In What Ratio Does The Line Of The Equation 4x + 5y = 21 Divide The Line Segment Joining The Points (-2,

In What Ratio Does The Line Of The Equation 4x + 5y = 21 Divide The Line Segment Joining The Points (-2,)

In what ratio does the line of the equation 4x + 5y = 21 divide the line segment joining the points (-2, y_1) and (x_2, y_2)? This question is a common problem in coordinate geometry that involves understanding how a line intersects and divides a segment connecting two points. Solving such problems enhances our understanding of concepts like section formula, ratio division, and the properties of straight lines. In this article, we will explore the methods to determine the ratio, step-by-step, with detailed explanations and examples.

Understanding the Problem

Breaking Down the Question

Before diving into calculations, it is essential to interpret what the question asks:

- You are given a line equation: 4x + 5y = 21.
- You have two points (which are partially given): (-2, y_1) and (x_2, y_2).
- The goal is to determine the ratio in which the line divides the segment connecting these two points.

Clarifying the Data

Often, the problem provides two points, say:

- Point A: (-2, y_1)
- Point B: (x_2, y_2)

However, in the provided statement, only one point is fully specified, and the other is incomplete. Usually, such problems specify both points to find the ratio. For a comprehensive understanding, we will consider a typical scenario:

- Point A: $(-2, y_1)$
- Point B: (x_2, y_2)

and explore how the line 4x + 5y = 21 interacts with this segment.

Fundamentals of Division of a Line Segment

Section Formula

The section formula helps find the coordinates of a point dividing a line segment in a given ratio.

For points $A(x_1, y_1)$ and $B(x_2, y_2)$, if a point $P(x, y)$ divides the segment in ratio $m:n$, then:

$$
x = \frac{mx_2 + nx_1}{m + n}
$$

$$
y = \frac{my_2 + ny_1}{m + n}
$$

where $m:n$ is the ratio in which the point divides the segment.

Using the Equation of Line to Find Intersection Point

Suppose the line $4x + 5y = 21$ intersects the segment AB at point P. The point P divides AB in a certain ratio $m:n$. Our goal is to find this ratio.

Step-by-Step Approach to Find the Ratio

1. Define the Coordinates of the Points

Assuming the points are:

- Point A: $(-2, y_1)$
- Point B: (x_2, y_2)

To proceed, we need specific values or expressions for y_1 and y_2. In many problems, the second point is given explicitly, or both points are on the line $4x + 5y = 21$.

2. Confirm if the Points Lie on the Given Line

Check if points A and B satisfy the line's equation:

- For A: $4(-2) + 5 y_1 = 21 \Rightarrow -8 + 5 y_1 = 21 \Rightarrow 5 y_1 = 29 \Rightarrow y_1 = \frac{29}{5}$

- For B: $4 x_2 + 5 y_2 = 21$

If points A and B lie on the line, then:

$$
A = \left(-2, \frac{29}{5}\right)
$$

and

$$
B = \left(x_2, y_2\right), \text{ where } 4 x_2 + 5 y_2 =21
$$

Calculating the Division Ratio

3. Find the Coordinates of the Dividing Point P

Suppose P divides AB in the ratio $m:n$. Then, using the section formula:

$$
x_P = \frac{m x_2 + n(-2)}{m + n}
$$

$$
y_P = \frac{m y_2 + n \frac{29}{5}}{m + n}
$$

Since P lies on the line $4x + 5y = 21$, it must satisfy:

$$
4 x_P + 5 y_P = 21
$$

Substituting the section formula expressions:

$$
4 \left(\frac{m x_2 + n(-2)}{m + n} \right) + 5 \left(\frac{m y_2 + n \frac{29}{5}}{m + n} \right) = 21
$$

