Graph On The Same Axes: Y =x^2-4 And Y=4-x^2. Write An Equation For The Part Of The Graph Which Is Above

Graph On The Same Axes: Y = x^2 - 4 And Y = 4 - x^2. Write An Equation For The Part Of The Graph Which Is Above

Understanding how to analyze and interpret graphs of functions is a fundamental skill in mathematics. When two functions are graphed on the same axes, it allows us to explore their intersections, relative positions, and the regions where one graph lies above or below the other. In this article, we focus on the specific pair of functions: Y = x^2 - 4 and Y = 4 - x^2, and how to write an equation for the part of the graph which is above the other. This exploration involves graphing the functions, finding their points of intersection, and determining the regions where one graph surpasses the other.

Understanding the Functions: Y = x^2 - 4 and Y = 4 - x^2

Basic Characteristics of Y = x^2 - 4

- This is a parabola opening upwards because the coefficient of x^2 is positive.
- The vertex of this parabola is at the point $(0, -4)$, which is the minimum point.
- The parabola intersects the y-axis at -4.
- It is symmetric with respect to the y-axis.

Basic Characteristics of Y = 4 - x^2

- This is a parabola opening downwards since the coefficient of x^2 is negative.
- The vertex of this parabola is at $(0, 4)$, which is the maximum point.
- It intersects the y-axis at 4.
- It is symmetric about the y-axis as well.

Graphing the Functions on the Same Axes

Visualizing the graphs helps in understanding their relative positions and points of intersection.

Plotting Y = x^2 - 4

- The parabola opens upward.
- Key points include:
- Vertex at (0, -4)
- Points at (1, -3), (-1, -3)
- Points at (2, 0), (-2, 0)
- Points at (3, 5), (-3, 5)

Plotting Y = 4 - x^2

- The parabola opens downward.

- Key points include:
- Vertex at (0, 4)
- Points at (1, 3), (-1, 3)
- Points at (2, 0), (-2, 0)
- Points at (3, -5), (-3, -5)

Points of Intersection

To find where the graphs intersect, set the functions equal:
$$ x^2 - 4 = 4 - x^2 $$

Solve for x:
$$
x^2 - 4 = 4 - x^2
$$
$$
x^2 + x^2 = 4 + 4
$$
$$
2x^2 = 8
$$
$$
x^2 = 4
$$
$$
x = \pm 2
$$

Calculate the corresponding y-values:
- At $x = 2$:
$$
y = (2)^2 - 4 = 4 - 4 = 0
$$
- At $x = -2$:
$$
y = (-2)^2 - 4 = 4 - 4 = 0
$$

Therefore, the points of intersection are $(2, 0)$ and $(-2, 0)$.

Determining Which Graph Is Above the Other

To write an equation for the part of the graph which is above, we need to analyze the regions between the points of intersection.

Interval Analysis

- Between $x = -\infty$ and -2:
- Test a point, say $x = -3$:
- $y_1 = (-3)^2 - 4 = 9 - 4 = 5$
- $y_2 = 4 - (-3)^2 = 4 - 9 = -5$
- Since $y_1 > y_2$, the first graph, $y = x^2 - 4$, is above $y = 4 - x^2$.

- Between -2 and 2:
- Test $x = 0$:

- $y_1 = 0 - 4 = -4$
- $y_2 = 4 - 0 = 4$
- Since $y_2 > y_1$, $y = 4 - x^2$ is above $y = x^2 - 4$.

- Between 2 and $+\infty$:
- Test $x = 3$:
- $y_1 = 9 - 4 = 5$
- $y_2 = 4 - 9 = -5$
- y_1 is again above y_2.

Writing the Equation for the Part of the Graph Which Is Above

Based on the interval analysis, the two functions switch which one is on top at the points of intersection.

- For $x < -2$:
- $y = x^2 - 4$ is above.
- Equation: $y = x^2 - 4$

- For $-2 < x < 2$:
- $y = 4 - x^2$ is above.
- Equation: $y = 4 - x^2$

- For $x > 2$:
- $y = x^2 - 4$ is above.
- Equation: $y = x^2 - 4$

Therefore, the equations describing the parts of the graphs which are above the other are:

$$
y =
\begin{cases}
x^2 - 4, & x < -2 \\
4 - x^2, & -2 < x < 2 \\
x^2 - 4, & x > 2
\end{cases}
$$

Alternatively, if you want a single equation with a piecewise definition:

Write an equation for the part of the graph which is above as:

$$
y =
\begin{cases}
x^2 - 4, & x < -2 \quad \text{or} \quad x > 2 \\
4 - x^2, & -2 \leq x \leq 2
\end{cases}
$$

Summary
Graphing the functions $y = x^2 - 4$ and $y = 4 - x^2$ on the same axes reveals their intersection points at $(\pm 2, 0)$. The parabola $y = x^2 - 4$ is above $y = 4 - x^2$ outside the interval $[-2, 2]$, while $y = 4 - x^2$ is above within $[-2, 2]$. The piecewise functions above provide a clear equation for the regions where each graph is on top, essential for understanding the shape and relative positioning of these parabolae. This approach is fundamental in analyzing intersections and relative positions of functions in

coordinate geometry.

