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If F (x, Y) = (3 + 2xy) I + (x^2 3y^2) J, Find A Function F, Evaluate The Line Integral F · dr, Where

Understanding how to evaluate line integrals of vector fields is a fundamental aspect of vector calculus, with widespread applications in physics, engineering, and mathematics. Suppose we are given a vector field $F(x, y) = (3 + 2xy) i + (x^2 + 3y^2) j$. The task involves finding a potential function F (also known as a scalar potential or potential function), and then calculating the line integral $\int_C F \cdot dr$ along a specified path C. This process combines concepts such as vector field properties, potential functions, and the evaluation of line integrals, which are essential for understanding fields like gravity, electromagnetism, and fluid flow.

In this article, we will explore how to determine the potential function F, analyze whether the vector field is conservative, and then evaluate the line integral over a given path. We will break down each step carefully to provide a comprehensive understanding suitable for students and professionals alike.

Understanding the Vector Field F(x, y)

Definition of the Vector Field

The vector field in question is:

$$\mathbf{F}(x, y) = (3 + 2xy) \mathbf{i} + (x^2 + 3y^2) \mathbf{j}$$

where:
- $P(x, y) = 3 + 2xy$
- $Q(x, y) = x^2 + 3y^2$

This representation indicates the components of the vector field along the x-axis (i-component) and y-axis (j-component).

Physical and Mathematical Significance

Such a vector field could represent various physical phenomena; for instance, a fluid flow with varying velocity components or an electromagnetic field. To analyze its properties, especially to evaluate line integrals, it is crucial to determine whether F is conservative.

Checking if the Vector Field is Conservative

What Does it Mean for a Vector Field to be Conservative?

A vector field F is called conservative if there exists a scalar potential function $\phi(x, y)$ such that:

$$ \mathbf{F} = \nabla \phi = \frac{\partial \phi}{\partial x} \mathbf{i} + \frac{\partial \phi}{\partial y} \mathbf{j} $$

This means:

$$ P = \frac{\partial \phi}{\partial x} $$
$$ Q = \frac{\partial \phi}{\partial y} $$

and the line integral of F over a path depends only on the endpoints, making evaluation straightforward.

Test for Conservativeness

The necessary condition in two dimensions is:

$$ \frac{\partial P}{\partial y} = \frac{\partial Q}{\partial x} $$

Calculating these derivatives:

- $\frac{\partial P}{\partial y} = \frac{\partial}{\partial y} (3 + 2xy) = 2x$
- $\frac{\partial Q}{\partial x} = \frac{\partial}{\partial x} (x^2 + 3y^2) = 2x$

Since:

$$ \frac{\partial P}{\partial y} = \frac{\partial Q}{\partial x} = 2x $$

the vector field F is conservative.

Finding the Potential Function F(x, y)

Integrating P with Respect to x

To find the potential function $\phi(x, y)$, integrate $P(x, y) = 3 + 2xy$ with respect to x:

$$\phi(x, y) = \int (3 + 2xy) dx$$

Performing the integration:

$$\phi(x, y) = 3x + y x^2 + C(y)$$

where $C(y)$ is an arbitrary function of y because the partial derivative with respect to x might include functions only dependent on y.

Determining $C(y)$ via $Q(x, y)$

Next, differentiate $\phi(x, y)$ with respect to y:

$$\frac{\partial \phi}{\partial y} = x^2 + C'(y)$$

Recall that $\frac{\partial \phi}{\partial y} = Q(x, y) = x^2 + 3 y^2$.

Set equal:

$$x^2 + C'(y) = x^2 + 3 y^2$$

which implies:

$$C'(y) = 3 y^2$$

Integrate with respect to y:

$$C(y) = y^3 + K$$

where K is a constant.

Final Potential Function

Thus, the potential function $\phi(x, y)$ is:

$$\boxed{
\phi(x, y) = 3 x + y x^2 + y^3 + \text{constant}
}$$

This scalar function encapsulates the entire vector field.

Evaluating the Line Integral of F · dr

Line Integral of a Conservative Vector Field

Since the vector field F is conservative, the line integral between two points A and B simplifies to:

$$\int_{C} \mathbf{F} \cdot d\mathbf{r} = \phi(B) - \phi(A)$$

where $A = (x_A, y_A)$ and $B = (x_B, y_B)$.

Choosing the Path and Endpoints

Suppose the problem specifies a path C from point A to point B. For example, consider:

- $A = (x_1, y_1)$
- $B = (x_2, y_2)$

Without specific points given, the general solution remains:

$$\text{Line integral} = \phi(x_2, y_2) - \phi(x_1, y_1)$$

which can be computed once the endpoints are known.

Example Calculation

Suppose the path goes from $A(0, 0)$ to $B(1, 2)$. The potential function evaluated at these points:

- $\phi(1, 2) = 3(1) + 2 \times 1^2 + 2^3 = 3 + 2 + 8 = 13$
- $\phi(0, 0) = 3(0) + 0 + 0 = 0$

Thus, the line integral along this path:

$$\int_{C} \mathbf{F} \cdot d\mathbf{r} = 13 - 0 = 13$$

Applications and Further Insights

Physical Significance of the Potential Function

The potential function $\phi(x, y)$ can represent potential energy, electric potential, or gravitational potential in physical systems. Its gradient yields the vector field, describing forces or flows.

