Part A:How Many Conduction Electrons Are There In A 3.50 Mm Diameter Gold Wire That Is 50.0 Cm Long?Part

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Understanding the number of conduction electrons in a gold wire is fundamental in solid-state physics and electrical engineering. This knowledge helps explain the wire's electrical conductivity, resistance, and overall behavior when used in circuits and electronic devices. In this article, we will explore how to determine the total number of conduction electrons in a gold wire with a diameter of 3.50 millimeters and a length of 50.0 centimeters. We will walk through the necessary calculations step-by-step, clarifying the concepts involved and providing a comprehensive guide to this interesting problem.

Understanding the Physical Parameters of the Gold Wire

Before diving into the calculations, it's essential to understand the physical dimensions and properties of the wire.

Dimensions of the Wire

- Diameter (d): 3.50 mm
- Length (L): 50.0 cm

To facilitate calculations, we convert these dimensions into SI units (meters):

- Diameter: 3.50 mm = 3.50×10^{-3} meters
- Radius (r): d/2 = 1.75×10^{-3} meters
- Length: 50.0 cm = 0.50 meters

Calculating the Volume of the Gold Wire

The volume of the cylindrical wire can be calculated using the formula:

Volume of a Cylinder

$$ V = \pi r^2 L $$

Substituting the known values:

$$ V = \pi \times (1.75 \times 10^{-3} \text{ m})^2 \times 0.50 \text{ m} $$

Calculating step-by-step:

1. Square the radius:

$$ (1.75 \times 10^{-3})^2 = 3.0625 \times 10^{-6} \text{ m}^2 $$

2. Multiply by the length:

$$ 3.0625 \times 10^{-6} \times 0.50 = 1.53125 \times 10^{-6} \text{ m}^3 $$

3. Multiply by π:

$$ V = \pi \times 1.53125 \times 10^{-6} \approx 4.81 \times 10^{-6} \text{ m}^3 $$

Thus, the volume of the gold wire is approximately 4.81×10^{-6} cubic meters.

Understanding the Electron Density in Gold

The number of conduction electrons depends on the atomic structure and electron configuration of gold.

Properties of Gold Relevant to Electron Count

- Atomic number (Z): 79
- Atomic mass: approximately 197 g/mol
- Density (ρ): 19,320 kg/m^3
- Number of atoms per unit volume (atomic density): to be calculated

Since gold is a metal, each atom contributes conduction electrons. Typically, in metallic gold, each atom contributes approximately 1 conduction electron, because its outermost electrons are delocalized and free to move within the metal.

Calculating Atomic Density

To find the total number of conduction electrons, we first need the number of gold atoms in the given volume.

1. Convert density to atomic density:

$$
\text{Number of moles per unit volume} = \frac{\rho}{\text{atomic mass}}
$$

$$
= \frac{19,320 \text{ kg/m}^3}{0.197 \text{ kg/mol}} \approx 98,095 \text{ mol/m}^3
$$

2. Number of atoms per cubic meter:

$$
\text{Atoms/m}^3 = \text{moles/m}^3 \times N_A
$$

where $N_A = 6.022 \times 10^{23} \text{ mol}^{-1}$:

$$
98,095 \times 6.022 \times 10^{23} \approx 5.9 \times 10^{28} \text{ atoms/m}^3
$$

This is the atomic density of gold, i.e., the number of atoms per cubic meter.

Calculating the Total Number of Conduction Electrons

Now, with the atomic density and the volume, we can determine the total number of atoms—and hence the number of conduction electrons, assuming one conduction electron per atom.

Total Atoms in the Wire

$$
N_{atoms} = \text{Atomic density} \times V
$$

\[

$$
= 5.9 \times 10^{28} \text{ atoms/m}^3 \times 4.81 \times 10^{-6} \text{ m}^3
$$

$$
= 2.84 \times 10^{23} \text{ atoms}
$$

Since each atom contributes approximately one conduction electron:

$$
N_{electrons} \approx N_{atoms} \approx 2.84 \times 10^{23}
$$

Therefore, the total number of conduction electrons in the gold wire is approximately 2.84×10^{23} electrons.

Implications of the Number of Conduction Electrons

Knowing the number of conduction electrons helps in understanding several electrical properties of the gold wire:

- **Electrical Conductivity:** More conduction electrons generally mean higher conductivity, assuming minimal electron scattering.
- **Resistance:** The resistance depends on how these electrons scatter within the metal lattice and is related to electron density.
- **Current Capacity:** The number of conduction electrons influences how much current the wire can carry without damage or overheating.

Summary and Key Takeaways

- The volume of the gold wire with a diameter of 3.50 mm and length of 50.0 cm is approximately 4.81×10^{-6} m³.
- The atomic density of gold is about 5.9×10^{28} atoms per cubic meter.
- The total number of gold atoms—and thus conduction electrons—is approximately 2.84×10^{23}.
- Each atom contributes roughly one conduction electron, making this the approximate number of conduction electrons in the wire.