Frequently Asked Questions

What are the equations of the graphs plotted on the same axes?

The equations are $y = x^2 - 4$ and $y = 4 - x^2$.

How do these two parabolas intersect on the coordinate plane?

They intersect where $x^2 - 4 = 4 - x^2$, which simplifies to $2x^2 = 8$, so $x = \pm 2$. The points of intersection are at (2, 0) and (-2, 0).

Which parts of the graphs are above the other?

The regions where $y = x^2 - 4$ is above $y = 4 - x^2$ are between the points of intersection, specifically for x between -2 and 2, where $x^2 - 4 > 4 - x^2$.

How do you find the portion of the graph that is above the other?

Set $y = x^2 - 4$ and $y = 4 - x^2$, then find where $x^2 - 4 > 4 - x^2$, which simplifies to $x^2 > 2$.

What is the inequality representing the part of the graph above the other?

The part of the graph above is where $x^2 > 2$, i.e., for x < -□ 2 or x > □ 2.

Can you write the equation for the region of the graph that is above the other?

Yes, the region of the graph above is bounded by the inequalities $y > 4 - x^2$ and for x in the interval (-□ , -□ 2) □ (□ 2, □).

What is the shape of the region where $y = x^2 - 4$ is above $y = 4 - x^2$?

It is the area between the two parabolas for x less than -□ 2 and greater than □ 2, where $x^2 > 2$.

How do the graphs relate to each other at the points of intersection?

At $x = \pm 2$, the graphs intersect and have the same y-value (y=0). For x between -2 and 2, $y = x^2 - 4$ is below $y = 4 - x^2$, and outside this interval, $y = x^2 - 4$ is above $y = 4 - x^2$.

What is the equation of the combined graph for the part that is above?

The combined region where $y = x^2 - 4$ is above $y = 4 - x^2$ is where $x^2 > 2$, so the inequality $y > 4 - x^2$ holds for x < -□ 2 or x > □ 2.

Can you provide a complete description of the region above in terms of inequalities?

Yes, the part of the graph above $y = 4 - x^2$ is described by the inequalities: $y > 4 - x^2$ with x □ (-□ , -□ 2) □ (□ 2, □).

Additional Resources

Graph On The Same Axes: $Y = x^2 - 4$ And $Y = 4 - x^2$

Introduction

In the realm of algebra and graphing, the visualization of functions provides vital insights into their behavior, intersections, and overall structure. Among such functions, quadratic equations hold a central place due to their parabolic shapes and the rich geometric and algebraic properties they exhibit. When two quadratic functions are graphed on the same axes, they often reveal intriguing intersections and symmetries, inviting deeper analysis.

This article delves into the detailed examination of the combined graph of the functions:

- $y = x^2 - 4$
- $y = 4 - x^2$

The focus is on understanding the regions where these graphs lie relative to each other, particularly identifying and deriving the equation for the part of the graph that lies above the other. This investigation not only clarifies the geometric relationship between these functions but also enhances comprehension of quadratic inequalities, intersections, and the derivation of equations describing specific regions.

Foundations of the Functions

The Parabolas: An Overview

The two functions in question are quadratic:

1. $y = x^2 - 4$

- It is a parabola opening upward.
- Vertex at $(0, -4)$.
- Axis of symmetry is the y-axis ($x=0$).

2. $y = 4 - x^2$

- It is a parabola opening downward.
- Vertex at $(0, 4)$.
- Axis of symmetry is also the y-axis ($x=0$).

These functions are symmetric with respect to the y-axis, which simplifies the analysis of their intersection points and the regions where one graph lies above the other.

Basic Properties and Symmetries

Given their standard forms, both functions are centered at the same axis of symmetry, which implies symmetry about the y-axis. The intersection points happen where:

$$
x^2 - 4 = 4 - x^2
$$

Rearranged to find the intersection points:

$$

$$
x^2 - 4 = 4 - x^2
$$

$$
x^2 + x^2 = 4 + 4
$$

$$
2x^2 = 8
$$

$$
x^2 = 4
$$

$$
x = \pm 2
$$

The corresponding y-values are obtained by substituting $x = \pm 2$:

$$
y = (2)^2 - 4 = 4 - 4 = 0
$$

or

$$
y = 4 - (2)^2 = 4 - 4 = 0
$$

Thus, the graphs intersect at $(2, 0)$ and $(-2, 0)$.

