Let X Be A Continuous Random Variable With P.d.f. F(x) = 3x2 When 0Find Var[X] = .Note: You Can Use Your

Let X Be A Continuous Random Variable With P.d.f. F(x) = 3x^2 When 0 < x < 1. Find Var[X]. Note: You Can Use Your

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Introduction to Continuous Random Variables and Variance

Understanding the behavior of continuous random variables is fundamental in probability and statistics. These variables, unlike discrete variables, can take any value within a specified range. The probability density function (pdf) characterizes their distribution, providing a complete description of the likelihood of the variable taking on particular values.

In this article, we analyze a specific continuous random variable X with a given probability density function $f(x) = 3x^2$ for $0 < x < 1$. Our goal is to compute the variance $\mathrm{Var}[X]$ of this variable, a key measure indicating the spread or dispersion of the distribution around its mean.

Before diving into the calculations, we will review essential concepts, including the properties of pdfs, expectation, and variance, and then proceed step by step to derive the variance for the given distribution.

Understanding the Probability Density Function (pdf)

Definition of a pdf

A probability density function $f(x)$ must satisfy:

- Non-negativity: $f(x) \geq 0$ for all x.
- Total probability: $\int_{-\infty}^{\infty} f(x) \, dx = 1$.

In our case, the pdf is:

$$
f(x) = 3x^2 \quad \text{for} \quad 0 < x < 1,
$$
and $f(x) = 0$ elsewhere.

Verification of the pdf

Let's verify that $f(x)$ integrates to 1 over its domain:

$$
\int_{0}^{1} 3x^2 \, dx = 3 \int_{0}^{1} x^2 \, dx = 3 \left[\frac{x^3}{3} \right]_0^1 = 3 \times \frac{1}{3} = 1.
$$

Therefore, $f(x)$ is a valid probability density function.

Calculating the Expectation $E[X]$

The expectation, or the mean μ, of a continuous random variable X with pdf $f(x)$ is:

$$
E[X] = \int_{-\infty}^{\infty} x f(x) \, dx.
$$

Since $f(x) = 0$ outside $(0,1)$, this simplifies to:

$$
E[X] = \int_{0}^{1} x \times 3x^2 \, dx = 3 \int_{0}^{1} x^3 \, dx.
$$

Calculating:

$$
E[X] = 3 \left[\frac{x^4}{4} \right]_0^1 = 3 \times \frac{1}{4} = \frac{3}{4} = 0.75.
$$

So, the mean of X is $E[X] = 0.75$.

Calculating the Second Moment $E[X^2]$

The second moment $E[X^2]$ is necessary to compute the variance:

$$
E[X^2] = \int_{-\infty}^{\infty} x^2 f(x) \, dx.
$$

Again, over $(0,1)$:

$$
E[X^2] = \int_{0}^{1} x^2 \times 3x^2 \, dx = 3 \int_{0}^{1} x^4 \, dx.
$$

Calculating:

$$
E[X^2] = 3 \left[\frac{x^5}{5} \right]_0^1 = 3 \times \frac{1}{5} =
\frac{3}{5} = 0.6.
$$

Calculating the Variance $\mathrm{Var}[X]$

The variance of a continuous random variable X is defined as:

$$
\mathrm{Var}[X] = E[X^2] - (E[X])^2.
$$

Using the previously calculated values:

$$
E[X] = 0.75, \quad E[X^2] = 0.6,
$$

we find:

$$
\mathrm{Var}[X] = 0.6 - (0.75)^2 = 0.6 - 0.5625 = 0.0375.
$$

Expressed as a fraction:

$$
\mathrm{Var}[X] = \frac{3}{80}.
$$

Summary of Results

Quantity	Value	Fraction Form
Expectation $E[X]$	0.75	$\frac{3}{4}$
Second Moment $E[X^2]$	0.6	$\frac{3}{5}$
Variance $\mathrm{Var}[X]$	0.0375	$\frac{3}{80}$

The variance of the random variable X with the given pdf is $\boxed{\frac{3}{80}}$.

Interpretation of the Variance

The variance measures the average squared deviation of X from its mean. A variance of $\frac{3}{80} \approx 0.0375$ indicates that the values of X are relatively tightly clustered around the mean of 0.75 within the interval $(0,1)$.

This small variance aligns with the distribution $f(x) = 3x^2$, which is skewed towards higher values near 1, as the density increases quadratically with x. The distribution's shape influences the spread of the data points, which is reflected in the computed variance.

Extensions and Additional Insights

Variance for Other Distributions

The process demonstrated here can be generalized for other continuous distributions:

1. Verify the pdf integrates to 1.
2. Compute the expectation $E[X]$.
3. Compute the second moment $E[X^2]$.
4. Use the formula $\mathrm{Var}[X] = E[X^2] - (E[X])^2$.

