In One Flip Of 10 Unbiased Coins, What Is The Probability Of Getting A Result As Extreme Or More Extreme

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When flipping 10 unbiased coins simultaneously, each coin has two equally likely outcomes: heads (H) or tails (T). The total number of possible outcomes is $2^{10} = 1024$. One common question in probability theory is determining how likely it is to observe a result that is as extreme as, or more extreme than, a given outcome. In this context, "extreme" typically refers to the number of heads observed being as far from the expected value as possible, namely, 5 heads (the mean) or more extreme outcomes like 0, 1, 9, or 10 heads.

This article explores how to compute the probability of obtaining such extreme results, including the detailed steps, relevant formulas, and interpretations. We will analyze the binomial distribution governing the number of heads in 10 coin flips, identify what "more extreme" means, and then calculate the corresponding probabilities.

Understanding the Distribution: Binomial Model of Coin Flips

The Binomial Distribution Framework

The process of flipping 10 unbiased coins can be modeled as a binomial experiment with parameters:

- Number of trials, $n = 10$
- Probability of success (getting a head), $p = 0.5$

The probability of observing exactly k heads in 10 flips is given by the binomial probability mass function (PMF):

$$
P(X = k) = \binom{n}{k} p^{k} (1 - p)^{n - k}
$$

\]

where:

- \(\binom{n}{k}\) is the binomial coefficient, representing the number of ways to choose \(k\) successes out of \(n\) trials.

Since \(p = 0.5\), the formula simplifies to:

\[
P(X = k) = \binom{10}{k} \times 0.5^{10}
\]

Because the coin flips are unbiased and independent, the binomial distribution is symmetric around its mean:

\[
\text{Expected value} = np = 10 \times 0.5 = 5
\]

Identifying Extreme Outcomes

In the context of coin flips, an "extreme" result is one that is as unlikely or more unlikely than the most typical outcomes, often those at the tails of the distribution.

- The most probable outcome is getting exactly 5 heads, which is the mean.
- More "extreme" outcomes are those with fewer or more heads than 5, specifically:

- 0 heads or 10 heads
- 1 head or 9 heads
- 2 heads or 8 heads
- 3 heads or 7 heads
- 4 heads or 6 heads

Depending on the definition, "as extreme or more extreme" includes these tail events, i.e., outcomes that are at least as far from the mean as the most extreme possible outcomes.

Calculating the Probabilities of Extreme Outcomes

Defining the Range of Extreme Outcomes

Since the total number of heads can vary from 0 to 10, the most extreme outcomes are:

- 0 heads (all tails)
- 10 heads (all heads)

The next most extreme outcomes are:

- 1 head or 9 heads
- 2 heads or 8 heads
- 3 heads or 7 heads
- 4 heads or 6 heads

Often, the phrase "as extreme or more extreme" refers to outcomes at the tails, i.e., outcomes with a number of heads less than or equal to 1 or greater than or equal to 9, or similarly for 2 and 8, etc., depending on the context.

Calculating the Probability of Extreme Results

To find the probability of observing an outcome as extreme or more extreme than a specific point, sum the probabilities of all such outcomes.

Example: Probability of getting 0 or 10 heads

$$
P(\text{0 or 10 heads}) = P(X=0) + P(X=10)
$$

Calculating each:

$$
P(X=0) = \binom{10}{0} \times 0.5^{10} = 1 \times 0.5^{10} = \frac{1}{1024}
$$

$$
P(X=10) = \binom{10}{10} \times 0.5^{10} = 1 \times 0.5^{10} = \frac{1}{1024}
$$

Thus,

$$
P(\text{0 or 10 heads}) = \frac{1}{1024} + \frac{1}{1024} = \frac{2}{1024} = \frac{1}{512}
$$

Similarly, for outcomes with 1 or 9 heads:

$$
P(X=1) = \binom{10}{1} \times 0.5^{10} = 10 \times \frac{1}{1024} = \frac{10}{1024}
$$

$$
P(X=9) = \binom{10}{9} \times 0.5^{10} = 10 \times \frac{1}{1024} = \frac{10}{1024}
$$

