

Let $A=(3,3,1)$, $B=(0,7,0)$, $C=(3,4,0)$, And $D=(0,0,1)$. Find The Area Of The Parallelogram Determined By These

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Understanding the geometry of three-dimensional figures is a fundamental aspect of vector calculus and analytical geometry. In particular, calculating the area of a parallelogram defined by vectors in space requires a grasp of vector operations such as the cross product. This article provides a comprehensive guide to solving the problem of finding the area of the parallelogram determined by the given points, A, B, C, and D.

Introduction to the Problem

When given points in three-dimensional space, determining the area of a parallelogram formed by vectors originating from these points is essential in various fields, including physics, engineering, and computer graphics. The problem involves:

- Recognizing the relevant vectors that define the parallelogram.
- Applying vector operations to compute the area.
- Understanding the geometric significance of the calculations.

The specific problem involves the points:

- $A = (3, 3, 1)$
- $B = (0, 7, 0)$
- $C = (3, 4, 0)$
- $D = (0, 0, 1)$

The goal is to find the area of the parallelogram determined by these points, which involves choosing appropriate vectors and applying the cross product to find the magnitude that corresponds to the area.

Step 1: Visualizing the Points and the

Parallelogram

Before diving into calculations, it's crucial to visualize the points:

- A and C lie on the plane $z=0$, with A at $(3,3,1)$ and C at $(3,4,0)$.
- B is at $(0,7,0)$, and D at $(0,0,1)$.

From the points, it appears that the parallelogram is formed by vectors originating from one of the points, say A, and extending to other points.

Step 2: Identifying the Vectors Defining the Parallelogram

To determine the area of the parallelogram, we need two adjacent sides represented as vectors in space. Typically, this involves:

- Choosing a common starting point (e.g., point A).
- Constructing vectors from A to other relevant points.

Key vectors:

1. Vector AB: from A to B
2. Vector AC: from A to C

Calculations:

- $AB = B - A = (0 - 3, 7 - 3, 0 - 1) = (-3, 4, -1)$
- $AC = C - A = (3 - 3, 4 - 3, 0 - 1) = (0, 1, -1)$

These vectors define two adjacent sides of the parallelogram originating from point A.

Step 3: Computing the Cross Product of the Vectors

The area of a parallelogram in three-dimensional space is given by the magnitude of the cross product of its adjacent side vectors:

$$\text{Area} = |\vec{AB} \times \vec{AC}|$$

\]

Calculating the cross product:

Given vectors:

$$\begin{aligned} \vec{u} &= (u_1, u_2, u_3) = (-3, 4, -1) \\ \vec{v} &= (v_1, v_2, v_3) = (0, 1, -1) \end{aligned}$$

The cross product is:

$$\vec{u} \times \vec{v} = (u_2 v_3 - u_3 v_2, u_3 v_1 - u_1 v_3, u_1 v_2 - u_2 v_1)$$

Calculations:

- x-component:

$$u_2 v_3 - u_3 v_2 = 4 \times (-1) - (-1) \times 1 = -4 + 1 = -3$$

- y-component:

$$u_3 v_1 - u_1 v_3 = (-1) \times 0 - (-3) \times (-1) = 0 - 3 = -3$$

- z-component:

$$u_1 v_2 - u_2 v_1 = (-3) \times 1 - 4 \times 0 = -3 - 0 = -3$$

So,

$$\vec{AB} \times \vec{AC} = (-3, -3, -3)$$

Step 4: Calculating the Magnitude of the Cross Product

The magnitude of the cross product vector gives the area of the parallelogram:

$$|\vec{AB} \times \vec{AC}| = \sqrt{(-3)^2 + (-3)^2 + (-3)^2} = \sqrt{9 + 9 + 9} = \sqrt{27} = 3\sqrt{3}$$

Therefore:

$$\boxed{\text{Area of the parallelogram} = 3\sqrt{3}}$$

Additional Considerations: Multiple Parallelograms and Confirmations

While the above calculation provides the area based on vectors from point A, it's worthwhile to verify if other pairs of points could define the same or different parallelograms.

Alternate approach:

- Consider vectors from B or C to other points.
- Check for consistency in the area calculations.

Using Vector BC and BD

- Vector BC: $C - B = (3 - 0, 4 - 7, 0 - 0) = (3, -3, 0)$
- Vector BD: $D - B = (0 - 0, 0 - 7, 1 - 0) = (0, -7, 1)$

Calculating the cross product:

$$\begin{aligned} \vec{BC} \times \vec{BD} &= \\ \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 3 & -3 & 0 \end{vmatrix} &= \end{aligned}$$

```
0 & -7 & 1
\end{vmatrix}
\]
```

- x-component:

```
\[
(-3) \times 1 - 0 \times (-7) = -3 - 0 = -3
\]
```

- y-component:

```
\[
0 \times 0 - 3 \times 1 = 0 - 3 = -3
\]
```

- z-component:

```
\[
3 \times (-7) - (-3) \times 0 = -21 - 0 = -21
\]
```

Magnitude:

```
\[
\sqrt{(-3)^2 + (-3)^2 + (-21)^2} = \sqrt{9 + 9 + 441} = \sqrt{459} = 3
\sqrt{51}
\]
```

This yields a different area, indicating these vectors define different parallelograms or that the initial vectors from A are the correct ones for the specific parallelogram in question.