Multiply through by $m + n$:

$$
4 [m x_2 + n(-2)] + 5 [m y_2 + n \frac{29}{5}] = 21 (m + n)
$$

Simplify:

$$
4 m x_2 - 8 n + 5 m y_2 + 29 n = 21 m + 21 n
$$

Group like terms:

$$
(4 m x_2 + 5 m y_2) + (-8 n + 29 n) = 21 m + 21 n
$$

$$
m (4 x_2 + 5 y_2) + n (21) = 21 m + 21 n
$$

Recall that $4 x_2 + 5 y_2 = 21$, so:

$$
m (21) + 21 n = 21 m + 21 n
$$

which simplifies to:

$$
21 m + 21 n = 21 m + 21 n
$$

This identity indicates that any point on the line between A and B satisfies the equation when divided in ratio $m:n$. Therefore, the division ratio depends on the specific location of P, which might be given or determined by additional data.

Special Case: When Point B is on the Line

If both points A and B lie on the line $4x + 5y = 21$, then the entire segment is on the line, and the division ratio depends solely on the positions of A and B. To determine the ratio:

- Identify the coordinates of B satisfying $4 x_2 + 5 y_2 = 21$.
- Use the section formula to find how P divides AB.

Example: Concrete Calculation

Suppose we have:

- Point A: $(-2, 29/5)$
- Point B: $(3, 3)$

Check if B is on the line:

$$
4(3) + 5(3) = 12 + 15 = 27 \neq 21
$$

So, B is not on the line. To find the point P dividing AB in a ratio $m:n$, and lying on the line, proceed as follows:

- Use the section formula:

$$
x_P = \frac{m \times 3 + n \times (-2)}{m + n}
$$
$$
y_P = \frac{m \times 3 + n \times \frac{29}{5}}{m + n}
$$

- Since P lies on the line:

$$
4 x_P + 5 y_P = 21
$$

- Substitute x_P and y_P:

$$
4 \left(\frac{3 m - 2 n}{m + n} \right) + 5 \left(\frac{3 m + \frac{29}{5} n}{m + n} \right) = 21
$$

Multiply through by $m + n$:

$$
4 (3 m - 2 n) + 5 \left(3 m + \frac{29}{5} n \right) = 21 (m + n)
$$

Simplify:

$$
12 m - 8 n + 15 m + 29 n = 21 m + 21 n
$$

Combine like terms:

$$
(12 m + 15 m) + (-8 n + 29 n) = 21 m + 21 n
$$
$$
27 m + 21 n = 21 m + 21 n
$$

Subtract $21 m + 21 n$ from both sides:

\[
(27 m - 21 m) + (21

Frequently Asked Questions

In what ratio does the line of the equation 4x + 5y = 21 divide the segment joining the points $(-2, y_1)$ and (x_2, y_2)?

To determine the ratio, we need the specific points $(-2, y_1)$ and (x_2, y_2). Assuming these points lie on the line $4x + 5y = 21$, we can find their coordinates by substituting their x or y values into the equation. Once the points are known, the section formula can be used to find the ratio in which the line divides the segment.

How do you find the point of division of a line segment by a given line?

To find the point where a line divides a segment, first find the coordinates of the segment's endpoints. Then, determine the ratios in which the division occurs using the section formula, which states that the dividing point divides the segment in a ratio m:n, calculated based on the coordinates and the line's equation.

What is the section formula used for in coordinate geometry?

The section formula is used to find the coordinates of a point dividing a line segment joining two points in a given ratio m:n. It helps determine the point of intersection or division along a segment.

How can the equation 4x + 5y = 21 be used to find the division ratio on a segment?

The equation represents a line. To find the division ratio of a segment by this line, identify the points on the segment that satisfy the line's equation or are given, then apply the section formula to determine the ratio in which the line divides the segment.

What are common methods to find the ratio in which a line divides a segment connecting two points?

Common methods include using the section formula, solving simultaneous equations if the points lie on the line, and applying the concept of similar triangles or ratios of distances from the intersection point to the endpoints.

Can the line 4x + 5y = 21 intersect the segment joining (-2, y_1) and (x_2, y_2)? How to determine this?

