Let F(x)= (-2 To X2-3x) E^t2 Dt. At What Value Of X Is F(x) A Minimum?A. For No Value Of XB. 1/2C. 3/2D.

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Understanding the Function F(x): An Introduction

In the realm of calculus and mathematical analysis, functions involving integrals often present intriguing challenges and insights. The function in question here, F(x), is defined as an integral expression:

$$ F(x) = \int_{-2}^{x^2 - 3x} e^{t^2} \, dt $$

This function involves integrating the exponential of t^2 over a variable upper limit dependent on x. The task is to determine the value of x at which $F(x)$ attains its minimum, considering the options provided:

- A. For no value of x (implying the function has no minimum)
- B. $x = \frac{1}{2}$
- C. $x = \frac{3}{2}$
- D. (The specific value is not provided here)

Understanding this problem requires a solid grasp of calculus fundamentals, especially the differentiation of integrals with variable limits, the behavior of exponential functions, and critical point analysis.

Dissecting the Function: Components and Meaning

Defining the integral and variable limits

The function:

$$ F(x) = \int_{-2}^{x^2 - 3x} e^{t^2} \, dt $$

is an integral with a fixed lower limit (-2) and an upper limit that depends on x. The integrand, e^{t^2}, is a well-known function that grows rapidly and is always positive for all real t.

The key to analyzing $F(x)$ lies in understanding how the upper limit, $u(x) = x^2 - 3x$, influences the behavior of $F(x)$. Since the integrand is positive everywhere, the value of $F(x)$ will be sensitive to the position of this upper limit relative to the lower limit.

Why is the integral important?

- The integral accumulates the area under e^{t^2} from -2 up to $u(x)$.
- If $u(x)$ increases, $F(x)$ increases.
- If $u(x)$ decreases, $F(x)$ may decrease or increase depending on the position relative to -2.

Understanding the behavior of $u(x)$ is crucial for identifying the minimum of $F(x)$.

Calculating the Derivative $F'(x)$: A Critical Step

To locate the minima of $F(x)$, we use the Fundamental Theorem of Calculus and chain rule to find $F'(x)$, the derivative of $F(x)$ with respect to x.

Applying Leibniz's Rule

Given:

$$F(x) = \int_{a}^{u(x)} e^{t^2} \, dt$$

then:

$$F'(x) = e^{u(x)^2} \cdot u'(x)$$

where:

- $u(x) = x^2 - 3x$
- $u'(x) = 2x - 3$

Thus, the derivative:

$$ F'(x) = e^{(x^2 - 3x)^2} \cdot (2x - 3) $$

Finding Critical Points: When Does $F'(x) = 0$?

Critical points occur where $F'(x) = 0$. Since $e^{(x^2 - 3x)^2}$ is always positive, the only way for $F'(x)$ to be zero is when:

$$ 2x - 3 = 0 $$

Solving for x:

$$ 2x = 3 \Rightarrow x = \frac{3}{2} $$

This critical point suggests that $x = \frac{3}{2}$ is a candidate for a minimum or maximum of $F(x)$.

Analyzing the Nature of the Critical Point at $x = \frac{3}{2}$

To determine whether $x = \frac{3}{2}$ corresponds to a minimum, we examine the second derivative or analyze the behavior of $F'(x)$ around this point.

Second derivative $F''(x)$

Differentiating $F'(x)$:

$$ F'(x) = e^{(x^2 - 3x)^2} \cdot (2x - 3) $$

Applying the product rule:

$$ F''(x) = \frac{d}{dx} \left[e^{(x^2 - 3x)^2} \right] \cdot (2x - 3) + e^{(x^2 - 3x)^2} \cdot 2 $$

Calculating $\frac{d}{dx} e^{(x^2 - 3x)^2}$:

$$ e^{(x^2 - 3x)^2} \cdot 2(x^2 - 3x) \cdot (2x - 3) $$

Putting it all together:

$$ F''(x) = e^{(x^2 - 3x)^2} \left[2(x^2 - 3x)(2x - 3)^2 + 2 \right] $$

At $x = \frac{3}{2}$:

- $x = 1.5$
- $x^2 - 3x = (1.5)^2 - 3 \times 1.5 = 2.25 - 4.5 = -2.25$
- $2x - 3 = 3 - 3 = 0$

Substituting into the second derivative:

$$ F''(1.5) = e^{(-2.25)^2} \left[2 \times (-2.25) \times 0^2 + 2 \right] = e^{5.0625} \times 2 > 0 $$

Since $F''(1.5) > 0$, the critical point at $x = 1.5$ is a local minimum.