Summary of Key Points

- The area of a parallelogram in 3D space can be found via the magnitude of the cross product of vectors defining its adjacent sides.
- Correctly choosing the vectors is crucial; in this case, vectors AB and AC from point A were used.
- The calculated area is $(3\sqrt{3})$, approximately 5.196 units.
- Multiple vector pairs can be tested to confirm the shape and size of the parallelogram.

Applications of Parallelogram Area Calculation in Real-World Scenarios

Understanding how to compute the area of a parallelogram in 3D space has numerous practical applications:

1. Physics: Calculating work done by a force vector over an area.
2. Engineering: Stress and strain analysis in materials.
3. Computer Graphics: Rendering surfaces and calculating surface areas.
4. Navigation and Geodesy: Determining surface areas of irregular terrains.

Conclusion

Calculating the area of a parallelogram in three-dimensional space involves a clear understanding of vector operations, particularly the cross product. Using the given points A, B, C, and D, the key steps involve constructing the relevant vectors, computing the cross product, and then finding its magnitude. The resulting area, $(3\sqrt{3})$, provides a precise measure of the parallelogram's size in space.

This approach can be extended to more complex geometries and serves as a foundational technique in vector calculus and spatial analysis. Mastery of these calculations enhances problem-solving skills in various scientific and engineering disciplines, emphasizing the importance of vector operations in understanding the geometry of our three-dimensional world.

Keywords: area of parallelogram in 3D, vector cross product, three-dimensional geometry, space vectors, calculating surface area, vector calculus, analytical geometry, spatial analysis

Frequently Asked Questions

How do you find the area of the parallelogram determined by vectors A and B with given points $A=(3,3,1)$, $B=(0,7,0)$, $C=(3,4,0)$, $D=(0,0,1)$?

First, identify the vectors forming the parallelogram, such as AB and AD. Then, compute their cross product and find its magnitude. The magnitude gives the area of the parallelogram.

What steps are involved in calculating the area of a parallelogram defined by points A, B, C, and D in 3D space?

1. Determine two adjacent sides as vectors, for example, AB and AD. 2. Calculate their cross product. 3. Find the magnitude of this cross product, which equals the area of the parallelogram.

Given points $A=(3,3,1)$, $B=(0,7,0)$, $C=(3,4,0)$, and $D=(0,0,1)$, which vectors should be used to find the area of the parallelogram?

Vectors AB and AD can be used, where $AB = B - A$ and $AD = D - A$. These vectors define two adjacent sides of the parallelogram.

How do you compute the cross product of vectors AB and AD for the given points?

Calculate $AB = B - A = (-3, 4, -1)$ and $AD = D - A = (-3, -3, 0)$. Then, compute their cross product using the determinant method to find a vector whose magnitude gives the area.

What is the final formula for the area of the parallelogram determined by the given points?

The area = $|AB \times AD|$, which is the magnitude of the cross product of vectors AB and AD. Compute this to find the exact area.

Additional Resources

Let $A=(3,3,1)$, $B=(0,7,0)$, $C=(3,4,0)$, and $D=(0,0,1)$. Find the Area of the Parallelogram Determined by These Points

Understanding the geometric relationships between points in three-dimensional space is fundamental in vector calculus, physics, and engineering. When given four points like $A=(3,3,1)$, $B=(0,7,0)$, $C=(3,4,0)$, and $D=(0,0,1)$, one common task is to determine the area of a parallelogram they define. This problem not only tests spatial visualization but also the application of vector operations such as the cross product. In this article, we will explore the problem step-by-step, analyze the methods involved, and discuss the mathematical principles underpinning the calculation of the parallelogram's area.

Understanding the Problem and the Geometric Setup

The first step in solving this problem is to interpret the given points and identify which three points define the parallelogram. The points are:

- $A = (3, 3, 1)$
- $B = (0, 7, 0)$
- $C = (3, 4, 0)$
- $D = (0, 0, 1)$

Given these four points, the task is to find the area of the parallelogram determined by them. Typically, in such problems, the key is to identify two adjacent sides that originate from a common vertex, forming the parallelogram.

Identifying the Parallelogram:

- Usually, the points A, B, C, D are used to specify the vertices of a parallelogram in space.
- Since the problem mentions "the parallelogram determined by these points," it suggests that three points (say, A, B, C) form one corner, with D possibly being the fourth vertex.
- Alternatively, the points could be arranged to form two vectors originating from the same point, which define the sides of the parallelogram.

To proceed, let's analyze the possibilities for which points form the sides of the parallelogram.

