Let X[n] Be A Periodic Signal With One Period Given By [1, -2, 3, 4, 5, -6] For 2 N 3. Given That X[n]is

Let X[n] Be A Periodic Signal With One Period Given By [1, -2, 3, 4, 5, -6] For 2 N 3. Given That X[n]is

Understanding the properties and analysis of periodic signals is fundamental in the field of signal processing. When dealing with discrete-time signals, especially periodic ones, their behavior, Fourier series representation, and spectral characteristics reveal essential insights into their nature. In this article, we explore the periodic discrete-time signal X[n], which has a fundamental period defined by the sequence [1, -2, 3, 4, 5, -6], and analyze its properties, Fourier series expansion, and implications for signal processing applications.

Defining the Periodic Signal X[n]

What is a Periodic Discrete-Time Signal?

A discrete-time signal X[n] is called periodic if there exists a positive integer N such that:

- X[n + N] = X[n] for all integers n

The smallest such N is called the fundamental period of the signal. Periodic signals are ubiquitous in engineering, physics, and communications, as they model repetitive phenomena such as oscillations, waveforms, and modulated signals.

Given Sequence as One Period of X[n]

In our specific problem, the fundamental period of the signal X[n] is given by the sequence:

- [1, -2, 3, 4, 5, -6]

This sequence repeats indefinitely for all integer n, such that:

- X[n + 6] = X[n]

since the sequence length is 6, the period N = 6.

Analyzing the Periodic Sequence

Sequence Representation

The sequence can be written mathematically as:

- X[n] = x[n mod 6]

where x[n] = [1, -2, 3, 4, 5, -6].

Explicitly, for any integer n, the value of the signal is determined by:

- x[n] =
  - 1 if n mod 6 = 0
  - -2 if n mod 6 = 1
  - 3 if n mod 6 = 2
  - 4 if n mod 6 = 3
  - 5 if n mod 6 = 4
  - -6 if n mod 6 = 5

This periodic structure allows for Fourier analysis and spectral decomposition, which are essential tools in understanding the frequency components of the signal.

Fourier Series Representation of X[n]

Why Fourier Series?

Fourier series decompose a periodic signal into a sum of sinusoids with different frequencies, amplitudes, and phases. This decomposition provides insights into the spectral content of the signal, which is critical for filtering, modulation, and demodulation processes.

Calculating Fourier Coefficients

Given the fundamental period N = 6, the Fourier series coefficients are given by:

- $C[k] = (1/N) \sum_{n=0}^{N-1} x[n] e^{-j 2\pi k n / N}$, for k = 0, 1, ..., N-1

Where:
- x[n] are the sequence elements within one period
- $e^{-j 2\pi k n / N}$ are the basis functions

Applying this formula:

- For each k, compute C[k] by summing over n=0 to 5
- Use the sequence x[n] = [1, -2, 3, 4, 5, -6]

This allows the complete spectral representation:

$X[n] = \sum_{k=0}^{N-1} C[k] e^{j 2\pi k n / N}$

Example: Calculating a Fourier Coefficient

For k=0,

- C[0] = (1/6) (1 + (-2) + 3 + 4 + 5 + (-6)) = (1/6) (1 - 2 + 3 + 4 + 5 - 6) = (1/6) (5) = 5/6

Similarly, other coefficients are computed for k=1, 2, ..., 5, resulting in a complete spectral profile.

Spectrum and Spectral Analysis

Frequency Components

The Fourier coefficients C[k] correspond to discrete frequency components at:

- f_k = k / N f_s,

where f_s is the sampling frequency. These components indicate the strength of various sinusoidal elements in the signal.

Implications of Spectral Content

Understanding the spectral content helps in:

- Designing filters to isolate or suppress certain frequencies
- Analyzing the signal’s behavior in the frequency domain
- Optimizing communication systems by understanding bandwidth requirements

Applications of Periodic Discrete-Time Signals

Signal Processing and Communications

Periodic signals like X[n] are fundamental in digital communication systems, where they serve as carriers, modulating information for transmission over channels.

Fourier Analysis in Engineering

The Fourier series enables engineers to analyze harmonic content, predict system response, and design filters tailored to specific frequency components.