Conclusion

Calculating the number of conduction electrons in a gold wire involves understanding the physical dimensions, converting units, calculating the volume, and applying atomic properties. This process reveals that a relatively small volume of gold contains on the order of 10^{23} conduction electrons, a number so large that it underscores the exceptional conductivity and electrical properties of metals like gold. Such calculations are fundamental in materials science and electrical engineering, influencing how we design and utilize metallic conductors in countless applications.

By mastering these calculations, engineers and physicists can better understand and predict the behavior of metallic conductors, optimize electrical systems, and develop new materials with tailored electrical properties.

Frequently Asked Questions

How do you calculate the number of conduction electrons in a gold wire?

To find the number of conduction electrons, first determine the volume of the wire, then multiply by the atomic density (atoms per unit volume), and finally multiply by the number of conduction electrons per atom (for gold, typically 1).

What is the formula to find the volume of a cylindrical wire?

The volume V of a cylindrical wire is given by $V = \pi \times (\text{radius})^2 \times \text{length}$.

What is the significance of the diameter in calculating conduction electrons?

The diameter helps determine the radius, which is used in the volume calculation to find the total number of atoms and, consequently, the conduction electrons.

What is the atomic density of gold used in calculations?

The atomic density of gold is approximately 5.90×10^{28} atoms per cubic meter.

How do you convert the given diameter from millimeters to meters?

Divide the diameter in millimeters by 1000 to convert it to meters; for example, 3.50 mm = 0.00350 meters.

What is the length of the wire in meters for this calculation?

The length is given as 50.0 cm, which converts to 0.50 meters.

How many conduction electrons are there per gold atom?

Each gold atom contributes one conduction electron, so the number of conduction electrons equals the number of atoms.

What is the step-by-step process to find the total number of conduction electrons in the wire?

First, calculate the volume of the wire; second, find the number of atoms by multiplying volume by atomic density; third, since each atom contributes one conduction electron, that result is the total number of conduction electrons.

Why is understanding conduction electrons important in electrical conductivity?

Conduction electrons are responsible for carrying electric current in metals; knowing their number helps explain a material’s conductivity and electrical behavior.

Can the calculation be approximated using known constants for gold?

Yes, using known values such as the atomic radius, atomic density, and the geometry of the wire allows for an accurate approximation of the total conduction electrons.

Additional Resources

How Many Conduction Electrons Are There In A 3.50 mm Diameter Gold Wire That Is 50.0 cm Long?

Understanding the number of conduction electrons in a given piece of metal, such as gold, is fundamental to grasping its electrical conductivity and related properties. This problem involves applying concepts from solid-state physics and materials science, specifically the free electron model, to determine the total number of conduction electrons in a gold wire of specified dimensions. In this detailed review, we'll explore the underlying principles, perform the calculations step-by-step, and discuss the significance of each component.

Introduction to Conduction Electrons in Metals

Metals are characterized by a crystalline lattice of metal atoms with a "sea" of free electrons that are not bound to any particular atom. These conduction electrons are responsible for the metal's electrical conductivity, thermal conductivity, and other electronic properties.

Key Concepts:
- Free Electron Model: Simplifies the behavior of conduction electrons as free particles within the

metal, ignoring interactions with ions and other electrons.
- Electron Density: The number of conduction electrons per unit volume, which is crucial for calculating total conduction electrons.

Gold (Au) is a noble metal with excellent electrical conductivity, making it a common choice in electronic applications. Its conduction electrons arise from the outermost electrons of each atom, typically one per atom for monovalent metals like gold.

Step 1: Gathering Material Properties of Gold

To determine the total number of conduction electrons, we need specific properties of gold:

- Atomic Number (Z): 79 (number of protons, and approximately the number of electrons in a neutral atom).
- Atomic Mass (M): approximately 197 g/mol.
- Density (ρ): about 19.32 g/cm^3.

Note: For conduction electrons, we often consider the number of free electrons per atom. Gold's electron configuration indicates one conduction electron per atom, as it is monovalent.

Step 2: Calculating the Number of Atoms in the Wire

The total number of conduction electrons corresponds directly to the total number of atoms times the number of conduction electrons per atom (which is typically 1 for gold).

