In A Bag Of 4 Dimes, 3 Nickels, 5 Quarters, 4 Coins Are Selected Find The Probability That All Are Dimes

Understanding the Problem: In A Bag Of 4 Dimes, 3 Nickels, 5 Quarters, 4 Coins Are Selected Find The Probability That All Are Dimes

In a scenario involving probability and basic combinatorics, we often encounter problems that require calculating the likelihood of specific outcomes when selecting items from a collection. The problem statement, **In A Bag Of 4 Dimes, 3 Nickels, 5 Quarters, 4 Coins Are Selected Find The Probability That All Are Dimes**, is a classic example that tests understanding of probability principles, especially the concepts of total possible outcomes and favorable outcomes.

This problem involves a bag containing different types of coins: dimes, nickels, and quarters. We are asked to find the probability that, upon randomly selecting four coins from this bag, all four coins are dimes. To solve this, we need to understand the basic definitions of probability, the principles of combinatorics involved in counting possible arrangements, and how to compute probabilities based on favorable outcomes over total outcomes.

Breaking Down the Problem: Key Concepts

Before diving into calculations, it's essential to clarify some foundational concepts:

What is Probability?

Probability measures the likelihood of a specific event occurring out of all possible events. It is expressed as a ratio:

\[
\text{Probability} = \frac{\text{Number of favorable outcomes}}{\text{Total number of possible outcomes}}
\]

In this context, the event of interest is "all four selected coins are dimes."

Understanding the Composition of the Bag

The bag contains:
- 4 dimes
- 3 nickels
- 5 quarters

Total coins in the bag:

$$
4 + 3 + 5 = 12
$$

The total coins are 12, from which we select 4 coins.

What Are Favorable Outcomes?

Favorable outcomes are those in which all four selected coins are dimes. Since there are only 4 dimes, the only way to have all four coins as dimes is to select all 4 dimes.

Step-by-Step Solution to the Problem

Let's now methodically analyze how to compute the probability.

Step 1: Total Number of Ways to Select 4 Coins from 12 Coins

The total possible outcomes represent all different combinations of 4 coins that can be selected from the 12 coins.

This is a combinations problem, which is calculated using the binomial coefficient:

$$
\text{Total outcomes} = \binom{12}{4}
$$

Calculating:

$$
\binom{12}{4} = \frac{12!}{4! \times (12-4)!} = \frac{12 \times 11 \times 10 \times 9}{4 \times 3 \times 2 \times 1} = 495
$$

So, there are 495 possible ways to select any 4 coins from the bag.

Step 2: Number of Favorable Outcomes (Selecting All Dimes)

Since all four coins must be dimes, and there are exactly 4 dimes in the bag, the only favorable way is to select all 4 dimes:

$$
\binom{4}{4} = 1
$$

There is only one way to select all 4 dimes.

Step 3: Calculate the Probability

Putting it all together:

$$
\text{Probability} = \frac{\text{Favorable outcomes}}{\text{Total outcomes}} = \frac{1}{495}
$$

Therefore, the probability that all four selected coins are dimes is $\boxed{\frac{1}{495}}$.

Additional Insights: Variations and Related Problems

Understanding this problem lays the groundwork for exploring other probability scenarios involving selecting coins or objects from a collection.

1. Probability of Selecting a Specific Number of Nickels or Quarters

For example, what is the probability of selecting exactly 2 nickels and 2 quarters in 4 coins? This involves calculating combinations for each group and considering overlaps.

2. Probability of Selecting At Least One Dime

Calculating the probability of selecting at least one dime involves considering the complement: the probability of selecting no dimes, and subtracting from 1.

3. Expected Values and Mean Number of Dimes in Multiple Draws

When making repeated selections or with replacement, you can analyze expected values to understand average outcomes.

Understanding Combinatorics in Probabilities

The calculation of probabilities in this problem relies heavily on combinatorics, specifically combinations.

What Are Combinations?

Combinations are selections of items where the order does not matter. The notation $\binom{n}{k}$ represents the number of ways to choose k items from a set of n items.

Formula for Combinations

$$
\binom{n}{k} = \frac{n!}{k!(n-k)!}
$$

In our problem:
- $\binom{12}{4}$ represents total ways to choose any 4 coins.
- $\binom{4}{4}$ represents the only way to select all 4 dimes.

Real-World Applications of Probability in Coin Selection

Understanding probabilities in coin selection scenarios is not just an academic exercise but has practical applications in various fields:

- Gambling and Gaming: Calculating odds in card and coin games.
- Quality Control: Estimating probabilities of selecting defective items.
- Statistics and Data Sampling: Random sampling techniques.
- Decision Making: Risk assessment based on probabilistic outcomes.

