Multiply The Previous Term By 3 And Subtract 4 The Second Term Of The Sequence Is 5. Find The Difference

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Understanding sequences and their patterns is fundamental in mathematics, especially when dealing with recursive formulas and progression problems. The phrase "Multiply The Previous Term By 3 And Subtract 4 The Second Term Of The Sequence Is 5. Find The Difference" hints at a specific recursive sequence where each term depends on the previous one. In this article, we will explore how to interpret this sequence, find its general form, determine its terms, and ultimately calculate the difference between its terms. Whether you're a student preparing for exams or someone interested in mathematical patterns, this guide will help clarify the steps involved in analyzing such sequences.

Deciphering the Sequence: Understanding the Recursive Formula

What Does the Sequence Rule Mean?

The instruction "Multiply The Previous Term By 3 And Subtract 4" indicates a recursive relationship. In mathematical terms, this can be expressed as:

$$ a_{n} = 3 \times a_{n-1} - 4 $$

where a_{n} is the current term, and a_{n-1} is the previous term.

The phrase "The Second Term Of The Sequence Is 5" suggests that:

$$ a_2 = 5 $$

However, to fully define the sequence, we need the initial term a_1. Since it’s not explicitly provided, we'll analyze the sequence based on the recursive rule and the given second term to find the initial term.

Finding the First Term of the Sequence

Using the Recursive Formula to Determine a_1

Given:

$$ a_2 = 3 \times a_1 - 4 = 5 $$

We can solve for a_1:

$$
\begin{aligned}
3 \times a_1 - 4 &= 5 \\
3 \times a_1 &= 5 + 4 \\
3 \times a_1 &= 9 \\
a_1 &= \frac{9}{3} = 3
\end{aligned}
$$

Thus, the first term of the sequence is:

$$ a_1 = 3 $$

Summary:
- $a_1 = 3$
- $a_2 = 5$

Generating the Sequence Terms

Calculating Subsequent Terms

Using the recursive rule:

$$ a_{n} = 3 \times a_{n-1} - 4 $$

We can compute the next few terms:

- $a_3 = 3 \times a_2 - 4 = 3 \times 5 - 4 = 15 - 4 = 11$
- $a_4 = 3 \times a_3 - 4 = 3 \times 11 - 4 = 33 - 4 = 29$
- $a_5 = 3 \times a_4 - 4 = 3 \times 29 - 4 = 87 - 4 = 83$

The sequence begins as:

$$ 3,\, 5,\, 11,\, 29,\, 83, \ldots $$

Finding the General Term of the Sequence

Deriving the Closed-Form Expression

Recursive sequences like this often have a closed-form formula. To find it, we analyze the recurrence relation:

$$ a_{n} = 3 a_{n-1} - 4 $$

This is a linear non-homogeneous recurrence relation. The standard approach involves solving the associated homogeneous recurrence and then finding a particular solution.

1. Homogeneous Part:

$$ a_{n}^{(h)} = 3 a_{n-1}^{(h)} $$

The characteristic equation suggests:

$$ a_{n}^{(h)} = C \times 3^{n-1} $$

2. Particular Solution:

Since the non-homogeneous term is constant (-4), assume a constant particular solution $ a_{n}^{(p)} = A $:

$$ A = 3A - 4 \Rightarrow -2A = -4 \Rightarrow A = 2 $$

3. General Solution:

$$ a_{n} = a_{n}^{(h)} + a_{n}^{(p)} = C \times 3^{n-1} + 2 $$

4. Determine C Using the Initial Condition:

Recall:

$$ a_1 = 3 $$

Substitute $ n = 1 $:

$$ a_1 = C \times 3^{0} + 2 = C + 2 $$

Set equal to 3:

$$ C + 2 = 3 \Rightarrow C = 1 $$

Final closed-form formula:

$$ a_{n} = 3^{n-1} + 2 $$

Verification:

- For $ n=2 $:

$$ a_2 = 3^{1} + 2 = 3 + 2 = 5 $$ (matches the given)

- For $ n=3 $:

$$ a_3 = 3^{2} + 2 = 9 + 2 = 11 $$ (matches earlier calculation)

Summary:

$$ a_n = 3^{n-1} + 2 $$

Calculating the Difference Between Terms

Finding the Difference $ a_{n+1} - a_{n} $

Using the explicit formula:

$$
\begin{aligned}
a_{n+1} &= 3^{n} + 2 \\

a_{n} &= 3^{n-1} + 2
\end{aligned}
\]

Subtract to find the difference:

\[
\begin{aligned}
a_{n+1} - a_{n} &= (3^{n} + 2) - (3^{n-1} + 2) \\
&= 3^{n} - 3^{n-1} \\
&= 3^{n-1} (3 - 1) \\
&= 3^{n-1} \times 2 \\
&= 2 \times 3^{n-1}
\end{aligned}
\]

Therefore:

\[\boxed{
a_{n+1} - a_{n} = 2 \times 3^{n-1}
} \]

This expression shows that the difference between consecutive terms increases exponentially, scaled by a factor of 2.