Implications of Conservative Fields

The fact that F is conservative simplifies calculations significantly. Instead of parameterizing the path or performing complex integrations, the potential difference suffices, making problems more manageable.

Extensions to Non-Conservative Fields

If the field were not conservative (i.e., $\frac{\partial P}{\partial y} \neq \frac{\partial Q}{\partial x}$), the line integral would depend on the path, requiring more advanced techniques such as line parameterizations or numerical methods.

Conclusion

In this comprehensive exploration, we have:

- Analyzed the vector field $\mathbf{F}(x, y) = (3 + 2xy) \mathbf{i} + (x^2 + 3y^2) \mathbf{j}$
- Confirmed that it is conservative by verifying the equality of mixed partial derivatives
- Derived the potential function $\phi(x, y) = 3x + xy^2 + y^3 + \text{constant}$
- Explained how to evaluate the line integral $\int_C \mathbf{F} \cdot d\mathbf{r}$ using the potential function
- Provided an example calculation for specific endpoints

Understanding these methods is crucial for tackling real-world problems involving vector fields, whether in physics, engineering, or applied mathematics. Recognizing when a field is conservative and knowing how to find the potential function streamline the process of evaluating line integrals, thereby offering deeper insights into the behavior of complex systems.

Keywords for SEO: vector field analysis, potential function, line integral calculation, conservative vector fields, gradient fields, physics applications, mathematical techniques, vector calculus,

Frequently Asked Questions

What is the vector function F(x, y) given as F(x, y) = (3 + 2xy) i + (x^2 3y^2) j?

The vector function is F(x, y) = (3 + 2xy) i + (x^2 + 3y^2) j.

How do you evaluate the line integral of a vector field F along a curve C?

To evaluate the line integral, parameterize the curve C as r(t), compute F(r(t)), find dr/dt, and then integrate F(r(t)) · dr/dt over the parameter interval.

What is the significance of identifying whether F is conservative in evaluating line integrals?

If F is conservative, the line integral over any path depends only on the endpoints, simplifying the calculation to a potential function difference.

How can you determine if the given vector field F(x, y) is conservative?

Check if the curl of F is zero; for 2D fields, verify if ∂P/∂y = ∂Q/∂x, where P and Q are the components of F.

If F is conservative, what is the general approach to evaluate the line integral from point A to B?

Find a potential function φ such that ∇φ = F, then evaluate φ at B minus φ at A: ∫_C F · dr = φ(B) - φ(A).

Given the vector field F(x, y) and a specific curve,

how do you explicitly compute the line integral?

Parameterize the curve, substitute into F, compute the dot product with the derivative of the parameterization, and integrate over the parameter interval.

Additional Resources

Investigative Analysis of the Vector Field F(x, y) = (3 + 2xy) I + (x^2 + 3y^2) J and Its Line Integral Evaluation

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Introduction

In the realm of vector calculus, the study of vector fields and their line integrals serves as a foundational tool in understanding physical phenomena, from fluid flow to electromagnetism. This article investigates the properties of a specific vector field F(x, y) defined by:

$$ \mathbf{F}(x, y) = (3 + 2xy) \mathbf{i} + (x^2 + 3 y^2) \mathbf{j} $$

Our goal is to evaluate the line integral of F along a specified curve C, denoted as:

$$ \int_C \mathbf{F} \cdot d\mathbf{r} $$

where $ d\mathbf{r} = dx\, \mathbf{i} + dy\, \mathbf{j} $. To do so comprehensively, we will explore the properties of F, determine whether it is conservative, identify potential functions if applicable, and then execute the integral calculation along a representative path.

Understanding the Vector Field F(x, y)

Structural Breakdown

The vector field is composed of two components:

- $F_1(x, y) = 3 + 2xy$
- $F_2(x, y) = x^2 + 3 y^2$

These components suggest a combination of polynomial functions, which often lends themselves to straightforward differentiation and potential scalar potential function analysis.

Physical Interpretation

While purely mathematical, such a field could model a variety of phenomena:

- The F_1 component, involving xy, might depict a flow with shear characteristics.
- The F_2 component, quadratic in x and y, could be representative of rotational or divergence patterns in a flow.

Understanding whether F is conservative is crucial because it influences whether the line integral depends solely on endpoints or the specific path taken.

Determining if F is Conservative

A vector field F in two dimensions is conservative if it can be expressed as the gradient of some scalar potential function $\phi(x, y)$:

$$\mathbf{F} = \nabla \phi = \frac{\partial \phi}{\partial x} \mathbf{i} + \frac{\partial \phi}{\partial y} \mathbf{j}$$

Criteria:

- The curl of F must be zero.

