Problem 9 (12 Pts.) Determine The Transfer Function For The Following ODE: 38 +30x + 63x = 5f (t) , X(0)

Problem 9 (12 Pts.) Determine The Transfer Function For The Following ODE: 38 +30x + 63x = 5f (t) , X(0)

Introduction to Transfer Function Analysis

Understanding the transfer function of a system is fundamental in the field of control systems and signal processing. It provides a mathematical relationship between the input and output in the frequency domain, enabling engineers to analyze system stability, response characteristics, and design appropriate controllers. In this problem, we are asked to determine the transfer function for a given ordinary differential equation (ODE), which models a dynamic system's behavior.

The specific ODE provided is:

38 + 30x + 63x = 5f(t), with initial condition X(0)

However, the notation here appears to be inconsistent or incomplete. Typically, an ODE involving a system's output x(t) and input f(t) would be expressed in terms of derivatives of x(t). Given the context, it is likely that the equation should involve derivatives of x(t), such as x'(t) or x''(t).

Assuming the intended ODE is:

38 + 30x(t) + 63x'(t) = 5f(t)

and the initial condition is:

x(0) = some value (possibly zero or unspecified)

This is a common form for a first-order linear differential equation. We will proceed with this assumption to derive the transfer function.

Understanding the Differential Equation

General Form of the System Equation

The typical form of a linear time-invariant (LTI) system's differential equation is:

$$a_n x^{(n)}(t) + a_{n-1} x^{(n-1)}(t) + ... + a_1 x'(t) + a_0 x(t) = b f(t)$$

where:

- $x(t)$ is the system output
- $f(t)$ is the input
- a_i are coefficients related to the system dynamics
- b is a coefficient relating input to output

Given the problem, and assuming the intended equation is:

$$38 + 30x(t) + 63x'(t) = 5f(t)$$

we can interpret this as a first-order differential equation with constant coefficients.

Rewriting the Equation

To analyze the transfer function, it is essential to express the equation in the standard differential form:

$$a_1 x'(t) + a_0 x(t) = b f(t)$$

or equivalently,

$$x'(t) + (a_0/a_1) x(t) = (b/a_1) f(t)$$

From the given, rearranged as:

$$63 x'(t) + 30 x(t) + 38 = 5 f(t)$$

Note: The constant term 38 is unusual in a standard differential equation involving derivatives. Usually, such terms are on the right or associated with initial conditions. If 38 is a constant offset, the system's differential equation is:

$$63 x'(t) + 30 x(t) = 5 f(t) - 38$$

However, for the purpose of transfer function derivation, constants like 38 are often treated as initial conditions or as offsets. Assuming the system is linear and time-invariant, and that we're analyzing the homogeneous part, we focus on the terms involving x(t) and its derivatives.

Therefore, the simplified differential equation becomes:

$63 x'(t) + 30 x(t) = 5 f(t)$

Deriving the Transfer Function

Step 1: Take the Laplace Transform of the Differential Equation

The Laplace Transform is a powerful tool for converting differential equations in the time domain into algebraic equations in the complex frequency domain (s-domain).

Applying the Laplace Transform to both sides:

$L\{63 x'(t) + 30 x(t)\} = L\{5 f(t)\}$

Using the properties:

- $L\{x'(t)\} = s X(s) - x(0)$
- $L\{x(t)\} = X(s)$

Assuming zero initial conditions ($x(0) = 0$) for simplicity, the equation becomes:

$63 [s X(s)] + 30 X(s) = 5 F(s)$

which simplifies to:

$(63 s + 30) X(s) = 5 F(s)$

Step 2: Solve for the Transfer Function H(s) = X(s)/F(s)

Rearranging:

$X(s)/F(s) = H(s) = 5 / (63 s + 30)$

This is the transfer function of the system, relating the Laplace transform of the output to that of the input.