Behavior of $F(x)$ at the Boundaries and Overall

Limits as $x \to \pm \infty$

- As $x \to +\infty$:

- $u(x) = x^2 - 3x \to +\infty$

- $F(x) \to \infty$, since the integral accumulates more area.

- As $x \to -\infty$:

- $u(x) = x^2 - 3x \to +\infty$

- $F(x) \to \infty$.

Thus, $F(x)$ tends to infinity at both ends, suggesting the existence of a minimum at some finite x.

Implication for the minimum

Since $F(x) \to \infty$ as $x \to \pm \infty$, and $F(x)$ has a critical point at $x = 1.5$, which is a local minimum, this critical point is indeed the global minimum.

Conclusion: At What Value of x Is $F(x)$ a Minimum?

Based on the derivative analysis, the critical point at $x = \frac{3}{2}$ (or 1.5) is where $F(x)$ attains its minimum value. The second derivative test confirms this is a minimum point, and the behavior at infinity supports this conclusion.

Therefore, the correct answer from the options provided is:

- C. $\frac{3}{2}$

Final Remarks and Additional Insights

- The integral's nature, involving e^{t^2}, which is always positive and increasing rapidly, makes the analysis of $F(x)$ reliant on the limits rather than the integrand's oscillations.
- The critical point at $x = 1.5$ is analytically derived and confirmed via the second derivative test.
- This problem exemplifies how calculus tools—derivatives, critical point analysis, and boundary behavior—combine to solve optimization problems involving integrals with variable limits.

Summary:

Step	Action	Result
1	Find $F'(x)$ using Leibniz rule	$F'(x) = e^{(x^2 - 3x)^2} (2x - 3)$
2	Set $F'(x) = 0$	

Frequently Asked Questions

What is the function F(x) given in the problem?

$F(x) = (-2x^2 - 3x) e^{t^2} dt$.

How do you find the value of x at which F(x) reaches its minimum?

By differentiating F(x) with respect to x, setting the derivative to zero, and solving for x to find critical points.

What is the significance of the options A. No value of x, B. 1/2, and C. 3/2?

These options suggest potential x-values where F(x) could attain its minimum, including the possibility that no such minimum exists.

Based on the options provided, which value of x minimizes F(x)?

The correct choice is C. 3/2, assuming it satisfies the conditions for a minimum after analysis.

Are there any specific steps to verify the minimum point for F(x)?

Yes, differentiate F(x), find critical points, and use the second derivative test or analyze the behavior of F(x) to confirm the minimum.

Why might the answer be 'No value of x' as per option A?

Because F(x) might not have any critical points or the derivative never equals zero, indicating no minimum exists for any real x.

Additional Resources

Understanding the Function and Its Significance

In the realm of calculus and mathematical analysis, functions serve as fundamental tools for modeling real-world phenomena, optimizing processes, and understanding relationships between variables. The function under consideration, $F(x) = \left(-2 \, \text{to} \, x^2 - 3x \right) e^{t^2} \, dt$, presents an intriguing case study. Although the notation appears somewhat unconventional, it opens a pathway to explore vital concepts such as function differentiation, critical points, and the determination of minima.

This article aims to dissect this complex function, interpret its components, and identify the value of x at which $F(x)$ attains its minimum. We will approach this systematically, breaking down the function's structure, applying calculus principles, and analyzing the options provided – A. No value of x, B. $1/2$, C. $3/2$, D. – to arrive at a well-informed conclusion.

Deciphering the Function $F(x)$: Components and Notation

Interpreting the Integral Expression

At the heart of $F(x)$ lies an integral expression:

$$
F(x) = \left(-2 \, \text{to} \, x^2 - 3x \right) e^{t^2} \, dt
$$

This notation suggests an indefinite integral or a definite integral with variable limits. Given the context, the most plausible interpretation is:

$$
F(x) = \int_{a(x)}^{b(x)} e^{t^2} dt
$$

where the limits depend on x, specifically:

- Lower limit: -2
- Upper limit: $x^2 - 3x$

Thus, the function becomes:

$$
F(x) = \int_{-2}^{x^2 - 3x} e^{t^2} dt
$$

This aligns with standard notation in calculus, where the integral of a function over a variable upper limit defines a new function of x. The exponential term e^{t^2} is a well-known, non-elementary function, but its properties are critical for analyzing $F(x)$.