Applications and Significance of the Sequence

Real-Life Examples

Sequences that grow exponentially are common in various fields:

- Population Growth: When populations grow at a rate proportional to their current size, similar recursive formulas apply.
- Financial Modeling: Compound interest calculations can mirror exponential sequences.
- Computer Science: Recursive algorithms often involve sequences with similar recurrence relations.

Mathematical Insights

Understanding how to derive explicit formulas from recursive relations is essential for analyzing sequences efficiently. Recognizing patterns and solving recurrence relations are foundational skills in algebra and discrete mathematics.

Summary and Key Takeaways

- The sequence is defined by the recursive rule: \(a_{n} = 3 a_{n-1} - 4 \).
- The initial term \(a_1 \) is 3, determined by the given second term \(

$a_2 = 5$.

- The explicit formula for the sequence is $a_{n} = 3^{n-1} + 2$.
- The difference between consecutive terms is $a_{n+1} - a_{n} = 2 \times 3^{n-1}$, which grows exponentially.

Understanding these concepts allows you to analyze similar recursive sequences effectively, predict future terms, and understand the growth patterns involved. Whether solving math problems or applying these principles in real-world scenarios, mastery of recursive relations and their explicit formulas is invaluable.

Frequently Asked Questions

What is the general approach to find the sequence where each term is multiplied by 3 and then 4 is subtracted?

You start with the first term and apply the operation repeatedly to generate subsequent terms, following the rule: next term = (previous term × 3) - 4.

If the second term of the sequence is 5, how can we find the first term?

Using the formula for the second term: 5 = (first term × 3) - 4. Solving for the first term gives first term = (5 + 4) / 3 = 3.

Given the sequence rule, what is the third term if the first term is 3?

The third term = (second term × 3) - 4. First, find the second term: 5. Then, third term = (5 × 3) - 4 = 15 - 4 = 11.

How do I find the difference between consecutive terms in this sequence?

Subtract the previous term from the current term to find the difference. For example, difference between second and first term is 5 - 3 = 2.

What is the difference between the second and first terms in the sequence?

The difference is 5 - 3 = 2.

Can the difference between consecutive terms be constant in this sequence?

No, because each term depends on the previous one multiplied by 3 and subtracting 4, so the differences change as the sequence progresses.

If the second term is 5, what is the third term, and what is the difference between the second and third terms?

Third term = (5 × 3) - 4 = 11. The difference between third and second terms = 11 - 5 = 6.

How do I determine the sequence if I know the second term is 5 and the rule is multiply by 3 and subtract 4?

First, find the first term using the second term: first term = (5 + 4) / 3 = 3. Then, generate further terms using the rule: each term = (previous term × 3) - 4.

Additional Resources

Multiply The Previous Term By 3 And Subtract 4 The Second Term Of The Sequence Is 5. Find The Difference

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Introduction

In the realm of mathematics, sequences are fundamental constructs that elucidate patterns, relationships, and growth behaviors within numbers. They serve as essential tools across various disciplines including algebra, calculus, computer science, and even financial modeling. The problem statement, "Multiply the previous term by 3 and subtract 4. The second term of the sequence is 5. Find the difference," encapsulates a typical recursive sequence challenge. Such problems require a precise understanding of recursive formulas, initial conditions, and the logical progression of terms to find the desired value—in this case, the difference between terms.

This article aims to dissect this problem comprehensively, exploring the underlying principles, solving methodically, and analyzing the significance of the sequence's structure. We will discuss the formation of the sequence, derive the terms explicitly, and interpret the "difference" in the context of sequences. The exploration will blend mathematical rigor with accessible explanations, making it suitable for students, educators, and enthusiasts eager to deepen their understanding of recursive sequences.

Understanding the Sequence Formula

The Recursion Rule

The core of the problem is a recursive rule:

> "Multiply the previous term by 3 and subtract 4."

Mathematically, if we denote the sequence as \(\{a_n\} \), then the recursive relation can be expressed as:

$$
a_{n} = 3a_{n-1} - 4
$$

where:
- a_{n} is the n^{th} term,
- a_{n-1} is the previous term.

Recursive relations like this are common in sequences where each term depends on its immediate predecessor, often leading to exponential or geometric growth behaviors.

Initial Condition

The problem provides a crucial initial condition:

> "The second term of the sequence is 5."

This implies:

$$
a_2 = 5
$$

However, the first term a_1 is not explicitly provided. To fully understand the sequence, we need to determine a_1. Given the recursive relation, we can express a_2 in terms of a_1:

$$
a_2 = 3a_1 - 4
$$

Since $a_2 = 5$:

$$
5 = 3a_1 - 4
$$

From which:

$$
3a_1 = 9 \implies a_1 = 3
$$

Thus, the initial term:

$$
a_1 = 3
$$

Developing the Sequence

Explicit Formula Derivation

Having the initial term $a_1 = 3$, and the recursive rule, we can develop an explicit formula for any term \(a_n \). This involves solving the recurrence relation.

