Match Each Of The Descriptions With The Correct Rule For The Transformation.Rotation 90 Counterclockwise

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Understanding geometric transformations is fundamental in the study of geometry, especially when analyzing how figures change position or orientation in a plane. One of the most common transformations is rotation, which involves turning a figure around a fixed point by a specified angle. Among these, a 90-degree counterclockwise rotation is particularly significant due to its simplicity and frequent application in various fields such as computer graphics, engineering, and design.

This article aims to help you master the skills of matching descriptions of figures with the correct rules for performing a 90-degree counterclockwise rotation. By the end of this guide, you'll be able to identify and apply the appropriate transformation rules confidently and accurately.

Understanding Rotation in Geometry

What Is Rotation?

Rotation is a type of transformation that turns a figure around a fixed point called the center of rotation. The figure maintains its size and shape but changes its position and orientation.

Key components of rotation:

- Center of Rotation: The fixed point around which the figure rotates.
- Angle of Rotation: The degree measure of how far the figure turns.
- Direction: Usually clockwise or counterclockwise.

In this context, we focus on a 90-degree counterclockwise rotation, which involves turning the figure 90 degrees to the left around a specific point.

Properties of a 90-Degree Counterclockwise Rotation

- The rotation turns the figure a quarter turn to the left.

- Coordinates of points are transformed based on their relative position to the center.
- The shape and size of the figure remain unchanged.
- The orientation of the figure is reversed in a way consistent with the rotation direction.

Understanding these properties will help in matching descriptions with the correct transformation rules.

Rules for Rotating Points 90 Degrees Counterclockwise

Rotation Rules About the Origin

When rotating a point (x, y) 90 degrees counterclockwise about the origin (0, 0), the rule is:

- New coordinates: (x', y') = (-y, x)

This rule is derived from the geometric interpretation of rotation and can be applied directly to any point relative to the origin.

Rotation About a Specific Point (h, k)

If the rotation is about a point other than the origin, the process involves:

1. Translate the figure so that the center of rotation becomes the origin.
2. Perform the 90-degree counterclockwise rotation using the rule (x, y) → (-y, x).
3. Translate back to the original position.

The combined transformation rule for a point (x, y) rotated about (h, k):

- Step 1: Translate point to origin: (x - h, y - k)
- Step 2: Rotate: (-(y - k), x - h)
- Step 3: Translate back: (-(y - k) + h, (x - h) + k)

Therefore, the coordinates after rotation are:

- New x: h - (y - k)
- New y: k + (x - h)

Simplified as:

- x' = h - (y - k) = h - y + k
- y' = k + x - h

This rule applies to any point and is essential for matching descriptions involving figures rotated about arbitrary centers.

Matching Descriptions With the Correct Rotation Rules

To effectively match descriptions to the correct transformation rules, it's important to analyze the given details carefully. Below are typical descriptions and the corresponding rules.

Common Descriptions and Their Corresponding Rules

Description	Correct Rule	Explanation
The figure is rotated 90 degrees counterclockwise about the origin.	$(x, y) \to (-y, x)$	The standard rule for rotation about (0, 0).
The figure is rotated 90 degrees counterclockwise about point (h, k).	$(x, y) \to (h - (y - k), k + (x - h))$	Rotation about an arbitrary point.
The vertices of the figure change positions such that their x and y coordinates are swapped with a negative sign on the new x-coordinate.	$(x, y) \to (-y, x)$	Indicates a 90-degree counterclockwise rotation centered at the origin.
The shape appears to have been rotated to the left around a point not at the origin, and the coordinates of points relative to that center are transformed accordingly.	Use the translation-rotation-translation rule for (h, k).	Rotation about a non-origin point involves translation before and after rotation.

Practical Examples

Example 1: Rotation About the Origin

Description:
A triangle with vertices at (2, 3), (4, 5), and (6, 7) is rotated 90 degrees counterclockwise about the origin.

Matching rule:
Use $(x, y) \to (-y, x)$.

Application:
- Point (2, 3): (-3, 2)
- Point (4, 5): (-5, 4)
- Point (6, 7): (-7, 6)

Example 2: Rotation About a Specific Point

Description:
A rectangle is rotated 90 degrees counterclockwise about the point (1, 2).

Matching rule:
Use the translation-rotation-translation rule:

$x' = h - (y - k)$
$y' = k + (x - h)$

Application:
Suppose a vertex at (3, 4):
- $x' = 1 - (4 - 2) = 1 - 2 = -1$
- $y' = 2 + (3 - 1) = 2 + 2 = 4$

Result: The vertex moves from (3, 4) to (-1, 4).

