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Introduction

In vector calculus, understanding how to manipulate vectors and compute distances between various vector points is fundamental. The problem presented involves three vectors in three-dimensional space: U, V, and W. The tasks require performing scalar multiplication and vector addition, followed by calculating the distance between specific vector points. These operations are common in geometry, physics, and engineering, where they help determine spatial relationships and measure how far apart points are in space.

This article will systematically explore the steps to solve the given problem. It will include detailed calculations, explanations of vector operations, and the geometric interpretation of the results. By the end, you will understand how to handle similar problems involving vector scaling, addition, and distance measurement.

Understanding the Given Vectors and Tasks

The Vectors

Let's first analyze the vectors provided:

- U = (-2, 0, 4)
- V = (3, -1, 6)
- W = (2, -5, -5)

Each vector is represented as an ordered triplet in 3D space, indicating its position relative to the origin.

The Tasks

The problem is divided into two parts:

1. Part A: Find the distance between -3u and V + 5w.
2. Part B: Compute an unspecified quantity (likely related to the vectors, but the original prompt seems incomplete; assuming it involves further operations such as dot product, cross product, or magnitude calculations).

In the following sections, we will perform the operations step-by-step to address Part A explicitly and then interpret Part B based on typical vector calculations.

Part A: Finding the Distance Between -3u and V + 5w

Step 1: Calculate -3u

To find -3u, multiply each component of vector U by -3:

- U = (-2, 0, 4)

- -3u = -3 (-2, 0, 4) = (-3 -2, -3 0, -3 4) = (6, 0, -12)

Result:

- -3u = (6, 0, -12)

Step 2: Calculate V + 5w

First, compute 5w:

- W = (2, -5, -5)

- 5w = 5 (2, -5, -5) = (10, -25, -25)

Next, add vector V:

- V = (3, -1, 6)

Adding V and 5w component-wise:

- $V + 5w = (3 + 10, -1 + (-25), 6 + (-25)) = (13, -26, -19)$

Result:

- $V + 5w = (13, -26, -19)$

Step 3: Find the Distance Between -3u and V + 5w

The distance between two points A and B in 3D space is given by the formula:

$$
d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}
$$

Where:

- $A = (x_1, y_1, z_1)$
- $B = (x_2, y_2, z_2)$

Using the computed vectors:

- $A = -3u = (6, 0, -12)$
- $B = V + 5w = (13, -26, -19)$

Calculating differences:

- $x_2 - x_1 = 13 - 6 = 7$
- $y_2 - y_1 = -26 - 0 = -26$
- $z_2 - z_1 = -19 - (-12) = -19 + 12 = -7$

Now, compute the squared differences:

- $7^2 = 49$
- $(-26)^2 = 676$
- $(-7)^2 = 49$

Sum of squared differences:

$$
49 + 676 + 49 = 774
$$

Finally, take the square root:

$$
d = \sqrt{774} \approx 27.82
$$

Answer for Part A:

$$
\boxed{
\text{Distance} \approx 27.82 \text{ units}
}
$$

Part B: Additional Computations (Interpretation)

The prompt indicates "Compute:" following Part A but doesn't specify the exact calculation. Common vector operations include:

- Dot product: measures the cosine of the angle between vectors.
- Cross product: finds a vector perpendicular to both.
- Magnitude: length of a vector.

Assuming the task is to compute the magnitude of the vector V, which is a common next step, or perhaps the dot product of vectors, let's explore both possibilities.

Option 1: Magnitude of V

The magnitude (or length) of a vector V = (x, y, z) is:

$$
|V| = \sqrt{x^2 + y^2 + z^2}
$$

Calculating:

$$
|V| = \sqrt{3^2 + (-1)^2 + 6^2} = \sqrt{9 + 1 + 36} = \sqrt{46} \approx 6.78
$$

Option 2: Dot Product of U and V

The dot product between U and V:

$$
U \cdot V = (x_1)(x_2) + (y_1)(y_2) + (z_1)(z_2)
$$

Calculating:

$$
(-2)(3) + (0)(-1) + (4)(6) = -6 + 0 + 24 = 18
$$

Summary of Results

Calculation	Result
-3u	(6, 0, -12)
V + 5w	(13, -26, -19)
Distance between -3u and V + 5w	approximately 27.82 units
Magnitude of V	approximately 6.78 units
Dot product of U and V	18

Geometric Interpretation and Applications

The calculations provide insights into the spatial relationships between vectors:

- The distance of approximately 27.82 units indicates how far apart the points -3u and V + 5w are in space. This could represent physical distances in real-world applications like engineering or physics simulations.
- The magnitude of V (about 6.78 units) shows the length of the vector V from the origin, helpful in normalization or understanding the vector's scale.
- The dot product (18) reveals the degree of alignment between U and V, with positive values indicating an acute angle.

These operations are fundamental in vector analysis, enabling the calculation of angles, projections, and distances critical to many scientific and engineering fields.

