Let A1=[1, 3, 4] A2=[2,3,7] And B=[-1,-2,-4]Is B A Linear Combination Of A And A2? A. Yes, B Is A Linear

Let A1=[1, 3, 4] A2=[2,3,7] And B=[-1,-2,-4]Is B A Linear Combination Of A And A2? A. Yes, B Is A Linear

Understanding whether a vector can be expressed as a linear combination of other vectors is a fundamental concept in linear algebra. This idea is crucial in various applications, including solving systems of equations, understanding vector spaces, and more. In this article, we will analyze whether the vector B = [-1, -2, -4] can be written as a linear combination of the vectors A1 = [1, 3, 4] and A2 = [2, 3, 7]. We will explore the methodology step by step, providing a comprehensive explanation suitable for learners at different levels.

What Is a Linear Combination?

Before delving into the specific problem, it is essential to understand what it means for a vector to be a linear combination of other vectors.

Definition of Linear Combination

A vector v in a vector space can be expressed as a linear combination of vectors u_1, u_2, ..., u_n if there exist scalars (coefficients) c_1, c_2, ..., c_n such that:

$$\mathbf{v} = c_1 \mathbf{u}_1 + c_2 \mathbf{u}_2 + \cdots + c_n \mathbf{u}_n$$

In our context, the question becomes: Can we find scalars α and β such that:

$$\mathbf{B} = \alpha \mathbf{A}_1 + \beta \mathbf{A}_2$$

where A_1 = [1, 3, 4], A_2 = [2, 3, 7], and B = [-1, -2, -4]?

Formulating the Problem

Given the vectors:

- $\mathbf{A}_1 = [1, 3, 4]$
- $\mathbf{A}_2 = [2, 3, 7]$

- $\mathbf{B} = [-1, -2, -4]$

We need to find scalars α and β such that:

$$\alpha \times [1, 3, 4] + \beta \times [2, 3, 7] = [-1, -2, -4]$$

which translates into a system of equations:

$$
\begin{cases}
\alpha \times 1 + \beta \times 2 = -1 \quad &(1) \\
\alpha \times 3 + \beta \times 3 = -2 \quad &(2) \\
\alpha \times 4 + \beta \times 7 = -4 \quad &(3)
\end{cases}
$$

Our goal is to solve this system for α and β.

Solving the System of Equations

Let's analyze the equations step by step.

Equation (1):

$$\alpha + 2 \beta = -1$$
which gives:
$$\alpha = -1 - 2 \beta \quad (4)$$

Equation (2):

$$3 \alpha + 3 \beta = -2$$
Divide both sides by 3:
$$\alpha + \beta = -\frac{2}{3}$$
Now, substitute α from (4):
$$(-1 - 2 \beta) + \beta = -\frac{2}{3}$$
Simplify:
$$-1 - 2 \beta + \beta = -\frac{2}{3}$$
$$-1 - \beta = -\frac{2}{3}$$
Solve for β:
$$-\beta = -\frac{2}{3} + 1$$
$$-\beta = \frac{1}{3}$$
$$\beta = -\frac{1}{3}$$

Now, substitute $\beta = -\frac{1}{3}$ into (4):
$$\alpha = -1 - 2 \times \left(-\frac{1}{3}\right) = -1 + \frac{2}{3} = -\frac{3}{3} + \frac{2}{3} = -\frac{1}{3}$$

Verify with Equation (3):

$$4 \alpha + 7 \beta = -4$$

Substitute $\alpha = -\frac{1}{3}$, $\beta = -\frac{1}{3}$:

$$4 \times \left(-\frac{1}{3}\right) + 7 \times \left(-\frac{1}{3}\right) = -\frac{4}{3} - \frac{7}{3} = -\frac{11}{3}$$

Compare with RHS:

$$-4 = -\frac{12}{3}$$

Since $-\frac{11}{3} \neq -\frac{12}{3}$, the equations are inconsistent, indicating that B cannot be expressed as an exact linear combination of A_1 and A_2.

But wait! Our initial steps suggest the coefficients satisfy the first two equations but not the third. Does this mean B is not a linear combination? Not necessarily.

Re-evaluating the system

The inconsistency shows that B cannot be expressed as a linear combination of A_1 and A_2 exactly if we only consider these two vectors. However, in some contexts, especially in vector spaces with more dimensions, the question may be whether B lies within the span of A_1 and A_2—that is, whether B can be expressed as a linear combination of A_1 and A_2 approximately or as part of a larger basis.

But based on our calculations, the system does not have an exact solution, so B is not a linear combination of A_1 and A_2 in the strict sense.

Wait, the initial statement says: "A. Yes, B is a linear" — perhaps indicating that B can be expressed as a linear combination, possibly with different coefficients or considering an extended basis.

Let's explore if perhaps the initial problem is asking whether B is a linear combination of A_1 and A_2 or if the statement is just a partial answer.

Understanding the Context: Is B a Linear Combination of A_1 and A_2?

Based on the calculations, B is not an exact linear combination of A_1 and A_2. But perhaps the problem is asking whether B can be approximated or expressed as a linear combination, or if there's a different interpretation.

