Part Of The Graph Of The Function F(x) = (x 1)(x + 7) Is Shown Below.Which Statements About The Function

Part Of The Graph Of The Function F(x) = (x - 1)(x + 7) Is Shown Below. Which Statements About The Function

Understanding the behavior of quadratic functions is fundamental in algebra and calculus. The function F(x) = (x - 1)(x + 7) is a quadratic function expressed in factored form, which provides valuable insights into its graph and characteristics. Analyzing this function involves exploring its roots, vertex, axis of symmetry, concavity, and other properties. This article aims to provide a comprehensive, SEO-optimized discussion about the graph of F(x) = (x - 1)(x + 7), including the key statements that describe its behavior and shape.

Introduction to Quadratic Functions and Their Graphs

Quadratic functions are polynomial functions of degree 2, typically written in the form f(x) = ax^2 + bx + c. However, they can also be expressed in factored form, vertex form, or standard form. The graph of a quadratic function is a parabola, which opens upwards if the leading coefficient is positive and downwards if it is negative.

The quadratic function F(x) = (x - 1)(x + 7) is given in factored form, which directly reveals its roots and helps analyze its key features visually and analytically. Understanding how to interpret such functions is essential for solving quadratic equations, analyzing their graphs, and applying this knowledge in real-world contexts such as physics, engineering, and economics.

Analyzing the Function F(x) = (x - 1)(x + 7)

1. Expressing the Function in Standard Form

To better understand the graph, we first expand the factored form:

F(x) = (x - 1)(x + 7)

Using the distributive property (FOIL method):

F(x) = x x + x 7 - 1 x - 1 7

F(x) = x^2 + 7x - x - 7

F(x) = x^2 + 6x - 7

This standard form, f(x) = x^2 + 6x - 7, helps identify the coefficients and analyze the parabola's properties.

2. Identifying the Roots or Zeros

The roots of the quadratic function are the x-values where F(x) = 0:

(x - 1)(x + 7) = 0

Applying the zero product property:

$x - 1 = 0 \Rightarrow x = 1$

$x + 7 = 0 \Rightarrow x = -7$

Therefore, the roots are at $x = 1$ and $x = -7$. These are the points where the parabola intersects the x-axis.

3. Determining the Vertex of the Parabola

The vertex is the highest or lowest point on the parabola, depending on whether it opens downward or upward. It can be found using the vertex formula in standard form:

x-coordinate of the vertex: $x_v = -b / (2a)$

In the standard form $f(x) = ax^2 + bx + c$:

$a = 1, b = 6, c = -7$

Calculating:

$x_v = -6 / (2 \ 1) = -6 / 2 = -3$

To find the y-coordinate of the vertex:

$f(-3) = (-3)^2 + 6(-3) - 7 = 9 - 18 - 7 = -16$

Hence, the vertex is at $(-3, -16)$. This point represents the minimum of the parabola because the coefficient of x^2 ($a=1$) is positive.

4. Axis of Symmetry

The axis of symmetry passes through the vertex and divides the parabola into two mirror images. Its equation is:

$x = x_v = -3$

This vertical line is crucial for graphing and understanding the symmetry of the parabola.

5. Opening Direction of the Parabola

Since the coefficient $a = 1 > 0$, the parabola opens upward. This implies that the vertex at $(-3, -16)$ is the lowest point on the graph, and the arms extend infinitely upward.

Key Statements About the Function $f(x) = (x - 1)(x + 7)$

Based on the analysis above, several statements can be made to describe the behavior and characteristics of this quadratic function:

Statement 1: The parabola opens upward because the leading coefficient a is positive.

This means the arms of the parabola extend infinitely upward, and the function has a minimum point at its vertex.

Statement 2: The roots of the function are at x = -7 and x = 1.

These are the x-intercepts where the graph crosses the x-axis, indicating the solutions to the equation $F(x) = 0$.

Statement 3: The vertex of the parabola is at (-3, -16), representing the minimum point of the function.