Simulation and Modeling

Periodic signals are used to model oscillatory phenomena, such as mechanical vibrations, electrical oscillations, and acoustic waves, providing a basis for simulation and control.

Conclusion

Understanding the periodic sequence X[n], with one period given by [1, -2, 3, 4, 5, -6], involves analyzing its structure, Fourier series representation, and spectral components. Recognizing that the sequence repeats every 6 samples allows for efficient computation of Fourier coefficients and spectral analysis, which are essential in numerous applications within signal processing, telecommunications, and systems engineering. Mastery of these concepts enables engineers and scientists to design better filters, analyze signals in the frequency domain, and develop advanced communication systems that leverage the properties of periodic signals.

Whether used in theoretical research or practical implementations, the principles outlined demonstrate the significance of periodic discrete-time signals like X[n] in modern technology and scientific analysis.

Frequently Asked Questions

What is the given period of the periodic signal X[n]?

The period of the signal X[n] is 6 samples, corresponding to the sequence [1, -2, 3, 4, 5, -6].

What is the fundamental period of the signal X[n]?

The fundamental period is 6, as the sequence repeats every 6 samples.

How is the sequence of X[n] defined over one period?

X[n] over one period is given by the sequence [1, -2, 3, 4, 5, -6].

What is the value of X[n] at n=2?

At n=2, X[n] = 3, which is the third element in the sequence.

What is the value of X[n] at n=7?

Since the sequence repeats every 6 samples, at n=7, X[n] = X[7 mod 6] = X[1] = -2.

How can you determine X[n] for any integer n?

X[n] can be found by taking n modulo 6 and referencing the corresponding value in the sequence [1, -2, 3, 4, 5, -6].

What is the significance of the sequence [1, -2, 3, 4, 5, -6] in analyzing X[n]?

This sequence defines one period of the signal, allowing us to analyze and compute X[n] for any n based on periodicity.

Is the sequence X[n] symmetric or antisymmetric over its period?

No, the sequence [1, -2, 3, 4, 5, -6] is neither symmetric nor antisymmetric.

What is the value of the discrete Fourier transform (DFT) of one period of X[n]?

Calculating the DFT involves applying the Fourier formula to the sequence [1, -2, 3, 4, 5, -6], which yields the frequency components of the signal.

How does the periodicity affect the spectral analysis of X[n]?

The periodicity results in discrete frequency components in the spectrum, with the fundamental frequency determined by the period of 6 samples.

Additional Resources

Let X[n] Be A Periodic Signal With One Period Given By [1, -2, 3, 4, 5, -6] For 2 N 3. Given That X[n]is a fundamental concept in signal processing, forming the backbone of many analyses in communications, control systems, and digital signal processing. Understanding the properties, behavior, and implications of such a periodic signal can unlock insights into system stability, frequency content, and signal reconstruction. This guide provides a comprehensive breakdown of what it means for a signal to be periodic with a specific period, how to analyze its properties, and what the notation suggests about its characteristics.

Introduction to Periodic Signals

A periodic signal is one that repeats itself after a fixed interval known as the period. Mathematically, a discrete-time signal $X[n]$ is called periodic if:

$$
X[n + N] = X[n] \quad \text{for all integers } n,
$$

where N is the fundamental period of the signal.

In the context of the problem statement, Let $X[n]$ be a periodic signal with one period given by [1, -2, 3, 4, 5, -6] for $2N \leq 3$, it indicates that the sequence repeats after a certain number of samples, with some additional conditions implied by the notation.

Analyzing the Given Sequence

The Basic Sequence

The sequence provided is:

$$[1, -2, 3, 4, 5, -6]$$

This sequence consists of 6 samples, which suggests that the period N of the signal is likely related to this sequence length. However, the statement includes "for 2N 3", which may be a typo or shorthand for a specific condition. Assuming that "for 2N \leq 3" or similar, the key idea is that the sequence repeats periodically.

Interpreting the Notation

- "Let $X[n]$ be a periodic signal": The signal repeats every N samples.
- "With one period given by [1, -2, 3, 4, 5, -6]": The fundamental period contains these 6 samples.
- "For 2N 3": Likely refers to a condition involving N, perhaps that the period N divides 6, or that the sequence repeats after certain intervals.

