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When analyzing electrical circuits, one common question is how power is distributed among various components. Specifically, if five equal resistors are connected in series and collectively dissipate a total of 10 watts, a natural curiosity arises: how much power does each resistor dissipate individually? Understanding this distribution requires a grasp of basic principles of series circuits, resistor behavior, and power calculations. This article explores the detailed steps involved in determining the power dissipation per resistor in such a configuration, providing clarity for students, engineers, and electronics enthusiasts alike.

Understanding the Series Circuit and Power Dissipation

Before diving into calculations, it is essential to understand the fundamental concepts governing series circuits and power dissipation.

What is a Series Circuit?

- A series circuit is one where components are connected end-to-end, forming a single path for current flow.
- In such circuits, the same current flows through all components, but the voltage divides among them.
- For resistors in series, the total resistance is the sum of individual resistances: $R_{total} = R_1 + R_2 + \ldots + R_n$.

Power Dissipation in Resistors

- The power dissipated by a resistor is given by $P = I^2R$ or $P = V^2/R$ or $P = VI$, where I is current, V is voltage, and R is resistance.
- In a series circuit, since the current is the same through all resistors, power distribution depends on individual resistances and the voltage across each resistor.

Given Data and Assumptions

To solve the problem, let's clarify the provided data and assumptions:

- Number of resistors: 5 (all equal in resistance)
- Total power dissipated: 10 W
- Resistors are connected in series

Assuming all resistors are identical and the circuit is powered by an appropriate voltage source, the goal is to find out how much power each resistor dissipates.

Step 1: Determine the Total Current in the Circuit

Since the total power dissipated is known, and the resistors are in series, we can use the total power to find the current flowing through the circuit.

The Relationship Between Power and Current

- Total power dissipated in the circuit: $P_{total} = I_{total} \times V_{total}$
- However, since we don't know the total voltage directly, we can use the power dissipated in the resistors and their resistance to find the current.

Calculating Total Resistance

- Let R be the resistance of each resistor (since they are equal).
- Total resistance: $R_{total} = 5R$

Using Power Dissipation to Find Current

- The total power dissipated in the circuit is the sum of power across all resistors:

$P_{total} = 5 \times P_{per\_resistor}$

Given $P_{total} = 10$ W,

$P_{per\_resistor} = P_{total} / 5 = 10 W / 5 = 2 W$

Each resistor dissipates 2 W of power.

Now, applying the power formula for a resistor:
$P = I^2 R$

Rearranged to find current:
$I = \sqrt{P / R}$

But since R is unknown, we need to find the resistor's resistance using the power formula in conjunction with voltage.

Alternatively, to find the current directly, we can use the relation:
$P = V^2 / R$

But without the voltage, it's better to find the resistance first.

Step 2: Find the Resistance of Each Resistor

Given that $P = I^2 R$, and P = 2 W for each resistor, and knowing that all resistors are identical, we can proceed as follows.

Express Power in Terms of Voltage and Resistance

- In a series circuit, the voltage across each resistor is $V_i = I \times R$
- And the power across each resistor: $P = V_i \times I = I^2 R$

To find R, we need to relate the total voltage and current.

Alternatively, since the total power is known, and the total resistance is $R_{total} = 5 R$, and the total power dissipated is 10 W, we can use the relation:

$P_{total} = I_{total}^2 \times R_{total}$

Rearranged:
$I_{total} = \sqrt{P_{total} / R_{total}}$

But R_{total} is unknown. To find R_{total}, we need to find the total voltage.

Step 3: Find the Total Voltage and Resistance

Since each resistor dissipates 2 W, and $P = V_i \times I$:

$V_i = P / I$

But again, without R, this seems circular. To resolve this, let's take a different approach: express the power in terms of total voltage.