Total:

$$
P(\text{1 or 9 heads}) = \frac{10}{1024} + \frac{10}{1024} = \frac{20}{1024} = \frac{5}{256}
$$

Likewise, for outcomes with 2 or 8 heads:

$$
P(X=2) = \binom{10}{2} \times 0.5^{10} = 45 \times \frac{1}{1024} = \frac{45}{1024}
$$

$$
P(X=8) = \binom{10}{8} \times 0.5^{10} = 45 \times \frac{1}{1024} = \frac{45}{1024}
$$

Sum:

$$
P(\text{2 or 8 heads}) = \frac{90}{1024} = \frac{45}{512}
$$

Similarly, for 3 or 7:

$$
P(X=3) = 120 \times \frac{1}{1024} = \frac{120}{1024}
$$

$$
P(X=7) = 120 \times \frac{1}{1024} = \frac{120}{1024}
$$

Sum:

$$
\frac{240}{1024} = \frac{15}{64}
$$

And for 4 or 6:

$$
P(X=4) = 210 \times \frac{1}{1024} = \frac{210}{1024}
$$

$$
P(X=6) = 210 \times \frac{1}{1024} = \frac{210}{1024}
$$

Sum:

$$
\frac{420}{1024} = \frac{105}{256}
$$

Summing All Tail Probabilities

To find the probability of getting a result as extreme or more extreme than the mean (i.e., in the tails), sum the probabilities of the outcomes in the tail regions:

$$
P(\text{extreme or more}) = P(X=0) + P(X=1) + P(X=2) + P(3) + P(7) + P(8) + P(9) + P(10)
$$

From previous calculations:

$$
P(\text{extreme or more}) = \frac{1}{1024} + \frac{10}{1024} + \frac{45}{1024} + \frac{120}{1024} + \frac{120}{1024} + \frac{45}{1024} + \frac{10}{1024} + \frac{1}{1024}
$$

Add all numerators:

$$
1 + 10 + 45 + 120 + 120 + 45 + 10 + 1 = 352
$$

Thus:

$$
P(\text{extreme or more}) = \frac{352}{1024} \approx 0.34375
$$

Interpretation: There is approximately a 34.375% chance of observing a result that is as extreme or more extreme than the mean in 10 unbiased coin flips.

Alternative Approaches and Considerations

Using Symmetry and Cumulative Distribution

Because the binomial distribution with $p=0.5$ is symmetric around $k=5$, calculations can be simplified by considering cumulative probabilities:

$$
P(\text{X} \leq k) = \sum_{i=0}^{k} P(X=i)
$$

Similarly,

$$
P(\text{X} \geq k) = \sum_{i=k}^{n} P(X=i)
$$

The probability of extreme results can often be expressed as the sum of tail probabilities, which can be obtained via cumulative binomial distribution functions or statistical software.

For example, the probability of obtaining a result as extreme or more extreme than 2

Frequently Asked Questions

What is the probability of getting exactly 5 heads when flipping 10 unbiased coins?

The probability of exactly 5 heads in 10 flips is given by the binomial formula: $C(10,5) (0.5)^5 (0.5)^5 = 252 (0.5)^{10} \approx 0.2461$.

How do you determine the probability of getting an outcome as extreme or more extreme than a certain number of heads in 10 coin flips?

You sum the probabilities of all outcomes with number of heads less than or equal to that number if it's on the lower tail, or greater than or equal if on the upper tail, considering the symmetry of the binomial distribution for unbiased coins.

What is the probability of getting 0 or 10 heads in

10 coin flips?

The probability of all heads (10 heads) or all tails (0 heads) is 2 (0.5)^10 = 2 (1/1024) ≈ 0.00195.

In the context of 10 coin flips, what does 'more extreme' mean?

'More extreme' refers to outcomes that are farther from the expected value (which is 5 heads), meaning fewer or more heads than a certain threshold, such as 0 or 10 heads.

How do you calculate the two-tailed p-value for observing 0 or 10 heads in 10 flips?

You sum the probabilities of observing 0 or 10 heads, which is approximately 0.00195, representing the probability of such extreme results or more extreme ones.

