 1 and F(x) = (x/5)^2

Introduction

In the world of mathematics, particularly in algebra and functions, understanding how composite functions behave and how their graphs look is fundamental. When dealing with transformations, the ability to interpret and visualize complex functions like Y = H(f(x)) provides insights into their properties, such as stretching, shifting, and reflection.

This article offers an in-depth exploration of the composite function Y = H(f(x)) where:

- $H(x) = 9x + 1$ (a linear function)
- $F(x) = \left(\frac{x}{5}\right)^2$ (a quadratic function)

Our goal is to analyze the graph of $Y = H(F(x))$ on a standard coordinate grid, breaking down the transformation process, understanding the resulting shape, and discussing key features that emerge from this composition.

Understanding the Component Functions

Before diving into the composite function, it's essential to understand the individual functions involved.

Linear Function: H(x) = 9x + 1

- Nature of H(x):

This is a linear function with a slope of 9 and a y-intercept at 1.

- Slope (m = 9): For every unit increase in x, y increases by 9 units, indicating a steep positive slope.
- Y-intercept: The point (0, 1) where the line crosses the y-axis.

- Graph Characteristics:
- Straight line with a steep incline.
- Rapidly increasing y-values for increasing x-values.
- Reflects a proportional relationship scaled by 9 and shifted upward by 1.

Quadratic Function: F(x) = (x/5)^2

- Nature of F(x):

This is a parabola opening upwards, scaled horizontally by a factor of 5.

- Vertex at (0, 0): The minimum point of the parabola.
- Shape: Symmetric around the y-axis, with the width governed by the division by 5 (i.e., wider parabola).
- Key points:
- At x = 0, F(0) = 0
- At x = ±5, F(±5) = (±5/5)^2 = 1
- At x = ±10, F(±10) = (±10/5)^2 = 4

- Graph Characteristics:

- U-shaped curve symmetric about the y-axis.

- The parabola widens as the divisor (5) increases, resulting in a flatter curve compared to a standard y = x^2.

Constructing the Composite Function: Y = H(F(x))

The composite function involves applying F(x) first, then transforming the result using H(x). This process is fundamental in understanding how functions combine and how their graphs transform.

Step 1: Compute F(x)

- For any input \(x \), \(F(x) = \left(\frac{x}{5}\right)^2 \).

- This yields a non-negative output since the square of any real number is non-negative.

Step 2: Apply H to F(x)

- \(Y = H(F(x)) = 9 \times F(x) + 1 \).

- Substituting F(x):

\[

Y = 9 \times \left(\frac{x}{5}\right)^2 + 1

\]

- Simplify:

\[

Y = 9 \times \frac{x^2}{25} + 1 = \frac{9}{25}x^2 + 1

\]

This simplifies our composite function to:

\[

\boxed{

Y = \frac{9}{25}x^2 + 1

}

\]

Graphical Analysis of Y = (9/25) x^2 + 1

The composite function reduces to a quadratic with a specific scale factor and shift, making it easier to analyze.

Key Features of the Graph

- Shape: Parabola opening upwards, similar to \(y = x^2 \) but scaled vertically.
- Vertex: Located at \((0, 1) \), since the function is shifted upward by 1.
- Axis of symmetry: The vertical line \(x = 0 \).
- Y-intercept: When x = 0, \(Y = 1 \).
- X-intercepts: Set \(Y = 0 \):

\[

0 = \frac{9}{25}x^2 + 1 \Rightarrow \frac{9}{25}x^2 = -1

\]

Since the right side is negative, there are no real x-intercepts—the parabola does not cross the x-axis, indicating the minimum point is above the x-axis.

- Vertex point: At $(0, 1)$, the minimum value of the function.

Understanding the Transformations

- Vertical scaling: The coefficient $\frac{9}{25}$ indicates the parabola is wider than the standard $y = x^2$, because $\frac{9}{25} \approx 0.36$, which is less than 1.
- Vertical shift: Upward by 1 unit, shifting the entire parabola above the x-axis.
- No horizontal shifts: The parabola is centered at the origin.

Visualizing the Graph on a Coordinate Plane

To truly grasp the shape and position, consider plotting key points:

x-value	$F(x) = (x/5)^2$	$Y = H(F(x))$	Notes
-10	4	$9 \times 4 + 1 = 37$	Far left, high y-value
-5	1	$9 \times 1 + 1 = 10$	Left vertex
0	0	1	Central point
5	1	10	Right mirror of x = -5
10	4	37	Far right

Plotting these points yields a smooth parabola opening upward, with the lowest point at (0, 1).

Implications of the Function Composition

Understanding the composite function's graph offers insights into the behavior of transformations and how combining functions affects shape and position.

Transformation Summary

- Scaling: The quadratic is scaled vertically by $\frac{9}{25}$, making it less steep than $y = x^2$.
- Translation: The entire parabola is shifted upward by 1 unit.
- Symmetry: The parabola remains symmetric around the y-axis, a property inherited from both the quadratic and the even nature of the original functions.

Behavioral Characteristics

- Growth rate: For large $|x|$, Y increases quadratically, dominating linear components.
- Minimum point: The vertex at $(0,1)$ indicates the lowest point of the parabola, important in optimization problems.
- No real x-intercepts: Since the parabola doesn't cross the x-axis, the minimum y-value is above zero, which could be relevant in certain applications requiring positive outputs.

Practical Applications and Further Insights

The analysis of such composite functions extends beyond pure mathematics, influencing fields like physics, economics, and engineering.

- Physics: Modeling motion where the displacement depends quadratically on time, scaled and shifted according to specific parameters.
- Economics: Cost functions or profit models that involve quadratic relationships combined with linear adjustments.
- Engineering: Designing systems with responses modeled by quadratic functions subjected to linear transformations.

Understanding these transformations enables professionals to predict behavior, optimize systems, and interpret complex data relationships effectively.

Conclusion

The composite function $Y = H(F(x))$, with given component functions, elegantly demonstrates how mathematical transformations work in tandem to produce a precise, predictable graph. By dissecting each step—from the individual functions to their combination—we see a parabola shifted upward by 1, scaled vertically to be wider and less steep than the standard quadratic.

This detailed analysis not only clarifies the shape and key features of the graph but also emphasizes the importance of understanding how transformations—stretching, shifting, and reflecting—affect the overall function. Mastery of these concepts enhances one's ability to interpret complex functions, predict their behavior, and apply such knowledge across various scientific and practical domains.

In summary:

- The composite function simplifies to $Y = \frac{9}{25}x^2 + 1$.
- Its graph is a parabola opening upwards, vertex at (0, 1).

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