The recurrence:

\[
a_n = 3a_{n-1} - 4
\]

is a linear non-homogeneous recurrence relation. To solve it:

1. Solve the homogeneous part:

\[
a_n^{(h)} = 3a_{n-1}^{(h)}
\]

which has the general solution:

\[
a_n^{(h)} = C \cdot 3^{n-1}
\]

where \(C \) is a constant to be determined later.

2. Find a particular solution:

Since the non-homogeneous part is a constant (-4), we look for a constant particular solution \(a_n^{(p)} = A \):

\[
A = 3A - 4
\]

which yields:

\[
A - 3A = -4 \implies -2A = -4 \implies A = 2
\]

3. Combine solutions:

The general solution:

\[
a_n = a_n^{(h)} + a_n^{(p)} = C \cdot 3^{n-1} + 2
\]

4. Determine the constant \(C \) using initial conditions:

From earlier, \(a_1 = 3 \):

\[
a_1 = C \cdot 3^{0} + 2 = C + 2
\]

Set equal to 3:

\[

C + 2 = 3 \implies C = 1
\]

Thus, the explicit formula:

\[
\boxed{
a_n = 3^{n-1} + 2
}
\]

Computing Specific Terms of the Sequence

Using the explicit formula:

\[
a_n = 3^{n-1} + 2
\]

we can compute the first few terms:

Term \(a_n \)	Calculation	Value
\(a_1 \)	\(3^{0} + 2 \)	3
\(a_2 \)	\(3^{1} + 2 \)	5 (matches given)
\(a_3 \)	\(3^{2} + 2 \)	\(9 + 2 = 11 \)
\(a_4 \)	\(3^{3} + 2 \)	\(27 + 2 = 29 \)
\(a_5 \)	\(3^{4} + 2 \)	\(81 + 2 = 83 \)

This pattern shows rapid growth, characteristic of exponential sequences.

Analyzing the "Difference" in the Sequence

The problem concludes with:

> "Find the difference."

In sequences, the term "difference" can refer to various concepts:

- First difference: The difference between consecutive terms, i.e., \(a_{n} - a_{n-1} \).
- Second difference: The difference of the differences, useful in polynomial sequences.
- Net change or specific difference between two terms.

Given the context, the most logical interpretation is the first difference, as it directly relates to the sequence's progression.

Calculating the First Difference

Using the explicit formula:

\[

$$
a_n = 3^{n-1} + 2
$$

the difference between consecutive terms:

$$
a_{n} - a_{n-1} = (3^{n-1} + 2) - (3^{n-2} + 2) = 3^{n-1} - 3^{n-2}
$$

Factor out 3^{n-2}:

$$
a_{n} - a_{n-1} = 3^{n-2} (3 - 1) = 2 \times 3^{n-2}
$$

Thus, the first difference is:

$$
\boxed{
\Delta a_n = 2 \times 3^{n-2}
}
$$

for $n \geq 2$.

Example calculations:

- For $n=3$:

$$
\Delta a_3 = 2 \times 3^{1} = 2 \times 3 = 6
$$

which matches $a_3 - a_2 = 11 - 5 = 6$.

- For $n=4$:

$$
\Delta a_4 = 2 \times 3^{2} = 2 \times 9 = 18
$$

which matches $29 - 11 = 18$.

This exponential growth in the differences underlines the rapid expansion of the sequence.

Significance of the Sequence and Its Difference

Understanding the difference provides insight into the sequence's growth rate. Since the first difference grows exponentially with base 3, the sequence's terms increase at an accelerating pace, characteristic of exponential functions. This behavior is typical in processes where each step compounds upon the previous, such as population growth models, compound interest calculations, and certain algorithms in computer science.

The explicit formula and difference analysis reveal:

- The sequence's initial value is 3.
- The second term is 5, confirming the initial condition.
- The difference between consecutive terms increases exponentially, specifically by a factor of 3 each step, scaled by 2.

This exponential nature makes the sequence a classic example of geometric progression with a constant additive shift.

Practical Applications and Broader Context

Sequences like these are not just theoretical constructs; they have numerous applications:

- Modeling exponential growth: Population dynamics, radioactive decay, and viral spread.
- Algorithm analysis: Recursive algorithms often exhibit similar exponential or recursive behavior.
- Financial calculations: Compound interest calculations and investment growth projections.
- Mathematical education: Teaching recursion, exponential functions, and explicit formula derivation.

Understanding how to transition from recursive definitions to explicit formulas is a critical skill in mathematics, enabling analysts and scientists to predict future terms efficiently and analyze long-term behaviors.

Summary and Conclusions

This comprehensive exploration of the sequence defined by "Multiply the previous term by 3 and subtract 4," with the second term being 5, demonstrates the power of recursive relations, explicit formulas, and difference analysis. By logically deducing the initial term and deriving an explicit formula, we uncovered the sequence:

$$
a_n = 3^{n-1} + 2
$$

which exhibits exponential growth. The first difference between terms:

$$
\Delta a_n = 2 \
$$

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