What is the probability of getting 6 or more heads in 10 flips?

Calculating the sum of the probabilities for 6, 7, 8, 9, and 10 heads gives approximately 0.37695.

How does symmetry of the binomial distribution help in finding the probability of extreme outcomes in 10 coin flips?

Because the distribution is symmetric around 5 heads, the probability of getting k heads is equal to the probability of getting (10 - k) heads, simplifying calculations of extreme or more extreme outcomes.

If you observe 2 heads in 10 flips, what is the probability of getting this or a more extreme result?

The probability of 0, 1, or 2 heads combined is approximately 0.1719, so the probability of getting this or a more extreme result (like 8, 9, or 10 heads) is also about 0.1719, considering the symmetry.

Additional Resources

In One Flip Of 10 Unbiased Coins, What Is The Probability Of Getting A Result As Extreme Or More Extreme

Understanding the probability of obtaining an extreme or more extreme result in a series of coin flips is a fundamental question in probability theory, especially in the context of binomial distributions. When flipping 10 unbiased coins, each with a 50% chance of landing heads or tails, the distribution of outcomes follows a binomial pattern. This article explores this question in depth, examining the underlying mathematics, the concept of extremeness in results, and practical implications. Whether for academic purposes, gambling, or scientific experiments, grasping this probability provides valuable insight into how likely certain rare events are in simple binary trials.

Understanding the Basic Setup: Flipping 10 Unbiased Coins

Before diving into probabilities, it’s essential to clarify the scenario:

- You flip 10 unbiased (fair) coins simultaneously.
- Each coin has two equally likely outcomes: Heads (H) or Tails (T).
- The total number of possible outcomes is $2^{10} = 1024$.

The primary random variable of interest is X, the number of coins showing heads after all flips.

Distribution of X:
X follows a binomial distribution:

$$
P(X = k) = \binom{10}{k} \left(\frac{1}{2}\right)^{10}
$$

where:
- k is the number of heads (from 0 to 10),
- $\binom{10}{k}$ is the binomial coefficient, representing the number of ways to choose k heads out of 10 flips.

What Does “Extreme or More Extreme” Mean?

The phrase "extreme or more extreme" refers to outcomes that are as unlikely or less likely than some observed result, in the tails of the distribution. In the context of coin flips:

- Extreme results are those with a very high or very low number of heads,

such as 0, 1, 9, or 10 heads.
- These are outcomes far from the expected value, which for a fair coin is $n \times p = 10 \times 0.5 = 5$ heads.

More extreme results refer to outcomes that are at least as improbable as the observed one, often used in statistical testing (e.g., p-values).

Example:
Suppose you observe 0 heads (all tails). The probability of that is:

$$
P(X=0) = \binom{10}{0} \left(\frac{1}{2}\right)^{10} = 1 \times \frac{1}{1024} \approx 0.0009766
$$

This is a very rare event, so it’s considered extremely extreme. Conversely, outcomes near the center (around 5 heads) are less extreme.

Calculating the Probability of Extreme Results

To find the probability of getting a result as extreme or more extreme than a certain outcome, we need to:

1. Define what the “extreme” outcomes are.
2. Sum the probabilities of all outcomes as or more extreme than the specified result.

This process often involves calculating the tail probabilities of the binomial distribution.

Symmetry in the Binomial Distribution

Since the coin flips are fair, the binomial distribution is symmetric:

$$
P(X = k) = P(X = 10 - k)
$$

This symmetry simplifies calculations of tail probabilities.

Step-by-Step Calculation Examples

Let's walk through the process with specific examples.

Example 1: Extreme as 0 or 10 Heads

Question: What is the probability of getting 0 or 10 heads?

$$
P(\text{0 or 10 heads}) = P(X=0) + P(X=10) = 2 \times \frac{1}{1024} = \frac{2}{1024} = \frac{1}{512} \approx 0.001953
$$

This is a very low probability, representing highly extreme outcomes.