Assumptions Based on Context

Given the sequence length of 6, the most straightforward assumption is:

$$
N = 6
$$

which means $X[n]$ repeats every 6 samples:

$$
X[n + 6] = X[n]
$$

for all n.

Properties of the Periodic Sequence

Basic Periodicity

Since the sequence repeats every 6 samples:

$$
X[n + 6] = X[n]
$$

The sequence is fully characterized by its 6 samples. Understanding this, we can analyze:

- Its fundamental period.
- Its discrete-time Fourier series (DTFS) coefficients.
- Its energy and power.

Signal Symmetry and Pattern

Examining the sequence:

$$ [1, -2, 3, 4, 5, -6] $$

we can observe:

- No obvious symmetry like even or odd symmetry.
- The sequence exhibits a mixture of positive and negative values.
- The sum of the samples:

$$
1 + (-2) + 3 + 4 + 5 + (-6) = 1 - 2 + 3 + 4 + 5 - 6 = 5
$$

which might influence the DC component in the Fourier analysis.

Fourier Analysis of the Periodic Sequence

Discrete-Time Fourier Series (DTFS)

Any periodic discrete-time signal with period N can be represented as a sum of complex exponentials:

$$
X[n] = \sum_{k=0}^{N-1} C_k e^{j 2 \pi k n / N}
$$

where C_k are the Fourier coefficients calculated as:

$$
\[

$$
C_k = \frac{1}{N} \sum_{n=0}^{N-1} X[n] e^{-j 2 \pi k n / N}
$$

Given the sequence $[1, -2, 3, 4, 5, -6]$, we can compute these coefficients explicitly to understand the frequency components.

Computing Fourier Coefficients

For $N=6$:

$$
C_k = \frac{1}{6} \sum_{n=0}^{5} X[n] e^{-j 2 \pi k n / 6}
$$

where $X[0] = 1$, $X[1] = -2$, $X[2] = 3$, $X[3] = 4$, $X[4] = 5$, $X[5] = -6$.

Calculations involve summing these weighted exponentials for each k, revealing the magnitude and phase of each harmonic component.

Implications and Applications

Spectral Content

Understanding the Fourier coefficients allows us to analyze:

- The dominant frequency components.
- How the signal might behave after passing through filters.
- Its spectral energy distribution.

Power and Energy

- Total energy:

$$
E = \sum_{n} |X[n]|^2
$$

- Average power for periodic signals:

$$
P = \frac{1}{N} \sum_{n=0}^{N-1} |X[n]|^2
$$

which can be calculated directly from the sequence.

Signal Reconstruction

Since the sequence is periodic, it can be reconstructed from its Fourier

coefficients, which is fundamental in digital signal processing applications such as:

- Signal compression.
- Noise filtering.
- Modulation schemes.

Special Cases and Generalizations

When N is a Divisor of 6

If N divides 6, the sequence can be viewed as a fundamental period or a multiple of smaller periods, leading to harmonic analysis with harmonics at specific frequencies.

Non-Uniform Periods

If the sequence's period is not exactly 6 but related to other conditions (e.g., $2N \leq 3$), then the analysis would require considering subharmonics, possibly leading into the realm of multi-periodic signals or quasi-periodicity.

Practical Example: Signal Processing Scenario

Suppose this sequence models a discrete-time signal in a communication system. Its periodicity means:

- The signal repeats every 6 samples.
- Its spectral content can be used to design filters that isolate or suppress specific frequency components.
- The sequence's energy distribution informs about the signal's power, important for power management.

Summary

- Let $X[n]$ be a periodic signal characterized by the sequence `[1, -2, 3, 4, 5, -6]`.
- The fundamental period is likely $N=6$, meaning the sequence repeats every 6 samples.
- Analyzing the Fourier coefficients provides insight into the spectral composition.
- Understanding these properties aids in filter design, spectral analysis, and signal reconstruction.
- Clarifying the notation "for 2N 3" requires context, but the core concept remains analyzing a periodic sequence with a known fundamental period.

Final Thoughts

Periodic signals like \(X[n] \) serve as foundational elements in digital signal processing. By dissecting their structure, Fourier content, and properties, engineers and scientists can manipulate, analyze, and utilize these signals effectively within various technological contexts. Whether in communications, control systems, or audio processing, mastering the analysis of such sequences is essential for advancing modern digital technology.

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