Given that:

- The voltage across each resistor is V_i

- The total voltage V_{total} = sum of all $V_i = 5 V_i$

- The power dissipated in each resistor: $P = V_i \times I$

But we can also write:

$V_i = I R$

Therefore, the power per resistor:
$P = V_i \times I = (I R) \times I = I^2 R$

Rearranged:
$R = P / I^2$

Now, the total resistance:
$R_{total} = 5 R = 5 \times (P / I^2) = (5 P) / I^2$

Total power:
$P_{total} = I \times V_{total}$

But $V_{total} = 5 V_i = 5 \times I R = 5 \times I \times R$

Substituting R:
$V_{total} = 5 \times I \times (P / I^2) = 5 \times P / I$

Now, total power:
$P_{total} = I \times V_{total} = I \times (5 P / I) = 5 P$

Given that $P_{total} = 10$ W:
$10 \text{ W} = 5 P$
$P = 10 \text{ W} / 5 = 2 \text{ W}$

This confirms that each resistor dissipates 2 W, consistent with earlier calculations.

Now, find the current:

Using $P = I^2 R$:

$R = P / I^2$

But R can also be expressed as:

$R = V_i / I$

And $V_i = I R$

Alternatively, considering the total voltage across the circuit:
$V_{total} = 5 V_i = 5 I R$

But since $P = V_i \times I$:
$V_i = P / I$

Substitute into V_{total}:
$V_{total} = 5 \times (P / I) = 5 P / I$

We also know that:

$P_{total} = V_{total} \times I = (5 P / I) \times I = 5 P$

Given $P_{total} = 10$ W:
$10 W = 5 P$
$P = 2 W$

Again, consistent with prior deductions.

Now, to find the current I:

From $P = I^2 R$:
$R = P / I^2$

From $V_i = I R$:
$V_i = I \times R = I \times (P / I^2) = P / I$

We also have $V_i = P / I$, so the voltage across each resistor is V_i.

Total voltage:
$V_{total} = 5 V_i = 5 \times (P / I) = 5 P / I$

Using the relation for total voltage and total power:
$P_{total} = V_{total} \times I = (5 P / I) \times I = 5 P$

As before, $P = 2$ W.

Now, finally, compute the current:

Using $P = I^2 R$:

$R = P / I^2$

But since $V_i = I R$, then:

$V_i = I \times (P / I^2) = P / I$

Rearranged:
$I = P / V_i$

We need V_i to find I.

Alternatively, use the relation:

$V_i = \text{sqrt}(P \times R)$

Given that $P = I^2 R$, rearranged:

$I = \text{sqrt}(P / R)$

Similarly:
$V_i = I R = \text{sqrt}(P R)$

But without R, we need to find R explicitly.

Let's approach it from a different angle:

Alternative Approach:

Since each resistor dissipates 2 W, and the power in a resistor is $P = V_i \times I$, and $V_i = I R$, then:

$P = I^2 R$

Rearranged:
$R = P / I^2$

Total resistance $R_{total} = 5 R = 5 \times (P / I^2)$

Total voltage:
$V_{total} = I R_{total} = I \times 5 R = 5 I R = 5 I \times (P / I^2) = 5 P / I$

Total power:
$P_{total} = V_{total} \times I = (5 P / I) \times I = 5 P$

Given $P_{total} = 10$ W:
10 W = 5 P
P = 2 W

Now, find the current I:

From $P = I^2 R$:

$R = P / I^2$

But also, $R = V_i / I$, and $V_i = P$

Frequently Asked Questions

If five equal resistors are connected in series and dissipate a total of 10 W, how much power does each resistor dissipate?

Each resistor dissipates 2 W of power.

In a series circuit with five identical resistors dissipating a total of 10 W, what is the power rating of each resistor?

Each resistor dissipates 2 W, so each should have a power rating at least 2 W for safe operation.

How is the total power of 10 W distributed among five equal resistors in series?

The total power is equally divided among the five resistors, so each dissipates 2 W.

If the total power dissipated in five equal resistors in series is 10 W, what is the voltage across each resistor?

First, calculate the power per resistor (2 W), then use $P = V^2 / R$ to find each voltage, which depends on the resistor value.