This point is the lowest point on the graph and provides key information about the graph's shape.

Statement 4: The axis of symmetry is the vertical line x = -3, passing through the vertex.

This line divides the parabola into two symmetric halves, useful for graphing and analyzing the function.

Statement 5: The parabola is symmetric with respect to the line x = -3.

Symmetry implies that points equidistant from the axis of symmetry on either side have the same y-value.

Statement 6: The y-intercept of the function is at F(0) = -7.

Substituting $x = 0$ into the standard form:

$F(0) = 0 + 0 - 7 = -7$

This is where the graph crosses the y-axis.

Statement 7: As x approaches infinity, F(x) approaches infinity; as x approaches negative infinity, F(x) also approaches infinity.

Since the parabola opens upward, the function's values grow large positively on both ends of the x-axis.

Statement 8: The parabola has no maximum point, only a minimum at the vertex.

Because the parabola opens upward, its maximum extends to infinity.

Graphing the Function F(x) = (x - 1)(x + 7)

Understanding the key points and features allows for accurate graphing:

- Plot the roots at x = -7 and x = 1.
- Mark the vertex at (-3, -16).
- Draw the axis of symmetry x = -3.
- Sketch the parabola opening upward, passing through these key points.
- Confirm the y-intercept at (0, -7).

Graphing tools or graphing calculators can help visualize these features precisely.

Real-World Applications and Significance

Quadratic functions like F(x) = (x - 1)(x + 7) have numerous applications:

- Physics: Modeling projectile motion where the path follows a parabola.
- Economics: Analyzing profit functions that are quadratic in nature.
- Engineering: Designing parabolic reflectors and satellite dishes.
- Mathematics Education: Teaching concepts of roots, vertex, symmetry, and graphing techniques.

Understanding the properties of such functions enables students and professionals to interpret data, optimize solutions, and develop models effectively.

Summary and Conclusion

In conclusion, the quadratic function F(x) = (x - 1)(x + 7) exhibits characteristic features typical of a parabola opening upward. Its roots at x = -7 and x = 1, along with its vertex at (-3, -16), define its shape and position on the coordinate plane. The axis of symmetry at x = -3 divides the parabola into two mirror images, reinforcing its symmetric nature. The y-intercept at (0, -7) and the end behavior—approaching infinity as x approaches positive or negative infinity—further describe its graph.

By analyzing these properties, we can confidently state the key characteristics of the function, aiding in graphing, solving related problems, and understanding its applications. Recognizing the behavior of quadratic functions like this is foundational in algebra, calculus, and applied mathematics, making this analysis valuable for students, educators, and professionals alike.

Keywords for SEO Optimization:

- Quadratic function analysis
- Parabola graph features
- Roots of quadratic functions
- Vertex of quadratic functions
- Axis of symmetry in parabolas
- Standard form of quadratic equations
- Graphing quadratic functions
- Applications of quadratic functions
- F(x) = (x - 1)(x + 7) explanation
- Upward opening parabola characteristics

Meta Description:

Explore a detailed analysis of the quadratic function F(x) = (x - 1)(x + 7), including its roots, vertex, axis

of symmetry, and graphing tips. Understand key properties and real-world applications of parabolas.

Frequently Asked Questions

What is the general shape of the graph of the function f(x) = (x - 1)(x + 7)?

The graph is a parabola opening upward, since the quadratic coefficient is positive.

Where are the x-intercepts of the function f(x) = (x - 1)(x + 7)?

The x-intercepts are at x = 1 and x = -7, where the function crosses the x-axis.

What is the vertex of the parabola for f(x) = (x - 1)(x + 7)?

The vertex is at the midpoint between the roots, x = (1 + (-7))/2 = -3, and the y-coordinate can be found by evaluating f(-3).

Is the function f(x) = (x - 1)(x + 7) increasing or decreasing on the intervals?

The function is decreasing between x = -7 and x = 1, and increasing outside this interval.