Calculations:

1. Determine the volume of the wire:

Given dimensions:

- Diameter, $d = 3.50\, \text{mm}$
- Length, $L = 50.0\, \text{cm}$

Convert units to SI (meters):

- $d = 3.50\, \text{mm} = 3.50 \times 10^{-3}\, \text{m}$
- $L = 50.0\, \text{cm} = 0.50\, \text{m}$

Calculate radius:

- $r = \frac{d}{2} = 1.75 \times 10^{-3}\, \text{m}$

Calculate cross-sectional area A:

$$
A = \pi r^2 = \pi \times (1.75 \times 10^{-3})^2 \approx \pi \times 3.0625 \times 10^{-6} \approx 9.62 \times 10^{-6}\, \text{m}^2
$$

Volume of the wire:

$$
V = A \times L = 9.62 \times 10^{-6} \times 0.50 = 4.81 \times 10^{-6}\, \text{m}^3
$$

2. Calculate the mass of the gold in the wire:

Using density:

$$
\rho = 19.32\, \text{g/cm}^3
$$

Convert density to SI units:

$$
\rho = 19.32\, \text{g/cm}^3 = 19.32 \times 10^{3}\, \text{kg/m}^3
$$

(Alternatively, since 1 g/cm^3 = 1000 kg/m^3, actually 19.32 g/cm^3 = 19.32×10^{3} kg/m^3 is incorrect; let's clarify the conversion:

- 1 g/cm^3 = 1000 kg/m^3

Thus,

$$
\rho = 19.32\, \text{g/cm}^3 = 19.32 \times 1000\, \text{kg/m}^3 = 19320\, \text{kg/m}^3
$$

Now, compute the mass:

$$
m = \rho \times V = 19320\, \text{kg/m}^3 \times 4.81 \times 10^{-6}\, \text{m}^3 \approx 0.093\, \text{kg}
$$

3. Calculate the number of atoms:

Number of moles:

$$
n_{mol} = \frac{m}{M} = \frac{0.093\, \text{kg}}{0.197\, \text{kg/mol}} \approx 0.471\,
$$

\text{mol}
\]

Number of atoms:

\[
N_{atoms} = n_{mol} \times N_A
\]

where \(N_A = 6.022 \times 10^{23}\, \text{atoms/mol} \):

\[
N_{atoms} \approx 0.471 \times 6.022 \times 10^{23} \approx 2.84 \times 10^{23}\, \text{atoms}
\]

Step 3: Calculating the Total Number of Conduction Electrons

Since each gold atom contributes approximately one conduction electron:

\[
N_{electrons} \approx N_{atoms} \approx 2.84 \times 10^{23}
\]

Therefore, the total number of conduction electrons in the wire is approximately \(2.84 \times 10^{23}\).

Additional Considerations and Refinements

While the above calculation provides a good approximation, several factors can refine the estimate:

- Valence electrons: Gold is monovalent, so one conduction electron per atom is a reasonable assumption.
- Impurities and alloying: In real-world scenarios, impurities can alter conduction electron counts slightly, but for pure gold, this is negligible.
- Quantum effects and electron screening: For macroscopic dimensions, classical calculations are sufficient; quantum effects are more significant at nanoscales.

Understanding Electron Density and Its Implications

The calculated number of conduction electrons allows us to derive the electron density:

$$
n = \frac{N_{electrons}}{V} = \frac{2.84 \times 10^{23}}{4.81 \times 10^{-6}\, \text{m}^3}
\approx 5.91 \times 10^{28}\, \text{electrons/m}^3
$$

This high electron density explains why gold is such an excellent conductor. The free electron density is a critical parameter in:

- Electrical conductivity calculations
- Plasma frequency estimations
- Modeling electron transport phenomena

Significance of the Calculation in Practical Contexts

Calculating the number of conduction electrons in a gold wire isn’t just an academic exercise; it has practical implications:

- Designing electronic components: Knowing the conduction electron count informs resistivity and current-carrying capacity.
- Understanding electrical resistance: Electron density directly influences resistivity through scattering mechanisms.
- Thermal management: Electron conduction also contributes to heat transfer in metals.
- Nanotechnology: At nanoscale dimensions, quantum effects modify conduction, but bulk electron counts serve as foundational references.

Summary of the Calculation Process

To succinctly recap:

1. Convert the physical dimensions of the wire into SI units.
2. Calculate the cross-sectional area and the volume.
3. Use the density and molar mass to find the mass and number of atoms.
4. Assume one conduction electron per atom (for gold).
5. Derive the total number of conduction electrons from the total atom count.

Final Answer:

$$
\boxed{
\text{Number of conduction electrons} \approx 2.84 \times 10^{23}
}
$$

Conclusion

This comprehensive exploration underscores how fundamental physical constants and material properties converge to determine the microscopic electronic structure of a macroscopic object. The calculation demonstrates the power of basic physics principles and material data in quantifying properties that influence real-world applications. Whether for designing electrical wiring, microelectronic components, or nanoscale devices, understanding conduction electrons remains central to advancing technology.

Note: Always consider the assumptions made in the free electron model and the idealizations involved in such calculations. For precise engineering applications, more detailed models accounting for band structure, electron scattering, and impurities may be necessary.

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