Yes, the line can intersect the segment if the segment's endpoints lie on opposite sides of the line or if the line passes through the segment. To check, substitute the endpoints' coordinates into the line's equation and compare the signs of the results. If they differ, the line intersects the segment.

How do you find the specific point where the line divides the segment joining two points?

First, determine the coordinates of the endpoints, then use the ratio in which the line divides the segment and apply the section formula to find the exact coordinates of the division point.

What is the significance of the ratio in dividing a line segment by a line?

The ratio indicates how the segment is divided—whether equally or unequally—by the line. It helps in understanding the division's relative position and is essential in geometric constructions and proofs.

What additional information is needed to calculate the division ratio of the line 4x + 5y = 21 on a segment joining (-2, y_1) and (x_2, y_2)?

The specific coordinates of the points (-2, y_1) and (x_2, y_2) are required to apply the section formula and determine the exact ratio in which the line divides the segment.

Additional Resources

Line Division Ratio: A Comprehensive Exploration of the Equation \(4x + 5y = 21\) and Its Role in Segment Division

---

Introduction

In the realm of coordinate geometry, understanding how a line interacts with points and segments is fundamental. Among the various concepts, the division of a line segment by a given line—specifically, determining the ratio in which the line divides the segment connecting two points—is a crucial skill. This article delves deeply into the question: In what ratio does the line of the equation \(4x + 5y = 21\) divide the line segment joining the points \((-2, y_1)\) and \((x_2, y_2)\)?

While this may seem straightforward at first glance, it encompasses several underlying principles: the equation of the line, the concept of internal and external division, and the application of the section formula. By exploring each component thoroughly, we aim to provide clarity and a comprehensive understanding of the problem.

Background and Fundamental Concepts

Before addressing the specific problem, it's essential to revisit some foundational ideas:

- Line Equation: The equation \(4x + 5y = 21\) represents a straight line in the xy-plane. Its slope-intercept form is \(y = -\frac{4}{5}x + \frac{21}{5} \), indicating a slope of \(-\frac{4}{5}\).

- Line Segment: The segment joining two points \(A(x_1, y_1)\) and \(B(x_2, y_2)\) is a finite part of a straight line.

- Division of a Line Segment: When a point \(P\) lies on segment \(AB\), it divides the segment in a certain ratio \(m:n\), where \(m\) and \(n\) are positive real numbers if \(P\) is between \(A\) and \(B\).

- Section Formula: If P divides the segment AB in the ratio $m:n$, then the coordinates of P are given by:

$$
P \left(\frac{mx_2 + nx_1}{m + n}, \frac{my_2 + ny_1}{m + n} \right)
$$

Clarification of the Problem

The problem as originally posed appears to have some missing information, particularly the points involved. Typically, such a problem asks:

Given two points $A(x_1, y_1)$ and $B(x_2, y_2)$, and a line L, find the ratio in which L divides the segment AB.

Alternatively, if the line and one point are specified, the question might be:

Find the ratio in which the line $4x + 5y = 21$ divides the segment joining a given point $A(-2, y_1)$ and some other point $B(x_2, y_2)$.

For the purpose of this detailed analysis, we will assume the following:

- The points are $A(-2, y_1)$ and $B(x_2, y_2)$, with x_2, y_1, y_2 unspecified.
- The line $4x + 5y = 21$ intersects the segment AB at point P.
- The goal is to find the ratio $AP:PB$.

If you have specific points in mind, please specify them. Otherwise, we'll proceed with a general approach, illustrating the methodology with symbolic points and then applying specific values for clarity.

Step-by-Step Analysis

1. Expressing the Points

Suppose:

- $A(-2, y_1)$
- $B(x_2, y_2)$

Point A has a fixed x-coordinate of -2. The point B is variable, but for an illustrative example, let's choose concrete values later.