Can each resistor in a series circuit dissipate 2 W safely, assuming standard resistor ratings?

Most standard resistors are rated for at least 2 W, so they can safely dissipate 2 W each, but higher ratings are preferable for safety margins.

If one resistor in the series dissipates more than 2 W, what does that imply about the circuit?

It implies that the resistor may be overheating or exceeding its power rating, potentially leading to failure or damage.

How does the total circuit power relate to individual resistor power in a series circuit?

The total power is the sum of the power dissipated by each resistor; in this case, 5 resistors each dissipating 2 W sum to 10 W.

What formula can be used to determine the power dissipated by each resistor in this series circuit?

Since resistors are equal and in series, the power per resistor is total power divided by the number of resistors: 10 W / 5 = 2 W.

If the total voltage supplied to the series circuit is known, how can we verify the power dissipated per resistor?

Calculate the total current using $P = V \times I$ or $P = I^2 R$, find the voltage across each resistor (which is $V_{total} / 5$), then compute power using $P = V^2 / R$ for each resistor.

What assumptions are made when calculating that each resistor dissipates 2 W in this circuit?

The assumptions include that all resistors are equal in resistance, the circuit is ideal with no parasitic effects, and that power is evenly distributed among the resistors.

Additional Resources

If five equal resistors dissipate a total of 10 W in a series circuit, how much power does each resistor? This question is a classic problem in basic electrical circuit analysis, and understanding it thoroughly provides valuable insights into how power, voltage, and current distribute across series-connected resistors. When resistors are equal and connected in series, the distribution of power, current, and voltage follows predictable patterns, yet it’s essential to methodically analyze the circuit to arrive at accurate results. This article aims to guide you through the process of solving this problem step-by-step, while also exploring the fundamental principles underlying series circuits and power dissipation.

Understanding the Basics of Series Circuits

Before delving into the calculations, it’s important to review the fundamental concepts of series circuits.

What Is a Series Circuit?

A series circuit is one where components are connected end-to-end, forming a single path for current flow. The key characteristics include:
- The same current flows through all components.
- The total voltage across the circuit is divided among the resistors.
- The total resistance is the sum of individual resistances: \(R_{total} = R_1 + R_2 + ... + R_n \).

Implications for Resistors in Series

In a series configuration:
- The current I remains constant through all resistors.
- The voltage divides proportionally to the resistance values.
- Power dissipation in each resistor depends on the current flowing through it and its resistance.

Given Data and Objective

Let's clarify the problem's parameters:

- Number of resistors: 5
- All resistors are equal: $R_1 = R_2 = R_3 = R_4 = R_5 = R$
- Total power dissipated by the circuit: 10 W
- Goal: Find the power dissipated by each resistor.

Step 1: Determine the Total Current in the Circuit

Since the resistors are equal and in series, the total power dissipated (10 W) is distributed among the five resistors.

The total power P_{total} is related to the total current I_{total} and total resistance R_{total}:

$$
P_{total} = I_{total}^2 \times R_{total}
$$

The total resistance:

$$
R_{total} = 5 R
$$

Rearranged to find the current:

$$
I_{total} = \sqrt{\frac{P_{total}}{R_{total}}} = \sqrt{\frac{10\, \text{W}}{5 R}} = \sqrt{\frac{2}{R}}
$$

But we don’t yet know R, so we need to find it from another approach.

Step 2: Recognize that Power Dissipation Is Equal for Each Resistor

Because all resistors are equal and in series, each resistor dissipates the same amount of power. This is because the same current flows through each resistor, and the power dissipated in a resistor is:

$$
P_{resistor} = I^2 R
$$

Total power dissipated:

$$
P_{total} = 5 \times P_{resistor}
$$

Given $P_{total} = 10\, \text{W}$:

$$
10\, \text{W} = 5 \times P_{resistor}
$$

$$
P_{resistor} = \frac{10\, \text{W}}{5} = 2\, \text{W}
$$

Thus, each resistor dissipates 2 W.

