One Of The Factors Of Mx2 +nx + 21 Is X+3. Find The Smallestpossible Value Of M + N. (m Andn Are Natural

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Understanding algebraic factors and polynomial relationships is a fundamental aspect of mathematics. In particular, problems involving quadratic polynomials and their factors provide insight into how algebraic expressions are constructed and decomposed. Today, we'll explore a problem that requires identifying the smallest possible sum of two natural numbers, M and N, given specific conditions on a quadratic polynomial and one of its factors. This problem not only enhances problem-solving skills but also deepens understanding of polynomial factorization and the properties of natural numbers.

Understanding the Problem Statement

Let's restate the problem to ensure clarity:

- The quadratic polynomial is given as $Mx^2 + nx + 21$, where M and N are natural numbers (positive integers).
- One of its factors (i.e., a polynomial that divides it exactly) is $x + 3$.
- The goal is to find the smallest possible value of M + N.

This problem involves polynomial factorization, specifically recognizing factors and their implications on the coefficients of the quadratic polynomial.

Key Concepts to Solve the Problem

To approach this problem, it's essential to understand several algebraic concepts:

1. Polynomial Factorization

- A quadratic polynomial can often be factored into two binomials.
- If $x + 3$ is a factor, then the quadratic polynomial can be expressed as:

$$Mx^2 + nx + 21 = (x + 3)(A x + B)$$

where A and B are constants to be determined.

2. Polynomial Expansion

- Expanding the factored form:

$$(x + 3)(A x + B) = A x^2 + B x + 3 A x + 3 B$$

Simplify:

$$A x^2 + (B + 3A) x + 3 B$$

- Equate this to the original polynomial:

$M x^2 + n x + 21$

- From the comparison, deduce relationships between A, B, M, n, and the constant term 21.

3. Coefficient Matching

- Match coefficients for each power of x:

- Coefficient of x^2: $M = A$
- Coefficient of x: $n = B + 3A$
- Constant term: $21 = 3B$

4. Solving for B and A

- From the constant term:

$21 = 3B \implies B = 21 / 3 = 7$

- Since $B = 7$, then:

$n = B + 3A = 7 + 3A$

- Recall $M = A$, with A being a natural number (positive integer).

Determining the Values of M and N

Based on the relationships:

- $M = A$ (a natural number)
- $N = n = 7 + 3A$

Since M and N are natural numbers, and A is a natural number, the problem reduces to:

- Find the smallest possible value of $M + N = A + (7 + 3A) = 4A + 7$

Minimizing M + N

To find the smallest possible value of M + N:

- Since A is a natural number ($A \geq 1$),
- Compute the sum for $A = 1$:

$M + N = 4(1) + 7 = 4 + 7 = 11$

- For $A = 2$:

$M + N = 8 + 7 = 15$

- For $A = 3$:

$M + N = 12 + 7 = 19$

- And so on.

The smallest value occurs when $A = 1$:

$M + N = 11$

Verifying the Solution

Let's verify whether the values $M=1$ and $N=10$ satisfy all conditions:

- $M = 1$ (natural number)
- $N = 10$ (since $N = n = 7 + 31 = 10$)
- Polynomial: $1 x^2 + 10 x + 21 = x^2 + 10 x + 21$

Factorization:

- Polynomial factors as $(x + 3)(A x + B)$:

With $A=1$, $B=7$:

$(x + 3)(x + 7) = x^2 + 7x + 3x + 21 = x^2 + 10x + 21$

which matches the original polynomial.

- Since $(x + 3)$ is a factor, the conditions are satisfied.

Thus, the minimal sum $M + N$ is 11.

Conclusion and Final Remarks

This problem elegantly demonstrates how polynomial factorization can be used to determine unknown coefficients given certain factors. By expressing the quadratic polynomial in factored form and matching coefficients, we derived relationships between the variables and identified the minimal sum of M and N that satisfy the conditions.

Key takeaways:

- Recognizing factors of polynomials helps in reverse-engineering coefficients.
- Expressing the polynomial as a product of binomials simplifies the problem.
- Ensuring coefficients are natural numbers constrains the possible solutions.
- The minimal sum of $M + N$ is achieved when the associated parameter A is minimized, which in turn minimizes the sum.