Visual Aids and Diagrams

Using diagrams can greatly assist in understanding how points move during rotation. Visual representations help in matching descriptions with the correct rule by illustrating how each point's position changes.

Tip: When analyzing a description, draw the figure and mark the center of rotation. Trace the movement of key points to visualize the rotation.

Practice Problems for Mastery

To solidify your understanding, here are some practice problems. Attempt to match each description with the correct rule.

1. A pentagon is rotated 90 degrees counterclockwise about the origin. Its vertex at (1, -2) moves to the position:

a) (-2, -1)
b) (2, -1)
c) (2, 1)
d) (-2, 1)

2. A parallelogram is rotated 90 degrees counterclockwise about the point (4, 3). One vertex at (6, 5) moves to:

a) (2, 4)
b) (2, 1)
c) (4, 2)
d) (7, 4)

3. A figure undergoes a rotation of 90 degrees counterclockwise about the origin,

transforming a point (x, y) to (-y, x). Which of the following is true?

a) Coordinates swap with a negative sign on the x-coordinate.
b) Coordinates swap with a negative sign on the y-coordinate.
c) Coordinates are inverted.
d) Coordinates are unchanged.

Answers:

1. a) (-2, -1)

2. a) (2, 4)

3. a) Coordinates swap with a negative sign on the x-coordinate.

Conclusion

Mastering the matching of descriptions with the correct rotation rules is essential for understanding geometric transformations. Whether rotating about the origin or an arbitrary point, knowing the fundamental rules allows for accurate and efficient analysis of figures.

Remember, the key steps involve identifying the center of rotation, understanding whether the rotation is about the origin or a different point, and applying the appropriate transformation rule. Visual aids, practice problems, and a solid grasp of the properties of rotation will help you excel in geometry and related fields.

By practicing these concepts regularly, you'll develop a keen intuition for rotational transformations, enabling you to approach complex problems with confidence and precision.

Frequently Asked Questions

What is the effect of rotating a shape 90 degrees counterclockwise around the origin?

The shape is rotated so that each point turns 90 degrees to the left, resulting in a new position where the original x-coordinate becomes the new y-coordinate (negative), and the original y-coordinate becomes the new x-coordinate.

How does the coordinates of a point change when

rotated 90 degrees counterclockwise?

The point (x, y) transforms to (-y, x) after a 90-degree counterclockwise rotation.

Which rule should be applied to a shape's vertices to perform a 90-degree counterclockwise rotation?

Replace each point (x, y) with (-y, x).

If a rectangle has vertices at (2, 3), (4, 3), (4, 5), and (2, 5), what are their new coordinates after a 90-degree counterclockwise rotation?

The new vertices are at (-3, 2), (-3, 4), (-5, 4), and (-5, 2).

What is the geometric significance of applying a 90-degree counterclockwise rotation to a figure?

It rotates the figure about the origin, turning it leftward by a quarter turn, effectively swapping axes and negating the original y-coordinate.

How does rotating a figure 90 degrees counterclockwise affect its orientation?

The figure is turned so that it faces in a new direction, rotated leftward by 90 degrees, changing its orientation from its original position.

Additional Resources

Match Each Of The Descriptions With The Correct Rule For The Transformation: Rotation 90° Counterclockwise

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Introduction

Transformations are fundamental operations in geometry, providing a framework for understanding how shapes can be manipulated in space. Among these, rotations are particularly interesting because they preserve the shape and size of figures while changing their orientation. The specific case of a 90-degree counterclockwise rotation is a common transformation that appears frequently in mathematics, computer graphics, engineering, and art.

In this article, we undertake a detailed investigation into the rules governing the rotation of geometric figures by 90 degrees counterclockwise. Our goal is to match various descriptions of transformation effects with the precise mathematical rule that defines a 90-degree counterclockwise rotation. This exercise not only clarifies the conceptual understanding of this transformation but also provides a practical reference for students, educators, and professionals working with geometric manipulations.

The Fundamentals of Rotation

Understanding Rotation in the Coordinate Plane

Rotation about the origin (0,0) involves turning a figure around that fixed point by a specified angle. For a point (x, y), rotating it by an angle θ counterclockwise can be represented by a transformation:

$$
(x', y') = (x \cos \theta - y \sin \theta, x \sin \theta + y \cos \theta)
$$

When $\theta = 90^\circ$ or $\pi/2$ radians, the transformation simplifies significantly because:

$$
\cos 90^\circ = 0,\quad \sin 90^\circ = 1
$$

Thus, the rotation rule becomes:

$$
(x', y') = (- y, x)
$$

This is the foundational rule for 90-degree counterclockwise rotation about the origin.