In two dimensions, the curl simplifies to:

$$\frac{\partial F_2}{\partial x} - \frac{\partial F_1}{\partial y}$$

Calculating each:

$$
\begin{aligned}
\frac{\partial F_2}{\partial x} &= \frac{\partial}{\partial x} (x^2 + 3 y^2) = 2x \\
\frac{\partial F_1}{\partial y} &= \frac{\partial}{\partial y} (3 + 2xy) = 2x
\end{aligned}
$$

Since:

$$\frac{\partial F_2}{\partial x} - \frac{\partial F_1}{\partial y} = 2x - 2x = 0$$

Conclusion: The curl is zero everywhere; thus, F is a conservative vector field.

Finding a Scalar Potential Function $\varphi(x, y)$

Given $F = \nabla\varphi$, and:

$$
\begin{cases}
\frac{\partial \phi}{\partial x} = 3 + 2xy \\
\frac{\partial \phi}{\partial y} = x^2 + 3 y^2
\end{cases}
$$

Integrate $\frac{\partial \phi}{\partial x}$ with respect to x:

$$
\phi(x, y) = \int (3 + 2xy) dx = 3x + y x^2 + h(y)
$$

where $h(y)$ is an arbitrary function of y.

Differentiate this expression with respect to y:

$$
\frac{\partial \phi}{\partial y} = x^2 + h'(y)
$$

Set this equal to F_2:

$$
x^2 + h'(y) = x^2 + 3 y^2
$$

Thus:

$$
h'(y) = 3 y^2
$$

Integrate:

$$
h(y) = y^3 + C
$$

Choosing $C = 0$ for simplicity, the scalar potential function is:

$$
\boxed{
\phi(x, y) = 3x + xy^2 + y^3
}
$$

Evaluating the Line Integral F · dr

Given the conservation property, the line integral over any curve C from point $\mathbf{r}_a = (x_a, y_a)$ to $\mathbf{r}_b = (x_b, y_b)$ depends solely on the endpoints:

$$
\int_C \mathbf{F} \cdot d\mathbf{r} = \phi(x_b, y_b) - \phi(x_a, y_a)
$$

Key Point: The path taken between points does not matter for conservative fields.

Application: Example Path and Evaluation

Suppose the curve C is from point $(x_a, y_a) = (0, 0)$ to $(x_b, y_b) = (1, 2)$.

Calculate:

$$
\phi(1, 2) = 3(1) + (1)(2)^2 + (2)^3 = 3 + 4 + 8 = 15
$$

And at the starting point:

$$
\phi(0, 0) = 3(0) + (0)(0)^2 + 0 = 0
$$

Therefore, the line integral:

$$
\boxed{
\int_C \mathbf{F} \cdot d\mathbf{r} = 15 - 0 = 15
}
$$

Broader Implications and Applications

Significance of the Potential Function

The scalar potential function $\varphi(x, y)$ aids in:

- Simplifying complex integral calculations.
- Understanding the nature of the field—whether it is irrotational, source-free, or exhibits other properties.
- Facilitating energy or work calculations in physics.

Generalization to Arbitrary Curves

Because F is conservative, the line integral between any two points is path-independent, simplifying analysis in various physics and engineering problems.

Additional Considerations

Critical Points and Field Behavior

- Points satisfying $\nabla \phi = 0$:

$$
\begin{cases}
\frac{\partial \phi}{\partial x} = 3 + 2xy = 0 \\
\frac{\partial \phi}{\partial y} = x^2 + 3 y^2 = 0
\end{cases}
$$

Solving:

$$
x^2 + 3 y^2 = 0 \implies x=0, y=0
$$

and

$$
3 + 2 \times 0 \times 0 = 3 \neq 0
$$

So, the only critical point is at the origin, which is not a stationary point for the potential but could indicate a saddle or other behavior in the field.

Limitations and Domain

- The field is well-defined and smooth over all $\mathbb{R}^2$.
- No singularities or discontinuities are present, making the analysis globally valid.

Conclusion

This in-depth exploration reveals that the vector field:

$$
\mathbf{F}(x, y) = (3 + 2xy) \mathbf{i} + (x^2 + 3 y^2) \mathbf{j}
$$

is conservative, possessing a scalar potential function:

$$
\boxed{
\phi(x, y) = 3x + xy^2 + y^3
}
$$

Consequently, the line integral of F along any path from point (x_a, y_a) to (x_b, y_b) reduces to the difference in potential at these points:

$$
\int_C \mathbf{F} \cdot d\mathbf{r} = \phi(x_b, y_b) - \phi(x_a, y_a)
$$

This fundamental property simplifies calculations, allows for versatile applications, and highlights the importance of potential functions in vector calculus. Whether analyzing physical systems or solving abstract mathematical problems, recognizing the conservative nature of a vector field is invaluable, as demonstrated through the detailed evaluation herein.

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Note: This article serves as a comprehensive review for students and practitioners interested in vector calculus, showcasing methods for identifying conservative fields, deriving potential functions, and executing line integrals efficiently

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