Additional Tips for Solving Similar Polynomial Problems

- Always start by expressing the polynomial in its factored form when a factor is known.
- Use coefficient comparison to set up equations relating unknowns.
- Remember that natural numbers are positive integers; this constrains the possible values.
- Verify solutions by expanding the factors to ensure they match the original polynomial.
- Explore the impact of varying parameters systematically to find minimal or maximal values.

Final Thoughts

Mathematics often involves exploring relationships and constraints to find optimal solutions. The problem of determining the smallest possible value of M + N, given a quadratic polynomial with a known factor, exemplifies this approach. By applying algebraic techniques such as factorization, coefficient matching, and systematic checking, students and enthusiasts can develop a deeper understanding of polynomial behavior and problem-solving strategies.

Whether you're preparing for exams, practicing algebra, or simply exploring mathematical relationships, mastering these techniques will serve you well in tackling a wide variety of algebraic problems with confidence and precision.

Frequently Asked Questions

Given the quadratic expression Mx^2 + Nx + 21, if (x + 3) is a factor, how can we find the relationship between M and N?

Since (x + 3) is a factor, substituting x = -3 into the quadratic gives zero: M(-3)^2 + N(-3) + 21 = 0, which simplifies to 9M - 3N + 21 = 0. This provides a relationship between M and N.

What equation do we get when we substitute x = -3 into Mx^2 + Nx + 21, given that (x + 3) is a factor?

Substituting x = -3 yields 9M - 3N + 21 = 0, which simplifies to 3M - N + 7 = 0.

How can we express N in terms of M from the equation 3M - N + 7 = 0?

Rearranging gives N = 3M + 7.

Given that M and N are natural numbers, what is the smallest possible value of M + N based on N = 3M + 7?

Since M is natural, the smallest M can be is 1, which gives N = 3(1) + 7 = 10. Therefore, M + N = 1 + 10 = 11, which is the smallest possible value.

What is the smallest possible value of M + N when M and N are natural numbers and (x + 3) is a factor of Mx^2 + Nx + 21?

The smallest possible value is 11, achieved when M = 1 and N = 10.

Are there other pairs of natural numbers for M and N that satisfy the factor condition and result in a smaller M + N?

No, because increasing M increases N (since N = 3M + 7), which in turn increases M + N. The minimal sum occurs when M = 1, N = 10.

Additional Resources

One of the Factors of $Mx^2 + Nx + 21$ is $x + 3$. Find the Smallest Possible Value of M + N (M and N are Natural Numbers)

Introduction

Mathematics often challenges students and enthusiasts alike with its intricate problems that blend algebraic concepts with logical reasoning. One such problem involves quadratic expressions and their factors, which are fundamental in understanding polynomial behavior, roots, and factorization. The problem statement—"One of the factors of $Mx^2 + Nx + 21$ is $x + 3$. Find the smallest possible value of M + N, where M and N are natural numbers"—serves as an excellent example of how algebraic manipulation can lead to the discovery of underlying relationships between variables.

This article aims to dissect this problem thoroughly, providing a step-by-step analytical approach, exploring relevant algebraic principles, and contributing to a deeper understanding of quadratic factorization. We will analyze the problem's core concepts, examine the implications of the given factor, and ultimately derive the minimal sum of M and N consistent with the constraints provided.

Understanding the Problem

The Core Elements

- The quadratic polynomial: $Mx^2 + Nx + 21$
- Known factor: $x + 3$
- Unknowns: M and N, both natural numbers (positive integers: 1, 2, 3, ...)

Objective

Find the smallest possible value of M + N, given the conditions that:

- $x + 3$ is a factor of the quadratic,
- M and N are natural numbers.

Significance of the Factor

In algebra, if $x + 3$ is a factor of the quadratic $Mx^2 + Nx + 21$, then $x = -3$ is a root of the polynomial. This fundamental property allows us to establish relationships between M and N.

Theoretical Framework

Polynomial Factorization and Roots

A quadratic polynomial $ax^2 + bx + c$ can be factored as:

$$ a(x - r_1)(x - r_2) $$

where r_1 and r_2 are roots of the polynomial.