Descriptive Effects of a 90° Counterclockwise Rotation

Before matching descriptions to the rules, it helps to understand what a 90° counterclockwise rotation does to geometric figures and their coordinates.

Visual and Geometric Effects

- Orientation Change: The figure turns to the left (counterclockwise), such that what was originally "up" becomes "left."
- Coordinate Exchange: The x and y coordinates are swapped, with the y coordinate becoming negative in the new position.
- Preservation of Shape and Size: The figure maintains its dimensions and shape, only its orientation changes.
- Rotation about the Origin: Unless specified otherwise, the rotation is centered at the origin.

Coordinate Transformation

For any point (x, y), after a 90° counterclockwise rotation around the origin, the new coordinates are:

$$
(x', y') = (- y, x)
$$

This rule underpins all other descriptions and is the basis for matching.

Matching Descriptions with the Transformation Rule

Below are typical descriptions or effects associated with a 90° counterclockwise rotation. For each, we analyze the statement and identify the corresponding mathematical rule.

1. The coordinates of each point are swapped, and the original y-coordinate becomes negative.

Analysis: Swapping x and y is part of the transformation, and the original y-coordinate becomes negative in the new position. This matches the rule:

$$
(x, y) \to (- y, x)
$$

which is precisely the formula for a 90° counterclockwise rotation about the origin.

Match: The rule where $(x, y) \to (- y, x)$.

---

2. The figure appears rotated to the left by a quarter turn, with each point moving to a position that was previously "above" or "to the right" of its original position.

Analysis: Rotation to the left (counterclockwise) by 90° turns the figure so that what was "up" becomes "left," and so forth. Mathematically, this corresponds directly to the coordinate transformation:

$$
(x, y) \to (- y, x)
$$

Match: The same as the previous—coordinate swap with y becoming negative.

---

3. The x-coordinate of each point becomes the negative of the y-coordinate, and the y-coordinate becomes the original x-coordinate.

Analysis: This description explicitly states the transformation:

$$
x' = - y,\quad y' = x
$$

which matches the core rule for a 90° counterclockwise rotation.

Match: $\boxed{(x, y) \to (- y, x)}$

---

4. The shape is rotated about the origin, such that points on the x-axis stay fixed, and points on the y-axis are reflected across the x-axis.

Analysis:

- Points on the x-axis ($y=0$):

$$
(x, 0) \to (0, x)
$$

which moves them to the y-axis, not fixed unless $x=0$.

- Points on the y-axis ($x=0$):

$$
(0, y) \to (- y, 0)
$$

which moves them onto the x-axis, changing their position significantly.

This behavior indicates a rotation, but not a reflection across axes. Since a rotation about the origin moves points on axes to different axes unless at the origin, this description aligns with the rotation rule.

Match: The fundamental rotation rule $\boxed{(x, y) \to (- y, x)}$.

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5. When applying the transformation, the entire figure turns to the left, and each point’s new position can be obtained by swapping its coordinates and negating the original y-coordinate.

Analysis: The statement explicitly describes swapping the x and y coordinates, with the y-coordinate becoming negative, matching the rotation rule:

$$
(x, y) \to (- y, x)
$$

Match: The standard rotation rule $\boxed{(x, y) \to (- y, x)}$.

---

6. The shape is rotated about the origin so that the point (x, y) moves to a new position where x becomes negative y, and y becomes x.

Analysis: Directly restates the transformation formula:

\[

$$
(x, y) \to (- y, x)
$$

Match: The core mathematical rule for a 90° counterclockwise rotation.

Special Cases and Additional Clarifications

While the above descriptions all align with the primary rotation rule, some descriptions can be mistaken for reflections or other transformations if not carefully analyzed. For example:

- Reflection over axes involves changing signs differently.
- Rotation about points other than the origin involves additional translation.

Therefore, the key takeaway is that the fundamental rule for a 90° counterclockwise rotation about the origin is:

$$
\boxed{
(x, y) \to (- y, x)
}
$$

Any description matching this algebraic rule corresponds to a 90° counterclockwise rotation.

Conclusion

Through this thorough investigation, we've established that the core mathematical rule for rotating a point (x, y) by 90 degrees counterclockwise about the origin is:

$$
(x, y) \to (- y, x)
$$

Descriptions involving swapping coordinates, negating the y-coordinate, or turning the figure to the left by a quarter turn all correspond to this rule. Recognizing this transformation's algebraic form helps clarify many geometric descriptions and supports accurate application in various fields.

Understanding these rules is vital for students and professionals dealing with geometric transformations, computer graphics, and spatial reasoning. By matching descriptions to the precise algebraic rule, one can confidently analyze and perform 90° counterclockwise rotations, ensuring accuracy and clarity in geometric operations.

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