Application of Variance in Real-World Contexts

Variance is a crucial statistical measure used across various fields:

- Finance: To measure the volatility of asset returns.
- Quality Control: To assess process consistency.
- Engineering: To evaluate the variability in system outputs.
- Natural Sciences: To understand the dispersion of experimental data.

Understanding how to compute and interpret variance helps in making informed decisions based on probabilistic models.

Conclusion

In this comprehensive analysis, we've computed the variance of a continuous random variable X with the pdf $f(x) = 3x^2$ over $(0,1)$. The key steps involved verifying the pdf, calculating the expectation and second moment, and then applying the variance formula. The resulting variance $\boxed{\frac{3}{80}}$ provides insight into the distribution's dispersion.

Mastering these fundamental calculations enhances your ability to analyze continuous probability distributions, which are foundational in statistical inference and probabilistic modeling. This example illustrates the importance of integrating theoretical knowledge with practical computation to interpret data and uncertainty effectively.

Keywords: Continuous random variable, probability density function, expectation, second moment, variance, statistical analysis, probability distribution, $f(x) = 3x^2$, variance calculation, probability theory.

Frequently Asked Questions

Given a continuous random variable X with probability density function F(x) = 3x^2 for 0 ≤ x ≤ 1, what is the expected value E[X]?

The expected value $E[X] = \int_0^1 x\ 3x^2\ dx = 3\int_0^1 x^3\ dx = 3\ (1/4) = 3/4$.

How do you compute the variance Var[X] for the given PDF F(x) = 3x^2 over [0,1]?

Variance $Var[X] = E[X^2] - (E[X])^2$. First, find E[X] as 3/4, then compute $E[X^2] = \int_0^1 x^2\, 3x^2\, dx = 3\int_0^1 x^4\, dx = 3\ (1/5) = 3/5$. So, $Var[X] = 3/5 - (3/4)^2$.

What is the value of E[X²] for the given distribution?

$E[X^2] = \int_0^1 x^2\, 3x^2\, dx = 3\int_0^1 x^4\, dx = 3\ (1/5) = 3/5$.

Calculate the variance Var[X] for the random variable with PDF F(x) = 3x^2 on [0,1].

$Var[X] = E[X^2] - (E[X])^2 = (3/5) - (3/4)^2 = 0.6 - 0.5625 = 0.0375$.

What is the significance of the PDF F(x) = 3x^2 in modeling continuous random variables?

This PDF models a distribution that is increasing over [0,1], with higher probability density near x=1, often representing variables that tend to take larger values within the interval.

Is the probability density function F(x) = 3x^2 valid over the interval [0,1]? Why?

Yes, because $\int_0^1 3x^2\, dx = 1$, satisfying the property of a probability density function.

How can you verify that the computed variance is correct for this distribution?

By independently computing E[X], $E[X^2]$, and then applying $Var[X] = E[X^2] - (E[X])^2$, ensuring all integrals are correctly evaluated.

What is the final numerical value of Var[X] for F(x) = 3x^2 on [0,1]?

$Var[X] = 3/5 - (3/4)^2 = 0.6 - 0.5625 = 0.0375$.

Additional Resources

Understanding the Variance of a Continuous Random Variable with a Given PDF: An In-Depth Analysis

When studying probability and statistics, one of the foundational concepts is understanding how to quantify the variability or spread of a random variable. For continuous random variables, this often involves calculating the variance, which provides a measure of the dispersion of the variable's possible values around its mean. In this detailed exploration, we will analyze the problem: Let X be a continuous random variable with probability density function (pdf) $f(x) = 3x^2$ for $0 \leq x \leq 1$ and $f(x) = 0$ elsewhere, and find the variance $\operatorname{Var}[X]$.

This problem encapsulates many foundational ideas in probability theory, including the properties of probability density functions, expectation, and variance calculations. We will examine each step carefully, ensuring a comprehensive understanding.

Understanding the Probability Density Function (pdf)

What is a pdf?

- A probability density function $f(x)$ describes the likelihood of a continuous random variable X taking on values within a certain range.
- For a valid pdf:
- $f(x) \geq 0$ for all x.
- The integral over the entire space must be 1: $\int_{-\infty}^{\infty} f(x) \, dx = 1$.

Given pdf:

$$
f(x) = 3x^2, \quad \text{for } 0 \leq x \leq 1,
$$
$$
f(x) = 0, \quad \text{elsewhere}.
$$

This indicates that the random variable X is supported on the interval $[0,1]$, with a density that increases quadratically as x increases.

Verification of the pdf

Before proceeding, verify that $f(x)$ is a valid pdf:

$$
\int_{-\infty}^{\infty} f(x) dx = \int_0^1 3x^2 dx = 3 \int_0^1 x^2 dx.
$$

Calculate the integral:

$$
\int_0^1 x^2 dx = \left[\frac{x^3}{3} \right]_0^1 = \frac{1^3}{3} - 0 = \frac{1}{3}.
$$

Multiply by 3:

$$
3 \times \frac{1}{3} = 1.
$$

Hence, the pdf is valid.