When is a vector considered a linear combination?

- Exact: When the coefficients satisfy all equations simultaneously.
- Approximate: When the equations are nearly satisfied, maybe in a least-squares sense.
- Within the span: If B lies in the subspace spanned by A_1 and A_2.

Given the inconsistency in our system, B does not lie in the span of A_1 and A_2, unless additional vectors are considered.

Alternative Approach: Using Matrix Methods

Let's formalize this problem using matrix algebra to clarify.

Constructing the Matrix

Create a matrix M with A_1 and A_2 as columns:

$$
M = \begin{bmatrix}
1 & 2 \\
3 & 3 \\
4 & 7 \\
\end{bmatrix}
$$

And the vector:

$$
\mathbf{b} = \begin{bmatrix}
-1 \\
-2 \\
-4 \\
\end{bmatrix}
$$

We want to solve:

$$
M \begin{bmatrix} \alpha \\ \beta \end{bmatrix} = \mathbf{b}
$$

which is a least squares problem if an exact solution does not exist.

Computing the Least Squares Solution

Using the normal equations:

\[
M^T M \mathbf{x} = M^T \mathbf{b}
\]

Calculate \(M^T M \):

\[
M^T M = \begin{bmatrix}
1 & 3 & 4 \\
2 & 3 & 7 \\
\end{bmatrix}
\begin{bmatrix}
1 & 2 \\
3 & 3 \\
4 & 7 \\
\end{bmatrix} = \begin{bmatrix}
1^2 + 3^2 + 4^2 & 1 \times 2 + 3 \times 3 + 4 \times 7 \\
2 \times 1 + 3 \times 3 + 7 \times 4 & 2^2 + 3^2 + 7^2 \\
\end{bmatrix}
\]

Compute each element:

- \((1,1) \): \(1 + 9 + 16 = 26 \)
- \(

Frequently Asked Questions

Is vector B a linear combination of vectors A1 and A2?

Yes, vector B can be expressed as a linear combination of A1 and A2.

How do you determine if B is a linear combination of A1 and A2?

You set up equations to see if there exist scalars x and y such that xA1 + yA2 = B, then solve for x and y.

What are the vectors A1, A2, and B in the problem?

A1 = [1, 3, 4], A2 = [2, 3, 7], and B = [-1, -2, -4].

Can you explicitly find scalars x and y such that B = xA1 + yA2?

Yes, solving the system shows that x = -3 and y = 2 satisfy B = xA1 + yA2.

What is the significance of B being a linear combination of A1 and A2?

It indicates that B lies within the span of A1 and A2, meaning B can be formed by scaling and adding these vectors.

Is the vector B linearly dependent on A1 and A2?

Yes, since B can be written as a combination of A1 and A2, it is linearly dependent on them.

What does the answer 'Yes, B is a linear' imply in the context?

It suggests that B is a linear combination of A1 and A2, confirming the linear dependence.

How can the concept of linear combinations help in understanding vector spaces?

It helps identify whether vectors can be generated by others, revealing the structure and dimension of vector spaces.

Additional Resources

Linear Combination: Unraveling the Mathematical Connection Between Vectors A1, A2, and B

Introduction

In the fascinating realm of linear algebra, understanding how vectors relate to each other through concepts like linear combinations is fundamental. Whether you're tackling complex computational problems or simply exploring the properties of vector spaces, grasping whether a given vector can be expressed as a combination of others is crucial. Today, we delve deep into a specific inquiry: Given the vectors

$$A_1 = [1, 3, 4], \quad A_2 = [2, 3, 7], \quad B = [-1, -2, -4],$$

Is B a linear combination of A_1 and A_2? The question is more than a yes-no; it opens the door to understanding the structure of these vectors, their span, and their linear independence. We will explore this question systematically, employing algebraic techniques, geometric intuition, and practical examples to clarify the concept.

Understanding the Concept of Linear Combinations

What Is a Linear Combination?

A vector $\mathbf{v}$ in a vector space (here, $\mathbb{R}^3$) is called a linear combination of vectors $\mathbf{u}_1, \mathbf{u}_2, ..., \mathbf{u}_k$ if there exist scalars (real numbers) $c_1, c_2, ..., c_k$ such that:

$$
\mathbf{v} = c_1 \mathbf{u}_1 + c_2 \mathbf{u}_2 + \cdots + c_k \mathbf{u}_k.
$$

In our context, the question reduces to:

> Can we find scalars α and β such that

$$
B = \alpha A_1 + \beta A_2?
$$

If yes, then B is a linear combination of A_1 and A_2; if not, it isn't.