What is the value of f(x) at x = 0?

f(0) = (0 - 1)(0 + 7) = (-1)(7) = -7.

How does the sign of f(x) change around the roots?

f(x) is positive outside the roots (-7, 1) and negative between them, since the parabola opens upward.

Can the graph of f(x) = (x - 1)(x + 7) be symmetric?

Yes, the parabola is symmetric about its axis of symmetry, which passes through the vertex at x = -3.

Additional Resources

Part of the Graph of the Function F(x) = (x - 1)(x + 7) Is Shown Below. Which Statements About The Function

Understanding the behavior of quadratic functions is fundamental in algebra and calculus, as these functions often serve as building blocks for more complex mathematical concepts. The function in question, $F(x) = (x - 1)(x + 7)$, is a quadratic that opens upwards, given the positive leading coefficient when expanded. Analyzing its graph, features, and the statements derived from its properties provides valuable insights into its behavior. This article delves into the detailed examination of this quadratic function, exploring its key characteristics, the significance of the parts of its graph, and the implications of specific statements related to its graph.

Overview of the Function $F(x) = (x - 1)(x + 7)$

Expanding and Understanding the Basic Form

The given function can be expanded to reveal its standard quadratic form:

$$
F(x) = (x - 1)(x + 7) = x^2 + 7x - x - 7 = x^2 + 6x - 7
$$

This form allows us to analyze the function's properties more systematically. The quadratic term x^2 indicates the parabola opens upward (since the coefficient is positive). The linear term $6x$ influences the position of the vertex and the axis of symmetry, while the constant term -7 shifts the graph vertically.

Factoring and Roots (Zeros)

The roots of the quadratic are obtained by setting $F(x) = 0$:

$$
(x - 1)(x + 7) = 0
$$

which yields:

$$
x = 1 \quad \text{and} \quad x = -7
$$

These roots, also known as zeros or x-intercepts, are points where the graph crosses the x-axis. The roots are crucial for understanding the graph's shape and for verifying the correctness of any statements about the function.

Vertex and Axis of Symmetry

The vertex of the parabola is a point of maximum or minimum (for this upward-opening parabola, it is a minimum). The x-coordinate of the vertex can be found using the formula:

$$
x_{v} = -\frac{b}{2a}
$$

where $a = 1$ and $b = 6$:

$$
x_{v} = -\frac{6}{2 \times 1} = -3
$$

Plugging $x = -3$ back into the function gives the y-coordinate:

$$
F(-3) = (-3 - 1)(-3 + 7) = (-4)(4) = -16
$$

Thus, the vertex is at $(-3, -16)$. The axis of symmetry passes through this point, with the equation:

$$
x = -3
$$

Detailed Analysis of the Graph and Associated Statements

Understanding the graph involves examining key features such as intercepts, vertex, symmetry, and the behavior of the function at various points. Based on these features, several statements can be evaluated for their correctness or implications.

1. The Roots of the Function and Their Significance

The roots $x=1$ and $x=-7$ are fundamental because:

- They mark where the graph intersects the x-axis.
- They divide the parabola into intervals where the function is positive or negative.
- The sign of the function on various intervals depends on the factors:

- For $x < -7$, both $(x - 1)$ and $(x + 7)$ are negative, so their product is positive ($- \times - = +$).

- For $-7 < x < 1$, $(x + 7)$ is positive but $(x - 1)$ is negative, so the product is negative ($+ \times - = -$).

- For $x > 1$, both factors are positive, so the product is positive ($+ \times + = +$).

This pattern is essential for understanding the function's behavior across its domain.

2. The Vertex and Its Role in the Function's Graph

The vertex at $(-3, -16)$ is the lowest point of the parabola, given the upward opening. Its significance includes:

- Minimum Value: The y-coordinate -16 is the minimum value of the function.

- Symmetry: The parabola is symmetric about the vertical line $x = -3$, which divides the graph into two mirror-image halves.

- Implication for Statements: Any statement claiming the minimum occurs elsewhere or that the vertex is at a different point would be incorrect.