Choosing the Proper Vertices:

A common approach is to select one point as the vertex and define two vectors emanating from it that form the sides of the parallelogram. For example, choosing point A as the common vertex:

- Side vector 1: $AB = B - A = (0 - 3, 7 - 3, 0 - 1) = (-3, 4, -1)$
- Side vector 2: $AC = C - A = (3 - 3, 4 - 3, 0 - 1) = (0, 1, -1)$

Alternatively, consider other combinations:

- From point A, the vectors to B and D:
- AB as above
- $AD = D - A = (0 - 3, 0 - 3, 1 - 1) = (-3, -3, 0)$

The key is to select two vectors that are adjacent sides of the parallelogram. Given the points, the most natural choice seems to be vectors

AB and AC, as they originate from the same point A and are likely to define the sides of the parallelogram.

Mathematical Approach: Using Vector Cross Product

The area of a parallelogram in three-dimensional space can be calculated using the cross product of two adjacent side vectors originating from the same vertex. The magnitude of the cross product gives the area of the parallelogram:

$$\text{Area} = |\mathbf{u} \times \mathbf{v}|$$

where $\mathbf{u}$ and $\mathbf{v}$ are the vectors representing adjacent sides.

Step 1: Define the vectors
As identified, choose:

$$\begin{aligned}\mathbf{u} &= \overrightarrow{AB} = (-3, 4, -1) \\ \mathbf{v} &= \overrightarrow{AC} = (0, 1, -1)\end{aligned}$$

Step 2: Compute the cross product $\mathbf{u} \times \mathbf{v}$

The cross product of two vectors $\mathbf{u} = (u_1, u_2, u_3)$ and $\mathbf{v} = (v_1, v_2, v_3)$ is:

$$\mathbf{u} \times \mathbf{v} = (u_2 v_3 - u_3 v_2, u_3 v_1 - u_1 v_3, u_1 v_2 - u_2 v_1)$$

Plugging in the values:

$$\mathbf{u} \times \mathbf{v} = (4 \times -1 - (-1) \times 1, \text{second component}, \text{third component})$$

Calculating each component:

- First component:

$$4 \times -1 - (-1) \times 1 = -4 + 1 = -3$$

- Second component:

$$-(u_1 v_3 - u_3 v_1) = -((-3) \times -1 - (-1) \times 0) = -(3 - 0) = -3$$

- Third component:

$$u_1 v_2 - u_2 v_1 = (-3) \times 1 - 4 \times 0 = -3 - 0 = -3$$

Thus,

$$\mathbf{u} \times \mathbf{v} = (-3, -3, -3)$$

Step 3: Find the magnitude of the cross product

$$|\mathbf{u} \times \mathbf{v}| = \sqrt{(-3)^2 + (-3)^2 + (-3)^2} = \sqrt{9 + 9 + 9} = \sqrt{27} = 3\sqrt{3}$$

This magnitude represents the area of the parallelogram formed by vectors $\mathbf{u}$ and $\mathbf{v}$.

Conclusion: The Area of the Parallelogram

The area of the parallelogram determined by points A, B, and C is $3\sqrt{3}$ square units. This calculation hinges on selecting appropriate side vectors and applying vector cross product properties, which is a standard approach in vector geometry for finding areas of parallelograms in three-dimensional space.

Additional Considerations and Clarifications

While the above calculation provides a clear answer, it's essential to understand some nuances and alternative interpretations:

- Choice of vectors: The calculation depends on choosing the correct adjacent sides. Different pairs of vectors could be used, but they should be from the same vertex and represent sides of the parallelogram.
- Position of points: The points provided do not necessarily form a flat parallelogram in the xy-plane; they are in 3D space, and the area calculation accounts for this spatial orientation.
- Verification: To confirm the parallelogram's shape, one could verify if points D and C, combined with A and B, form a parallelogram via midpoint checks or by examining vector sums.

Features and Pros/Cons of the Vector Cross Product Method

Features:

- Provides an exact measure of the area in 3D space.
- Easy to implement computationally with vector components.
- Handles arbitrary orientations of the shape in space.

Pros:

- Applicable to any planar figure in space, not limited to simple shapes.
- Uses straightforward vector operations, which are computationally efficient.

Cons:

- Requires proper identification of adjacent sides; incorrect choices can lead to wrong results.
- Assumes the points indeed define a parallelogram; if they do not, the method computes a "parallelogram" area based on chosen vectors.

Summary and Final Remarks

Calculating the area of a parallelogram determined by four points in three-dimensional space involves understanding the geometric relationships between these points, selecting appropriate vectors, and applying the cross product.

In this specific case, choosing vectors $\vec{AB}$ and $\vec{AC}$ from vertex A led us to the area calculation, which resulted in an area of $3\sqrt{3}$. This approach exemplifies the power of vector calculus in solving spatial problems efficiently and accurately.

Understanding these methods enhances not only your ability to solve geometric problems but also deepens your comprehension of spatial relationships in three dimensions, which is invaluable across multiple scientific and engineering disciplines.

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