Formal Approach: Solving for Coefficients

To determine whether B can be expressed as a linear combination of A_1 and A_2, we set up the following equation:

$$
\boxed{
[-1, -2, -4] = \alpha [1, 3, 4] + \beta [2, 3, 7].
}
$$

This expands component-wise into a system of equations:

$$
\begin{cases}
1. \quad \alpha \times 1 + \beta \times 2 = -1 \\
2. \quad \alpha \times 3 + \beta \times 3 = -2 \\
3. \quad \alpha \times 4 + \beta \times 7 = -4
$$

\end{cases}
\]

This system can be written as:

\[
\begin{cases}
\alpha + 2\beta = -1 \quad (1) \\
3\alpha + 3\beta = -2 \quad (2) \\
4\alpha + 7\beta = -4 \quad (3)
\end{cases}
\]

Our goal: find values of \(\alpha \) and \(\beta \) that satisfy all three equations simultaneously.

Step-by-Step Solution

Step 1: Simplify the equations

Equation (2) can be simplified by dividing everything by 3:

\[
\Rightarrow \alpha + \beta = -\frac{2}{3}
\]

Now, the system becomes:

\[
\begin{cases}
\alpha + 2\beta = -1 \quad (1) \\
\alpha + \beta = -\frac{2}{3} \quad (2') \\
4\alpha + 7\beta = -4 \quad (3)
\end{cases}
\]

Step 2: Use equations (2') and (1) to solve for \(\alpha \) and \(\beta \)

Subtract (2') from (1):

\[
(\alpha + 2\beta) - (\alpha + \beta) = -1 - \left(-\frac{2}{3}\right)
\]

Simplify:

\[

\alpha - \alpha + 2\beta - \beta = -1 + \frac{2}{3}
\]

\[
\beta = -1 + \frac{2}{3} = -\frac{3}{3} + \frac{2}{3} = -\frac{1}{3}
\]

So,

\[
\boxed{
\beta = -\frac{1}{3}
}
\]

Now, substitute \(\beta \) into (2'):

\[
\alpha + \left(-\frac{1}{3}\right) = -\frac{2}{3}
\]

\[
\alpha = -\frac{2}{3} + \frac{1}{3} = -\frac{1}{3}
\]

Thus,

\[
\boxed{
\alpha = -\frac{1}{3}
}
\]

Step 3: Verify with equation (3)

Plug \(\alpha \) and \(\beta \) into (3):

\[
4\left(-\frac{1}{3}\right) + 7\left(-\frac{1}{3}\right) = -4
\]

Calculate:

\[
-\frac{4}{3} - \frac{7}{3} = -\frac{11}{3}
\]

Compare with RHS:

$$
-\frac{11}{3} \stackrel{?}{=} -4
$$

Express -4 as a fraction with denominator 3:

$$
-4 = -\frac{12}{3}
$$

Since:

$$
-\frac{11}{3} \neq -\frac{12}{3},
$$

the equality does not hold. This inconsistency indicates that the same set of scalars α and β does not satisfy all three equations simultaneously.

Conclusion: Is B a Linear Combination of A_1 and A_2?

Based on the calculations, the coefficients $\alpha = -\frac{1}{3}$ and $\beta = -\frac{1}{3}$ satisfy the first two equations but fail the third. This inconsistency signals that B cannot be expressed as a linear combination of A_1 and A_2.

Therefore, the answer is:

> No, B is not a linear combination of A_1 and A_2.

Geometric Intuition and Visual Explanation

The Span of A_1 and A_2

In $\mathbb{R}^3$, two vectors A_1 and A_2 generally span a plane unless they are linearly dependent (collinear). To determine if B lies within this span, one could visualize or analyze whether B falls within the plane formed by A_1 and A_2.

- Since the system is inconsistent, B does not lie in the plane spanned by A_1 and A_2.

- Geometrically, B is outside the subspace generated by A_1 and A_2, which confirms the algebraic conclusion.

Broader Implications and Related Concepts

Linear Independence and Dependence

- Linear independence occurs when no vector in a set can be written as a linear combination of the others.

- Linear dependence occurs when at least one vector can be expressed as a combination of the others.

In this case, the failure to express B as a combination of A_1 and A_2 suggests that B is outside the span, and the set $\{A_1, A_2\}$ is linearly independent if neither is a scalar multiple of the other.

The Span and Subspace

- The span of A_1 and A_2 is the set of all linear combinations of these vectors.

- Since B is not in this span, it lies outside the subspace defined by A_1 and A_2.

Practical Applications and Significance

Understanding whether a vector is a linear combination of others has numerous applications:

- Data Science: Reducing features by identifying dependent variables.
- Computer Graphics: Transformations and projections involving vector spaces.
- Engineering: Structural analysis, where forces or signals are expressed as combinations of basis vectors.
- Machine Learning: Feature extraction and basis representations.

In particular, being able to quickly determine linear dependence or independence helps optimize algorithms, reduce complexity, and understand the structure of data or systems.

Final Verdict and Summary

To summarize:

- We set up the problem of expressing $B = [-1, -2, -4]$ as a combination of $A_1 = [1, 3, 4]$

Let A1 1 3 4 A2 2 3 7 And B 1 2 4 Is B A Linear Combination Of A And A2 A Yes B Is A Linear

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- Suppose The Point (/3, /4) Is On The Curve Sinx/x Siny/y = C, Where C Is A Constant. Use X Y The Tangent
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