Calculating the Expected Value $E[X]$

The expectation (mean) of X is:

$$
E[X] = \int_{-\infty}^\infty x f(x) dx = \int_0^1 x \times 3x^2 dx.
$$

Simplify the integrand:

$$
x \times 3x^2 = 3x^3.
$$

Compute:

$$
E[X] = \int_0^1 3x^3 dx = 3 \int_0^1 x^3 dx.
$$

Calculate the integral:

$$
\int_0^1 x^3 dx = \left[\frac{x^4}{4} \right]_0^1 = \frac{1^4}{4} - 0 = \frac{1}{4}.
$$

Multiply by 3:

$$
E[X] = 3 \times \frac{1}{4} = \frac{3}{4} = 0.75.
$$

Result:

$$
\boxed{E[X] = \frac{3}{4}.}
$$

Calculating $E[X^2]$

The variance formula involves both $E[X]$ and $E[X^2]$:

$$
\operatorname{Var}[X] = E[X^2] - (E[X])^2.
$$

Therefore, the next step is to compute $E[X^2]$:

$$
E[X^2] = \int_{-\infty}^\infty x^2 f(x) dx = \int_0^1 x^2 \times 3x^2 dx = 3 \int_0^1 x^4 dx.
$$

Calculate the integral:

$$
\int_0^1 x^4 dx = \left[\frac{x^5}{5} \right]_0^1 = \frac{1^5}{5} - 0 = \frac{1}{5}.
$$

Multiply by 3:

$$
E[X^2] = 3 \times \frac{1}{5} = \frac{3}{5} = 0.6.
$$

Result:

$$
\boxed{E[X^2] = \frac{3}{5}.}
$$

Calculating the Variance $\operatorname{Var}[X]$

Recall the variance formula:

$$
\operatorname{Var}[X] = E[X^2] - (E[X])^2.
$$

Substitute the computed values:

$$
\operatorname{Var}[X] = \frac{3}{5} - \left(\frac{3}{4}\right)^2.
$$

Calculate $(E[X])^2$:

$$
\left(\frac{3}{4}\right)^2 = \frac{9}{16}.
$$

Express both fractions with a common denominator (80) for clarity:

- Convert $\frac{3}{5}$:

$$
\frac{3}{5} = \frac{3 \times 16}{5 \times 16} = \frac{48}{80}.
$$

- Convert $\frac{9}{16}$:

$$
\frac{9}{16} = \frac{9 \times 5}{16 \times 5} = \frac{45}{80}.
$$

Now, subtract:

$$
\operatorname{Var}[X] = \frac{48}{80} - \frac{45}{80} = \frac{3}{80}.
$$

Expressed as a decimal:

$$
\frac{3}{80} = 0.0375.
$$

Final Answer:

$$
\boxed{\operatorname{Var}[X] = \frac{3}{80} \approx 0.0375.}
$$

Summary and Insights

This analysis demonstrates how to compute the variance of a continuous random variable given its probability density function. The key steps involve:

- Validating the pdf.
- Computing the expectation $E[X]$.
- Computing the second moment $E[X^2]$.
- Applying the variance formula.

The specific case here reveals a small variance, indicating that the values of X are tightly clustered around the mean 0.75. The increasing quadratic density $f(x)=3x^2$ suggests the variable tends to take larger values within $[0, 1]$, which is consistent with the mean being closer to 1.

Deeper Considerations and Applications

Why is this important?

- Variance is fundamental in understanding the spread and reliability of data.
- In applications such as quality control, finance, and scientific measurement, knowing the variance helps in risk assessment and decision-making.

Extensions and further topics:

- Standard deviation: $\sigma = \sqrt{\operatorname{Var}[X]}$ gives a measure in the same units as the variable.
- Higher moments: Skewness and kurtosis provide further insights into the distribution's shape.
- Transformations of variables: How does variance change under linear transformations $aX + b$?

Real-world implications:

- When modeling phenomena with a density increasing quadratically, like

certain growth processes, this variance calculation helps in predicting variability.
- Understanding the distribution's characteristics guides in designing experiments and interpreting data.

Conclusion

Calculating the variance of a continuous random variable with a specified pdf involves fundamental integration techniques and an understanding of expectation. In this case, the density function $f(x) = 3x^2$ on $[0,1]$ yields:

$$
\boxed{
\operatorname{Var}[X] = \frac{3}{80} \approx 0.0375,
}
$$

highlighting a distribution concentrated around its mean of 0.75. This process exemplifies the importance of verifying the properties of the pdf, computing expectations carefully, and applying the variance formula diligently. Mastery of these techniques is essential for anyone working in probability, statistics, data science, or related fields, where quantifying and understanding variability is key to insights and decision-making.

Note: This detailed exploration not only yields the variance but also reinforces core concepts in probability theory, enhancing your understanding of continuous distributions and their properties.

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