Summary and Conclusion

To summarize, the problem of finding the probability that all four coins selected from a bag containing 4 dimes, 3 nickels, and 5 quarters are dimes involves understanding basic probability principles and combinatorics. The key steps include:

- Calculating total possible outcomes ($\binom{12}{4} = 495$)
- Identifying favorable outcomes (selecting all 4 dimes, which is $\binom{4}{4} = 1$)
- Computing the probability as the ratio of favorable to total outcomes:

$$
\boxed{\frac{1}{495}}
$$

This low probability reflects the rarity of randomly selecting all four dimes when there are many other coins in the bag.

Mastering such problems enhances your understanding of probability, combinatorics, and their practical applications—all fundamental skills in statistics, mathematics, and data analysis.

Additional Resources for Learning Probability and Combinatorics

- Books:
- "Introduction to Probability" by Joseph K. Blitzstein and Jessica Hwang
- "Discrete Mathematics and Its Applications" by Kenneth Rosen

- Online Courses:
- Khan Academy's Probability and Combinatorics courses
- Coursera's "Introduction to Probability and Data"

- Practice Problems:
- Websites like Brilliant.org and Mathway offer interactive probability exercises.

Final Thoughts

Probability problems involving simple scenarios like coin selection are excellent for building foundational understanding. They develop critical thinking skills and mathematical reasoning that are applicable in complex real-world situations. Remember, the key is to carefully analyze the problem, identify total and favorable outcomes, and apply the principles of combinatorics to arrive at accurate solutions.

Frequently Asked Questions

What is the probability of selecting all dimes from the bag containing 4 dimes, 3 nickels, and 5 quarters when choosing 4 coins?

The probability is calculated as the number of ways to select 4 dimes divided by the total number of ways to select any 4 coins. Since there are only 4 dimes, the numerator is 1 (choosing all 4 dimes), and the total ways are C(12,4). Therefore, the probability is 1 / C(12,4) = 1 / 495.

How do you determine the total number of ways to select 4 coins from the bag?

The total number of ways is the combination of all coins: C(12, 4), since there are 12 coins in total (4 dimes, 3 nickels, 5 quarters).

Why is the probability of drawing all dimes so low in

this scenario?

Because there are only 4 dimes in the bag and the total number of ways to select any 4 coins is much larger, resulting in a low probability of all selected coins being dimes.

What is the significance of calculating this probability in real-world contexts?

Calculating this probability helps understand the likelihood of specific outcomes in random selections, which is useful in scenarios like quality control, games of chance, or understanding randomness in sampling processes.

Can the probability of selecting all dimes be more than zero in this scenario?

Yes, the probability is more than zero because there is a chance of selecting all 4 dimes; it is simply very small, specifically 1 / 495.

How would the probability change if there were more than 4 dimes in the bag?

If there were more than 4 dimes, the numerator (ways to select all dimes) would increase, thus increasing the overall probability of selecting all dimes when choosing 4 coins.

Additional Resources

Understanding the Probability of Selecting All Dimes from a Bag of Coins

When dealing with probability problems involving coins, it's essential to understand the fundamental principles that govern random selection and probability calculation. The problem at hand provides a fascinating scenario: a bag containing a specific mix of coins—4 dimes, 3 nickels, and 5 quarters—totaling 12 coins, from which a subset of 4 coins is selected. The question posed is, "What is the probability that all four coins selected are dimes?"

This problem is a classic illustration of combinatorial probability, which involves calculating the likelihood of a specific event occurring within a finite set of possible outcomes. To approach this systematically, we'll break down the problem into manageable parts, explore relevant concepts, and perform detailed calculations to arrive at the correct probability.

Fundamental Concepts in Probability and Combinatorics

Before delving into the specifics of this problem, it's important to establish a foundational understanding of key concepts:

Probability Basics

- Probability measures the likelihood of an event occurring, expressed as a ratio between 0 and 1, or as a percentage.
- The probability $P(E)$ of an event E happening is given by:

$$
P(E) = \frac{\text{Number of favorable outcomes}}{\text{Total number of possible outcomes}}
$$

- When all outcomes are equally likely, this simple ratio suffices. This is known as the classical probability.

Combinatorics in Probability

- Combinations (denoted as $C(n, k)$ or $\binom{n}{k}$) represent the number of ways to choose k items from n items without regard to order.
- Permutations are used when the order matters, but in this problem, combinations are more relevant because the order of selected coins does not matter.
- The formula for combinations:

$$
\binom{n}{k} = \frac{n!}{k!(n - k)!}
$$

where $n!$ (n factorial) is the product of all positive integers up to n.

Analyzing the Composition of the Bag of Coins

The problem specifies the contents of the bag as follows:
- Dimes (10 cents each): 4 coins
- Nickels (5 cents each): 3 coins
- Quarters (25 cents each): 5 coins

Total number of coins:

$$
\text{Total coins} = 4 + 3 + 5 = 12
$$

This composition is crucial because it defines the sample space for the selection process.