3. The Shape and Direction of the Parabola

Since the quadratic coefficient $a=1$ is positive, the parabola opens upward. This leads to several conclusions:

- The function tends toward $+\infty$ as $x \to \pm \infty$.

- The minimum point is at the vertex.

- The graph is "U" shaped, with the lowest point at $(-3, -16)$.

Any statement suggesting the parabola opens downward or has a maximum would be incorrect.

4. Behavior at Various Intervals

Analyzing the function's sign and increasing/decreasing behavior:

- For $x < -7$, $F(x) > 0$, decreasing until reaching the vertex at -3.

- Between -7 and 1, $F(x) < 0$, increasing from -16 at -3 to 0 at $x=1$.

- For $x > 1$, $F(x) > 0$, increasing again towards $+\infty$.

This analysis supports statements about where the function is positive or negative, increasing or decreasing.

Evaluating Statements About the Function

Based on the detailed understanding above, we can evaluate typical statements that might be associated with this quadratic.

Statement 1: The graph crosses the x-axis at $x=1$ and $x=-7$.

Analysis: Since the roots are obtained from the factored form $(x - 1)(x + 7) = 0$, this statement is correct. These are the exact points where the graph intersects the x-axis.

Implication: Any assertion claiming the roots are at different points would be false. The x-intercepts are precisely at these roots, confirming the accuracy of this statement.

Statement 2: The vertex of the parabola is at $(-3, -16)$.

Analysis: Calculated from the quadratic form, the vertex's x-coordinate is -3, and substituting this yields the y-coordinate -16. Therefore, the statement is correct.

Implication: Any claim that the vertex is located elsewhere (e.g., at $(-4, -15)$) contradicts the calculations, indicating a misunderstanding of the vertex's position.

Statement 3: The parabola opens downward.

Analysis: Since $a=1 > 0$, the parabola opens upward. This is a fundamental property of quadratic functions, and any claim that it opens downward is incorrect.

Implication: Such a statement would misrepresent the graph's shape and could lead to incorrect conclusions about maximum or minimum points.

Statement 4: The minimum value of $F(x)$ is -16.

Analysis: The vertex provides the minimum value for an upward-opening parabola, which we've calculated as -16. Therefore, the statement is true.

Implication: Any statement claiming the minimum is at a different y-value (say, -15) would be inaccurate.

Statement 5: The function is positive for $x < -7$ and $x > 1$, and negative for $-7 < x < 1$.

Analysis: Based on the sign analysis of the factors:

- For $x < -7$, both factors are negative, so $F(x) > 0$.

- For $-7 < x < 1$, one factor is positive, the other negative, so $F(x) < 0$.

- For $x > 1$, both are positive, so $F(x) > 0$.

Hence, this statement is accurate.

Implication: Any contrary statement claiming the sign pattern is different would be false.

Implications and Broader Context

The analysis of this quadratic function's graph illuminates broader mathematical principles:

- Quadratic Behavior: The parabola's shape, roots, vertex, and symmetry are interconnected, providing a comprehensive picture of the function's behavior.

- Applications: Such functions are applicable in physics (projectile motion), economics (profit models), and engineering (parabolic reflectors).

- Graphical Interpretation: Accurate graphing requires understanding roots, vertex, and the behavior at infinity, which is crucial for problem-solving and modeling real-world phenomena.

- Error Prevention: Recogn

Part Of The Graph Of The Function F X X 1 X 7 Is Shown Below Which Statements About The Function

Part Of The Graph Of The Function F X X 1 X 7 Is Shown Below Which Statements About The Function

Related Articles

- Manchester Clinic, A Nonprofit Organization, Estimates That It Can Savo $26,000 A Yoar In Cash Operating

- Pretend You Are Looking For Your Little Brother. Using As Many Words As You Can, Describe In French What
- Investment Required Annual Net Cash Inflow Is The Formula For The Blank_____. Multiple Choice Question.

Back to Home