Graphical Analysis and Region Identification

Visualizing the Parabolas

Plotting these functions reveals:

- The upward opening parabola $y = x^2 - 4$ dips below the x-axis, reaching its minimum at $(0, -4)$. It intersects the x-axis at $x = \pm 2$.

- The downward opening parabola $y = 4 - x^2$ reaches its maximum at $(0, 4)$, crossing the x-axis at $x = \pm 2$.

The two parabolas intersect at $(\pm 2, 0)$, and their shapes are mirror images with respect to the y-axis.

Regions of Interest

The key regions are:

- Region A: Where $y = x^2 - 4$ is above $y = 4 - x^2$.

- Region B: Where $y = 4 - x^2$ is above $y = x^2 - 4$.

Since the graphs are symmetric, the analysis can focus on one side and then generalize.

Determining Which Graph Is Above the Other

To establish where one graph lies above the other, consider the difference:

$$
D(x) = (x^2 - 4) - (4 - x^2) = x^2 - 4 - 4 + x^2 = 2x^2 - 8
$$

Simplify:

$$
D(x) = 2x^2 - 8
$$

- When $D(x) > 0$, the graph $y = x^2 - 4$ is above $y = 4 - x^2$.

- When $D(x) < 0$, the opposite is true.

Set $D(x) = 0$ to find the boundary points:

$$
2x^2 - 8 = 0
$$
$$
x^2 = 4
$$
$$
x = \pm 2
$$

This confirms our earlier intersection points.

- For $|x| < 2$:

$$
D(x) = 2x^2 - 8 < 0
$$
since $x^2 < 4$, so $2x^2 < 8$, implying $D(x) < 0$.

- For $|x| > 2$:

$$
D(x) > 0
$$

However, outside the intersection points, the graphs do not overlap; the actual "above" region depends on the specific domain considered.

The Region Where One Graph Is Above the Other

Focus on the Region $-2 \leq x \leq 2$

Within this interval, the difference $D(x)$ is negative, indicating:

$$
x^2 - 4 < 4 - x^2
$$
which means:

$$
y = x^2 - 4 \quad \text{is below} \quad y = 4 - x^2
$$

Therefore, in this domain, the graph of $y = 4 - x^2$ is above $y = x^2 - 4$.

Visual Summary

- From $x = -2$ to $x = 2$, $y = 4 - x^2$ is above $y = x^2 - 4$.
- Outside this interval, $y = x^2 - 4$ is above $y = 4 - x^2$.

Deriving the Equation for the Part of the Graph Which Is Above

The core of this investigation is to formulate an explicit equation representing the region where one graph is above the other.

Given the functions:

$$
f(x) = x^2 - 4
$$
$$
g(x) = 4 - x^2
$$

and the analysis above, the region where $g(x)$ is above $f(x)$ is characterized by:

$$
g(x) \geq f(x)
$$
which simplifies to:

$$
4 - x^2 \geq x^2 - 4
$$
or equivalently,

$$
2x^2 \leq 8
$$
$$
x^2 \leq 4
$$
$$
|x| \leq 2
$$

Within this interval, the part of the graph of the upper parabola $y = 4 - x^2$ that is above the lower parabola $y = x^2 - 4$ can be represented by the set of points:

$$
\boxed{
\text{Above region:} \quad \left\{ (x, y) \mid x \in [-2, 2], \quad y \in [x^2 - 4,\, 4 - x^2] \right\}
}
$$

Explicit Equation for the Region

To formalize the description of the region "above", the key is to identify the upper boundary function:

$$
y = 4 - x^2
$$

and the lower boundary function:

$$
y = x^2 - 4
$$

The region of interest is the set of all points (x, y) such that:

$$
x \in [-2, 2]
$$
$$
x^2 - 4 \leq y \leq 4 - x^2
$$

In terms of inequalities, the region above the lower parabola within the intersection interval is:

$$
\boxed{
x \in [-2, 2], \quad y \text{ satisfying } x^2 - 4 \leq y \leq 4 - x^2
}
$$

Formulating a Single Equation for the Above Region

While the region is best described via inequalities, sometimes it's useful to express the "above" part via a combined equation or inequality that captures the region's boundary.

- The upper boundary:

$$
y = 4 - x^2
$$

- The lower boundary:

$$
y = x^2 - 4
$$

- The domain of x:

$$
x \in [-2, 2]
$$

Thus, the equation that captures the region above the lower parabola and below the upper parabola is:

\[

$x^2 - 4 \leq y$

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