2. Parametric Representation of the Segment

To analyze the division, express any point P on AB parametrically:

$$
P = A + t(B - A) = \left(-2 + t (x_2 + 2), y_1 + t (y_2 - y_1) \right),
\quad t \in [0,1]
$$

Here, $t=0$ corresponds to A, and $t=1$ corresponds to B. The coordinate of P is:

$$
P_x = -2 + t (x_2 + 2) \\
P_y = y_1 + t (y_2 - y_1)
$$

---

3. Applying the Line Equation

Since P lies on the line $4x + 5y = 21$:

$$
4 P_x + 5 P_y = 21
$$

Substitute the parametric expressions:

$$
4 \left(-2 + t (x_2 + 2) \right) + 5 \left(y_1 + t (y_2 - y_1) \right) = 21
$$

Simplify:

$$
4(-2) + 4 t (x_2 + 2) + 5 y_1 + 5 t (y_2 - y_1) = 21
$$

$$
-8 + 4 t (x_2 + 2) + 5 y_1 + 5 t (y_2 - y_1) = 21
$$

Group terms:

$$
(4 t (x_2 + 2) + 5 t (y_2 - y_1)) + (5 y_1 - 8) = 21
$$

Factor t:

$$
t \left(4 (x_2 + 2) + 5 (y_2 - y_1) \right) = 21 - 5 y_1 + 8
$$

$$
t \left(4 x_2 + 8 + 5 y_2 - 5 y_1 \right) = 29 - 5 y_1
$$

Define:

$$
\text{Coefficient of } t: \quad C = 4 x_2 + 8 + 5 y_2 - 5 y_1
$$
$$
\text{Constant term:} \quad D = 29 - 5 y_1
$$

Therefore,

$$
t = \frac{D}{C} = \frac{29 - 5 y_1}{4 x_2 + 8 + 5 y_2 - 5 y_1}
$$

This value of t indicates the position of P on AB.

---

4. Determining the Division Ratio

The division ratio relates to the parameter t:

- If $t \in (0,1)$, then P lies inside the segment AB, and the ratio is:

$$
AP : PB = t : (1 - t)
$$

- If $t < 0$ or $t > 1$, then P lies outside the segment on the line extended beyond A or B.

Assuming $t \in (0,1)$, the ratio is:

$$
\boxed{
\frac{AP}{PB} = \frac{t}{1 - t}
}
$$

---

5. Special Cases and Examples

Let's illustrate with specific points for clarity:

Suppose:

- $A(-2, 3)$
- $B(4, 1)$

Calculate t:

$$
C = 4(4) + 8 + 5(1) - 5(3) = 16 + 8 + 5 - 15 = 14
$$

$$
D = 29 - 5(3) = 29 - 15 = 14
$$

Thus,

$$
t = \frac{14}{14} = 1
$$

This indicates P coincides with B, meaning the line $4x + 5y = 21$ passes through point $B(4,1)$. Check:

$$
4(4) + 5(1) = 16 + 5 = 21
$$

Yes, point B lies on the line; hence, the division ratio is effectively:

$$
AP : PB = t : (1 - t) = 1 : 0
$$

which means the line intersects the segment at B, dividing it at its endpoint. The division occurs externally at A and internally at B.

General Insights and Conclusions

- The ratio in which the line divides the segment depends critically on the positions of A and B relative to the line.
- The parameter t is derived from substituting the parametric form into the line equation.
- Once t is known, the division ratio is straightforward:

\[
\boxed{

```
\frac{AP}{PB} = \frac{t}{1
```

In What Ratio Does The Line Of The Equation 4x 5y 21 Divide The Line Segment Joining The Points 2

In What Ratio Does The Line Of The Equation 4x 5y 21 Divide The Line Segment Joining The Points 2

Related Articles

- Non-disjunction Can Occur In Which Of The Following Cell Divisions? A. Melosis I Only B. Meiosis II Only
- Let , ,, X X1 2 X64 Be A Random Sample From A Poisson Distribution With A Mean Of 4. Find An Approximate
- Prior To Surgery, A Patient Is To Receive Versed 3 Mg. It Is Available As Versed 5 Mg/ml. In Ml, Howmuch

Back to Home