Conclusion

This comprehensive analysis demonstrates how to manipulate vectors algebraically and geometrically in three-dimensional space. Starting with scalar multiplication and vector addition, we calculated the vectors -3u and V + 5w. Using the distance formula, we found the Euclidean distance between these points. Further, exploring the magnitude and dot product provided additional insights into the vectors' properties.

Understanding these core operations allows for more advanced studies in vector calculus, physics, computer graphics, and related disciplines. Mastery of such techniques is essential for analyzing spatial relationships, optimizing designs, and solving complex real-world problems.

By practicing similar calculations, you will develop a strong intuition for vectors and their applications in multidimensional space.

Frequently Asked Questions

What is the vector -3U if U = (-2, 0, 4)?

-3U = (6, 0, -12)

What is the vector V + 5W if V = (3, -1, 6) and W = (2, -5, -5)?

V + 5W = (3 + 10, -1 + (-25), 6 + (-25)) = (13, -26, -19)

How do you find the distance between two vectors in 3D space?

The distance between vectors A and B is given by the magnitude of their difference: $|A - B| = \sqrt{(A_x - B_x)^2 + (A_y - B_y)^2 + (A_z - B_z)^2}$.

Calculate the distance between -3U and V + 5W given U, V, and W as specified.

First, compute -3U = (6, 0, -12), and V + 5W = (13, -26, -19). Then, find the difference: (13 - 6, -26 - 0, -19 - (-12)) = (7, -26, -7). The distance is $\sqrt{7^2 + (-26)^2 + (-7)^2} = \sqrt{49 + 676 + 49} = \sqrt{774} \approx 27.81$.

What is the computed value of V + 5W with the given vectors?

V + 5W = (13, -26, -19).

What is the magnitude of the vector difference between -3U and V + 5W?

The magnitude is approximately 27.81 units.

Can you verify the calculations for the distance between -3U and V + 5W?

Yes. Calculations: -3U = (6, 0, -12); V + 5W = (13, -26, -19); Difference = (7, -26, -7); Magnitude = sqrt(7^2 + (-26)^2 + (-7)^2) = sqrt(49 + 676 + 49) = sqrt(774) ≈ 27.81.

What are the steps to compute the distance between the two vectors in the problem?

1. Calculate -3U. 2. Calculate V + 5W. 3. Find the difference between these two vectors. 4. Compute the magnitude of the resulting vector to find the distance.

What is the significance of understanding vector operations in such problems?

Understanding vector operations allows for precise calculation of distances, directions, and relationships between points in space, which is fundamental in physics, engineering, and mathematics.

Additional Resources

Understanding the Geometric Foundations of Vectors and Their Distances in Three-Dimensional Space

In the realm of mathematics, particularly in vector calculus and analytic geometry, the concepts of vectors and their associated operations form the backbone of understanding spatial relationships. Vectors enable us to represent points, directions, and magnitudes in multi-dimensional space, facilitating complex calculations that underpin fields as diverse as physics, engineering, computer graphics, and data science. A fundamental aspect of vector analysis involves computing distances between points or between scaled and combined vectors—an essential skill for solving real-world problems involving spatial positioning, force analysis, and optimization.

This comprehensive exploration delves into two specific problems involving vectors in three-dimensional space, denoted as U, V, and W. We will meticulously analyze each problem, starting from foundational principles, then progressing through detailed calculations, and finally interpreting the results within the geometric context. The objective is to provide clarity, insight, and a thorough understanding of how vector operations translate into measurable distances and combined vector quantities.

Vectors in Three-Dimensional Space: Definitions and Operations

What Are Vectors?

Vectors are mathematical entities characterized by both magnitude (length) and direction. They are represented as ordered tuples of numbers in three-dimensional space, typically written as (x, y, z). For example, the vector U is given as:

U = (-2, 0, 4)

which indicates that U extends 2 units in the negative x-direction, 0 units in the y-direction, and 4 units in the z-direction.

Basic Operations on Vectors

Understanding vectors involves mastering several key operations:

- Addition: Combining two vectors by adding their corresponding components:

$$
\mathbf{A} + \mathbf{B} = (A_x + B_x, A_y + B_y, A_z + B_z)
$$

- Scalar Multiplication: Scaling a vector by a real number k:

$$
k\mathbf{A} = (kA_x, kA_y, kA_z)
$$

- Dot Product: An operation that yields a scalar, useful for calculating angles and projections:

$$
\mathbf{A} \cdot \mathbf{B} = A_x B_x + A_y B_y + A_z B_z
$$

- Magnitude (Length): The measure of a vector's length, calculated as:

\[

$$
|\mathbf{A}| = \sqrt{A_x^2 + A_y^2 + A_z^2}
$$

- Distance Between Vectors: The Euclidean distance between two points (represented as vectors) $\mathbf{A}$ and $\mathbf{B}$ is:

$$
d(\mathbf{A}, \mathbf{B}) = |\mathbf{A} - \mathbf{B}| = \sqrt{(A_x - B_x)^2 + (A_y - B_y)^2 + (A_z - B_z)^2}
$$

Having established these foundational operations, we are well-equipped to analyze the specific problems involving vectors U, V, and W.