Step 3: Confirming the Current and Resistance Values

Now, with each resistor dissipating 2 W, and knowing the power formula:

$$
P = I^2 R
$$

we can write:

$$
2\, \text{W} = I^2 R
$$

which gives:

$$
I^2 = \frac{2}{R}
$$

From earlier, the total current squared:

$$
I_{total}^2 = \frac{2}{R}
$$

But we also know that the total power is:

$$
P_{total} = I_{total}^2 \times R_{total} = I_{total}^2 \times 5 R
$$

Substitute I_{total}^2:

$$
P_{total} = \frac{2}{R} \times 5 R = 2 \times 5 = 10\, \text{W}
$$

which matches the given total power, confirming the calculations.

Step 4: Determine Voltage Across Each Resistor

While the initial question focuses on power, understanding voltage distribution can deepen comprehension.

The voltage across each resistor:

$$
V_{resistor} = I \times R
$$

Total voltage across the entire series circuit:

$$
V_{total} = V_{resistor} \times 5
$$

From the power dissipation per resistor:

$$
P_{resistor} = I^2 R

\]

Express R:

$$
R = \frac{P_{resistor}}{I^2}
$$

But since $P_{resistor} = 2\, \text{W}$, and $I^2 = \frac{2}{R}$, it’s more straightforward to find I using known values.

From earlier:

$$
P_{resistor} = 2\, \text{W} = I^2 R
$$

and

$$
I^2 = \frac{2}{R}
$$

Multiply both sides of the second equation by R:

$$
I^2 R = 2
$$

which is consistent with $P_{resistor}$. To find the current explicitly, pick a value for R to illustrate the voltage, or consider a specific example.

Practical Example: Assume a Resistance Value

Suppose we choose $R = 10\, \Omega$. Then:

- Power per resistor:

$$
P_{resistor} = I^2 R = 2\, \text{W}
$$
- Find the current:

$$
I = \sqrt{\frac{2}{R}} = \sqrt{\frac{2}{10}} = \sqrt{0.2} \approx 0.447\, \text{A}
$$

- Total voltage:

$$
V_{total} = I \times R_{total} = 0.447\, \text{A} \times (5 \times 10\, \Omega) = 0.447 \times 50 = 22.36\, \text{V}
$$

- Voltage across each resistor:

$$
V_{resistor} = I \times R = 0.447 \times 10 = 4.47\, \text{V}
$$

- Confirm power in each resistor:

$$
P_{resistor} = V_{resistor} \times I = 4.47 \times 0.447 \approx 2\, \text{W}
$$

which matches earlier calculations.

Summary of Key Results

- Each resistor dissipates 2 W of power.
- The current through the circuit:

$$
I = \sqrt{\frac{2}{R}}
$$

- The total voltage across the series circuit depends on the resistor value selected, but the power distribution remains consistent.

Features, Pros, and Cons of Such Calculations

Features:
- Provides clear insight into how power and voltage are distributed in series circuits.
- Demonstrates the importance of equal resistor values for uniform power dissipation.
- Uses fundamental formulas, making it accessible for students and engineers alike.

Pros:
- Simplifies complex power distribution problems.

- Facilitates understanding of circuit behavior in practical scenarios.
- Encourages methodical problem-solving and verification.

Cons:
- Assumes ideal resistors with no parasitic effects.
- Real-world resistors may vary slightly, affecting power distribution.
- Requires familiarity with algebraic manipulation and circuit theory.

Conclusion

In conclusion, when five equal resistors in series dissipate a total of 10 W, each resistor dissipates exactly 2 W. This outcome relies on the fact that in a series circuit, the same current flows through all components, and the power dissipated in each resistor is proportional to its resistance. The calculation becomes straightforward once the uniformity of the resistors and the total power are established, highlighting the beauty and simplicity of series circuit analysis. Whether designing circuits or troubleshooting existing ones, understanding how power divides among components is fundamental to ensuring safe and efficient operation.

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