The Selection Process and Event Definition

The problem involves selecting 4 coins randomly from the 12 coins in the bag. Assuming that each coin has an equal chance of being selected and that the selection is random and without replacement, the total number of possible outcomes can be calculated using combinatorics.

The specific event of interest is:
- "All four coins selected are dimes."

This event is quite specific, as it restricts the selection to only the 4 dimes in the bag. Therefore, understanding the favorable outcomes and total possible outcomes is essential.

Calculating the Total Number of Possible Outcomes

To understand the probability, first, determine the total number of ways to select any 4 coins from the 12 coins.

Using combinations:

$$
\text{Total outcomes} = \binom{12}{4}
$$

Calculating $\binom{12}{4}$:

$$
\binom{12}{4} = \frac{12!}{4! \times (12 - 4)!} = \frac{12!}{4! \times 8!}
$$

Breaking down:

$$
12! = 12 \times 11 \times 10 \times 9 \times 8!
$$

So:

$$
\binom{12}{4} = \frac{12 \times 11 \times 10 \times 9 \times 8!}{4! \times 8!} = \frac{12 \times 11 \times 10 \times 9}{4 \times 3 \times 2 \times 1}
$$

Calculating numerator:

$$
12 \times 11 = 132
$$

$$
132 \times 10 = 1320
$$

$$
1320 \times 9 = 11880
$$

Calculating denominator:

$$
4 \times 3 \times 2 \times 1 = 24
$$

Thus:

$$
\binom{12}{4} = \frac{11880}{24} = 495
$$

Total possible outcomes = 495.

Calculating Favorable Outcomes: Selecting All Dimes

Since we want all four selected coins to be dimes, the favorable outcomes are the number of ways to select all 4 dimes from the 4 dimes available.

Number of ways to choose all 4 dimes:
$$
\binom{4}{4} = 1
$$
because there's only one way to choose all four dimes—the only possible combination is the entire set of 4 dimes.

No other coins are involved in this favorable event, as selecting more than 4 coins of the same type isn't possible here.

Calculating the Probability

The probability P that all four selected coins are dimes is:
$$
P = \frac{\text{Number of favorable outcomes}}{\text{Total outcomes}}
$$
which simplifies to:
$$
P = \frac{\binom{4}{4}}{\binom{12}{4}} = \frac{1}{495}
$$

Expressed as a decimal:
$$
P \approx 0.00202
$$
or approximately 0.202%.

This indicates that there is a very low chance of randomly selecting all four coins as dimes from the bag.

Deeper Insights and Variations

While the above calculation provides the exact probability for the specific event, it's instructive to explore related scenarios and insights:

1. Probability of Selecting Exactly 3 Dimes

- To find the probability of selecting exactly 3 dimes and 1 other coin:
- Number of ways to choose 3 dimes:

$$
\binom{4}{3} = 4
$$

- Number of ways to choose 1 coin from remaining 8 coins (3 nickels + 5 quarters):

$$
\binom{8}{1} = 8
$$

- Total favorable outcomes:

$$
4 \times 8 = 32
$$

- Probability:

$$
P = \frac{32}{495} \approx 0.0646
$$

or about 6.46%.

Implication: The chance of drawing 3 dimes and 1 other coin is significantly higher than drawing all four as dimes, which makes sense intuitively.

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2. Probability of Selecting No Dimes

- To find the probability of selecting 4 coins with no dimes:
- Only nickels and quarters are available, totaling $3 + 5 = 8$ coins.
- Number of ways to choose 4 coins from these 8:

$$
\binom{8}{4} = 70
$$

- Probability:

$$
P = \frac{70}{495} \approx 0.1414
$$

or about 14.14%.

Observation: The probability of not selecting any dimes is higher than the probability of selecting all four dimes, which aligns with the fact that there are more non-dime coins.

Real-World Applications and Teaching Value

This problem isn't just an abstract mathematical exercise; it has practical significance in various fields:

- Educational Contexts: Teaching students the concepts of probability, combinatorics, and how to approach discrete probability problems systematically.
- Game Theory and Gambling: Understanding odds in card and coin games.

- Quality Control in Manufacturing: Estimating probabilities of defective items in batches.
- Statistics and Data Science: Modeling random sampling and understanding bias or likelihood of certain outcomes.

Furthermore, the problem reinforces critical thinking about how the composition of a set influences the likelihood of specific events, a principle applicable in many real-world decision-making scenarios.

Summary of Key Findings

- Total number of ways to select 4 coins from 12: 495
- Number of favorable outcomes (all dimes): 1
- Probability that all four selected coins are dimes:

$$
\boxed{\frac{1}{495} \approx 0.00202}
$$

or approximately 0.202%.

This indicates a very low likelihood, as expected, given the small number of dimes relative to the total coins.

Conclusion

The problem of calculating the probability that all four coins selected from a bag containing 4 dimes, 3 nickels, and 5 quarters are dimes exemplifies

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