Given Vectors and Their Definitions

Let us revisit the vectors provided:

- U = (-2, 0, 4)
- V = (3, -1, 6)
- W = (2, -5, -5)

These vectors will serve as the basis for two distinct calculations: the distance between scaled vectors and the computation of combined vectors.

Part A: Finding the Distance Between $-3\mathbf{U}$ and $\mathbf{V} + 5\mathbf{W}$

Step 1: Calculate $-3\mathbf{U}$

Scalar multiplication involves multiplying each component of U by -3:

$$
-3\mathbf{U} = -3 \times (-2, 0, 4) = (6, 0, -12)
$$

This new vector points in the same direction as U but scaled by a factor of -3, effectively reversing its direction and tripling its magnitude.

Step 2: Calculate $5\mathbf{W}$

Similarly, scale W by 5:

$$
5\mathbf{W} = 5 \times (2, -5, -5) = (10, -25, -25)
$$

Step 3: Compute $\mathbf{V} + 5\mathbf{W}$

Adding vectors involves component-wise addition:

$$
\mathbf{V} + 5\mathbf{W} = (3 + 10, -1 + (-25), 6 + (-25)) = (13, -26, -19)
$$

This combined vector represents the resultant position or effect of moving along V and then along 5W.

Step 4: Find the distance between $-3\mathbf{U}$ and $\mathbf{V} + 5\mathbf{W}$

The Euclidean distance formula applies:

$$
d = |\mathbf{A} - \mathbf{B}| = \sqrt{(A_x - B_x)^2 + (A_y - B_y)^2 + (A_z - B_z)^2}
$$

Where:

$$
\mathbf{A} = (-3\mathbf{U}) = (6, 0, -12)
$$
$$
\mathbf{B} = (\mathbf{V} + 5\mathbf{W}) = (13, -26, -19)
$$

Calculate component differences:

- $6 - 13 = -7$
- $0 - (-26) = 26$
- $-12 - (-19) = 7$

Now, compute the squares:

- $(-7)^2 = 49$
- $26^2 = 676$
- $7^2 = 49$

Sum these:

$$
49 + 676 + 49 = 774
$$

Finally, take the square root:

$$
d = \sqrt{774} \approx 27.82
$$

Result for Part A: The distance between $-3\mathbf{U}$ and $\mathbf{V} + 5\mathbf{W}$ is approximately 27.82 units.

Part B: Computing and Analyzing Vector Combinations

Although the prompt appears to be incomplete after "Compute:", it likely refers to calculating a specific vector expression or property involving U, V, and W. Given the context, a common next step in vector analysis includes computing the sum or difference of vectors, their magnitudes, or projections. For a comprehensive review, let us consider the potential computations that could be involved:

- Sum or difference of vectors such as $\mathbf{U} + \mathbf{V} + \mathbf{W}$
- Magnitudes of these combined vectors
- Dot products to analyze angles or projections
- Scalar multiples or linear combinations relevant to the problem

For illustrative purposes, we will focus on computing the sum of the vectors and their magnitudes, which are standard operations in vector analysis.

Calculating $\mathbf{U} + \mathbf{V} + \mathbf{W}$

Component-wise addition:

$$
(-2, 0, 4) + (3, -1, 6) + (2, -5, -5) = ((-2)+3+2, 0+(-1)+(-5), 4+6+(-5))

\]

Calculate each component:

- $(-2)+3+2 = 3$
- $0 - 1 - 5 = -6$
- $4 + 6 - 5 = 5$

Resultant vector:

$$
\mathbf{U} + \mathbf{V} + \mathbf{W} = (3, -6, 5)
$$

Magnitude of the Resultant Vector

Compute the length:

$$
|\mathbf{U} + \mathbf{V} + \mathbf{W}| = \sqrt{3^2 + (-6)^2 + 5^2} = \sqrt{9 + 36 + 25} = \sqrt{70} \approx 8.37
$$

This magnitude indicates the net effect or resultant displacement when combining all three vectors.

Additional Analysis: Dot Product and Angle Calculations (Optional)

To deepen the analysis, one might compute the dot product between vectors to find angles:

$$
\cos \theta = \frac{\mathbf{A} \cdot \mathbf{B}}{|\mathbf{A}||\mathbf{B}|}
$$

For example, between U and V:

$$
\mathbf{U} \cdot \mathbf{V} = (-2)(3) + (0)(-1) + (4)(6) = -6 + 0 + 24 = 18
$$

Magnitudes:

\[
|\mathbf{U}| = \sqrt{(-2)^2 + 0^2 + 4^2} = \sqrt{4 + 0 + 16} =

Let U 2 0 4 V 3 1 6 And W 2 5 5 A Find The Distance Between 3u And V 5w B Compute

Let U 2 0 4 V 3 1 6 And W 2 5 5 A Find The Distance Between 3u And V 5w B Compute

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