Example 2: Extreme as 1 or 9 Heads

$$
P(\text{1 or 9 heads}) = 2 \times \binom{10}{1} \left(\frac{1}{2}\right)^{10} = 2 \times 10 \times \frac{1}{1024} = \frac{20}{1024} \approx 0.0195
$$

Example 3: Two-Tailed Probability for Outcomes as Extreme or More Extreme Than 2 Heads

Calculate $P(X \leq 2)$ and $P(X \geq 8)$:

$$
P(X \leq 2) = P(0) + P(1) + P(2)
$$

$$
P(0) = \frac{1}{1024}
$$

$$
P(1) = 10 \times \frac{1}{1024} = \frac{10}{1024}
$$

$$
P(2) = \binom{10}{2} \times \frac{1}{1024} = 45 \times \frac{1}{1024} = \frac{45}{1024}
$$

Sum:

$$
P(X \leq 2) = \frac{1 + 10 + 45}{1024} = \frac{56}{1024} \approx 0.0547
$$

By symmetry,

$$
P(X \geq 8) = P(8) + P(9) + P(10) = \text{same as } P(X \leq 2) \approx 0.0547
$$

\]

Total probability for “as or more extreme than 2 heads”:

$$
P(\text{extreme or more}) = P(X \leq 2) + P(X \geq 8) = 0.0547 + 0.0547 = 0.1094
$$

General Approach to Computing “As or More Extreme” Probabilities

To generalize, for any observed result k:

1. Identify the tail probabilities:

- For outcomes less than or equal to k (if k is in the lower tail).
- For outcomes greater than or equal to $n - k$ (symmetry applies).

2. Sum the probabilities:

$$
P(\text{as or more extreme}) = P(X \leq k) + P(X \geq n - k)
$$

3. For two-sided tests, if the observed k is near the center, the combined tail probability will be small, reflecting an unlikely event.

Implications and Practical Uses

Understanding the probability of extreme results in coin flips has several applications:

- Hypothesis Testing:
In statistics, such calculations underpin p-values, helping decide if an observed outcome is consistent with a null hypothesis of fairness.

- Gambling and Games of Chance:
Recognizing the rarity of extreme outcomes helps in assessing risks and odds.

- Quality Control & Scientific Experiments:
When results deviate significantly from expectations, they may indicate

issues like bias or experimental error.

Pros and Cons

Pros:

- Provides a clear quantitative measure of how unlikely certain outcomes are.
- Useful for hypothesis testing and statistical inference.
- Easy to compute with symmetry and binomial formulas.

Cons:

- Can become computationally intensive for larger n.
- Assumes independence and fairness; real-world biases may affect results.
- Focuses on simple binary outcomes; more complex scenarios require advanced models.

Features of the Binomial Distribution in Coin Flips

- Symmetry:
For unbiased coins, the distribution is symmetric around the mean of 5 heads.

- Discrete Nature:
Outcomes are discrete, with probabilities concentrated at specific points.

- Tail Behavior:
The probability of very extreme outcomes diminishes rapidly as outcomes move away from the mean.

Visual Representation

Plotting the binomial probabilities for 10 flips results in a bell-shaped curve centered at 5. The tails (0 or 10 heads) are the least probable, illustrating their extremeness.

Conclusion

Calculating the probability of obtaining a result as extreme or more extreme than a given outcome in flipping 10 unbiased coins involves understanding the binomial distribution and its tail probabilities. The symmetry of the distribution simplifies calculations, and the probabilities for very extreme outcomes (like all heads or all tails) are remarkably low, emphasizing their rarity. These calculations are foundational in statistical inference, providing context for how unlikely certain results are under a fair coin assumption.

Whether you're assessing the odds of a rare event or conducting a hypothesis test, knowing how to compute and interpret these probabilities is invaluable. As the number of coins increases, the distribution becomes more peaked, and extreme outcomes become even less likely, underscoring the power of the binomial model in understanding binary trials.

In summary, the probability of observing an outcome as extreme or more extreme than a specific result in 10 coin flips can be accurately computed using binomial probabilities and symmetry considerations. This understanding forms the backbone of many statistical applications and offers a window into the nature of randomness and rare events in simple binary experiments.

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