Significance of the Limits:

- The lower limit is constant at -2.

- The upper limit is a quadratic function in x, $x^2 - 3x$.

Analyzing the Behavior of $F(x)$

Applying the Fundamental Theorem of Calculus

To determine where $F(x)$ attains its minimum, we need to analyze its derivative $F'(x)$. The Fundamental Theorem of Calculus states:

$$
F'(x) = \frac{d}{dx} \left[\int_{-2}^{x^2 - 3x} e^{t^2} dt \right] = e^{b(x)^2} \cdot b'(x)
$$

where:

$$
b(x) = x^2 - 3x
$$

and

$$
b'(x) = \frac{d}{dx} (x^2 - 3x) = 2x - 3
$$

Therefore,

$$
F'(x) = e^{(x^2 - 3x)^2} \cdot (2x - 3)
$$

This derivative encapsulates the rate of change of $F(x)$ with respect to x.

Critical Points and Minima

To find the extrema of $F(x)$, set $F'(x) = 0$:

$$
e^{(x^2 - 3x)^2} \cdot (2x - 3) = 0
$$

\]

Since $e^{(x^2 - 3x)^2}$ is always positive (exponential of a real number), the critical points occur where:

$$
2x - 3 = 0 \quad \Rightarrow \quad x = \frac{3}{2}
$$

Thus, the only critical point is at:

$$
x = \frac{3}{2}
$$

To determine whether this critical point corresponds to a minimum, maximum, or saddle point, analyze the second derivative or the behavior of $F'(x)$ around $x = 3/2$.

Second Derivative and Nature of the Critical Point

Calculating $F''(x)$

Given:

$$
F'(x) = e^{(x^2 - 3x)^2} \cdot (2x - 3)
$$

Use the product rule:

$$
F''(x) = \frac{d}{dx} \left[e^{(x^2 - 3x)^2} \right] \cdot (2x - 3) + e^{(x^2 - 3x)^2} \cdot \frac{d}{dx} (2x - 3)
$$

Calculate $\frac{d}{dx} \left[e^{(x^2 - 3x)^2} \right]$:

$$
e^{(x^2 - 3x)^2} \cdot \frac{d}{dx} \left[(x^2 - 3x)^2 \right]
$$

Using the chain rule:

$$
\frac{d}{dx} \left[(x^2 - 3x)^2 \right] = 2(x^2 - 3x) \cdot (2x - 3)
$$

Putting it all together:

$$
F''(x) = e^{(x^2 - 3x)^2} \cdot 2(x^2 - 3x)(2x - 3) \cdot (2x - 3) + e^{(x^2 - 3x)^2} \cdot 2
$$

Simplify:

$$
F''(x) = e^{(x^2 - 3x)^2} \left[2(x^2 - 3x)(2x - 3)^2 + 2 \right]
$$

Now, evaluate at $x = 3/2$:

- $x = 1.5$
- $x^2 - 3x = (1.5)^2 - 3 \times 1.5 = 2.25 - 4.5 = -2.25$
- $2x - 3 = 3 - 3 = 0$

Substitute into the second derivative:

$$
F''(1.5) = e^{(-2.25)^2} \left[2 \times (-2.25) \times 0^2 + 2 \right] = e^{5.0625} \times 2 > 0
$$

Since $F''(1.5) > 0$, the critical point at $x = 1.5$ is a minimum.

Conclusion: The Value of x at Which $F(x)$ Is Minimized

Based on the calculus analysis:

- The critical point occurs at $x = 3/2$.
- The second derivative test confirms it's a minimum.
- The behavior of the function around this point indicates a minimum value.

Therefore, the function $F(x)$ attains its minimum at $x = \frac{3}{2}$.

Implications and Final Remarks

Understanding the behavior of functions defined via integrals with variable limits is crucial in various fields — from physics to economics. This particular analysis underscores the importance of applying the Fundamental Theorem of Calculus and second derivative tests to identify extrema.

Summary points:

- The integral form $F(x) = \int_{-2}^{x^2 - 3x} e^{t^2} dt$ defines a function of x.
- Differentiating $F(x)$ reveals the critical point at $x = 3/2$.
- The second derivative confirms this critical point is a minimum.
- The options provided in the multiple-choice question align with this mathematical conclusion.

Final answer:

C. 3/2

This choice correctly identifies the value of x where $F(x)$ reaches its minimum.

Let F X 2 To X2 3x E T2 Dt At What Value Of X Is F X A Minimum A For No Value Of Xb 1 2c 3 2d

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