Given that x + 3 is a factor, it implies:

$$ r_1 = -3 $$

and the quadratic can be written as:

$$ Mx^2 + Nx + 21 = (x + 3)(A x + B) $$

for some constants A and B.

Linking the Factorization to M and N

Expanding the right-hand side:

$$ (x + 3)(A x + B) = A x^2 + B x + 3A x + 3B = A x^2 + (B + 3A) x + 3B $$

Matching coefficients with $Mx^2 + Nx + 21$, we get:

1. Coefficient of x^2: $M = A$
2. Coefficient of x: $N = B + 3A$
3. Constant term: $21 = 3B$

From the constant term:

$$ 3B = 21 \Rightarrow B = 7 $$

Now, plugging $B = 7$ into the N expression:

$$ N = 7 + 3A $$

Recall that A corresponds to M, which must be a natural number, and N must also be a natural number.

Deriving Constraints and Relationships

Expressing N in Terms of M

Since $M = A$ (from coefficient matching), and $N = 7 + 3A$:

$$ N = 7 + 3 M $$

Given that both M and N are natural numbers:

- $M \in \mathbb{N}$
- $N = 7 + 3 M \in \mathbb{N}$

Minimizing $M + N$

Formulating the Sum

Expressed in terms of M:

$$ M + N = M + (7 + 3 M) = 4 M + 7 $$

Our goal is to find the smallest possible value of $M + N$, which simplifies to finding the minimal $M \in \mathbb{N}$.

Considering the Constraints

Since M is a natural number (positive integer), the smallest possible value is:

$$ M = 1 $$

Substituting into the sum:

$$ M + N = 4 \times 1 + 7 = 4 + 7 = 11 $$

Correspondingly, N would be:

$$ N = 7 + 3 \times 1 = 10 $$

which is also a natural number.

Validating the Solution

- Both $M = 1$ and $N = 10$ are positive integers.
- The polynomial becomes:

$$ 1 \times x^2 + 10 x + 21 $$

- Check if $x + 3$ is a factor:

Calculate $P(-3)$:

$$ P(-3) = (-3)^2 + 10 \times (-3) + 21 = 9 - 30 + 21 = 0 $$

Since $P(-3) = 0$, $x + 3$ is indeed a factor.

- The polynomial can be factored as:

$$ (x + 3)(x + 7) $$

which aligns with the factorization:

$$ (x + 3)(x + 7) = x^2 + 7 x + 3 x + 21 = x^2 + 10 x + 21 $$

matching our polynomial with $M = 1$ and $N = 10$.

Conclusion: The Smallest Possible Value of $M + N$

Based on the above derivation, the minimal sum:

$$ M + N = 11 $$

achieves all the problem's constraints, including the positivity of M and N, the factorization condition, and the polynomial's structure.

This problem exemplifies the power of algebraic factorization and root analysis in solving polynomial equations with constraints. It demonstrates how understanding the implications of known factors simplifies the process of identifying variable relationships.

Broader Implications and Mathematical Insights

The Role of Roots in Polynomial Factorization

The core insight in this problem revolves around recognizing that a known factor translates directly into a root of the polynomial. This root-based approach is fundamental in algebra, providing a straightforward method to relate coefficients when partial factorization information is available.

Constraints on Coefficients and Their Impact

The stipulation that M and N are natural numbers imposes integrality constraints that narrow the solution space significantly. This highlights the importance of considering domain restrictions in algebraic problems.

Optimization in Algebra

Finding the minimal sum $M + N$ is an example of an optimization problem within algebraic constraints. Such problems often appear in applied mathematics, economics, and engineering, emphasizing the importance of algebraic reasoning in practical decision-making.

Final Remarks

This problem underscores the elegance and utility of algebraic principles such as factorization, roots, and coefficient comparisons. It illustrates how a seemingly complex quadratic expression can be systematically analyzed to uncover minimal or optimal solutions under given constraints. The solution—where $M = 1$ and $N = 10$—not only satisfies all conditions but also exemplifies how foundational algebraic concepts underpin problem-solving strategies in mathematics.

Whether for students honing their algebra skills or enthusiasts exploring polynomial properties, this problem offers a clear pathway from hypothesis to solution, reinforcing the importance of understanding polynomial roots and factorization techniques in mathematical reasoning.

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