Step 3: Express the Transfer Function in Standard Form

To make the transfer function more interpretable, factor out constants:

$H(s) = (5) / (63 s + 30)$

Dividing numerator and denominator by 3:

$H(s) = (5) / (3 (21 s + 10)) = (5/3) / (21 s + 10)$

Alternatively, express it as:

$H(s) = K / (\tau s + 1)$

where:

- K is the system gain
- τ is the time constant

Matching the denominator:

$21 s + 10 = 21 (s + 10/21)$

Thus:

$H(s) = (5/3) / [21 (s + 10/21)] = (5/3) / 21 1 / (s + 10/21)$

Simplify:

$H(s) = (5/3) / 21 1 / (s + 10/21) = (5/3 \times 1/21) / (s + 10/21) = (5/63) / (s + 10/21)$

Therefore,

Final transfer function:

$H(s) = (5/63) / (s + 10/21)$

Interpreting the Transfer Function

System Gain (K)

The numerator coefficient (5/63) indicates the system's static gain.

Time Constant (τ)

The pole of the transfer function is at:

$s = -10/21 \approx -0.476$

which indicates the system’s time constant:

$\tau = 1 / (\text{pole magnitude}) = 21 / 10 = 2.1$ seconds

This suggests the system responds with a time constant of approximately 2.1 seconds, meaning it takes about 2.1 seconds for the response to reach approximately 63.2% of its final value after a step input.

System Type and Stability

Since the pole is in the left-half of the s-plane (negative real part), the system is stable. Its response to inputs will decay over time, and it will not exhibit oscillations in the absence of complex conjugate poles.

Additional Considerations

Initial Conditions and Their Effect

While the derivation assumed zero initial conditions ($x(0) = 0$), in real systems, initial states can influence the transient response. When initial conditions are non-zero, they contribute additional terms in the Laplace domain, but the transfer function itself remains unchanged, as it is a ratio of output to input assuming zero initial conditions.

Effect of Constant Terms

The constant term 38 in the original ODE might represent an offset or bias. To analyze the system's response accurately, consider whether this term influences the steady-state output or initial response. If necessary, include it as a particular solution component or adjust initial conditions accordingly.

Practical Application of the Transfer Function

Knowing the transfer function enables engineers to:

- Predict how the system responds to various inputs.
- Design controllers to modify system behavior.
- Analyze stability margins and transient response characteristics.
- Simulate system behavior in the frequency domain.

Summary and Conclusions

- The given ODE was interpreted as 63 x'(t) + 30 x(t) = 5 f(t).
- Taking the Laplace Transform and solving for the ratio X(s)/F(s) yields the transfer function H(s).
- The derived transfer function is:

H(s) = (5/63) / (s + 10/21)

- The system has a single pole at s = -10/21, indicating stability with a time constant of 2.1 seconds.
- This transfer function provides a foundation for analyzing system response, designing controllers, and understanding system dynamics.

By understanding how to derive and interpret the transfer function from a differential equation, engineers can effectively analyze a wide range of physical systems, from mechanical to electrical and beyond.

References and Further Reading

- Ogata, K. (2010). Modern Control Engineering. Prentice Hall.
- Dorf, R. C., & Bishop, R. H. (2011). Modern Control Systems. Pearson.
- Nise, N. S. (2015). Control Systems Engineering. Wiley.
- Laplace Transform Theory and Applications.
- System Response and Transfer Function Analysis Tutorials.

Note: The derivation assumes the intended differential equation involves derivatives of x(t). If the original problem intended a different form, adjustments in the derivation would be necessary. Always verify the exact form of the ODE before proceeding with transfer function

Frequently Asked Questions

What is the general approach to finding the transfer function from the given differential equation 38x'' + 30x' + 63x = 5f(t)?

The approach involves taking the Laplace transform of the differential equation, assuming zero initial conditions, and then solving for the ratio of output X(s) to input F(s), which gives the transfer function G(s) = X(s)/F(s).

How do initial conditions affect the derivation of the transfer function in this problem?

If initial conditions are zero, the Laplace transform of derivatives simplifies, making it straightforward to derive the transfer function. In this case, since initial conditions are not specified, we assume zero initial conditions for simplicity.

What is the Laplace transform of the differential equation 38x'' + 30x' + 63x = 5f(t)?

Applying the Laplace transform (assuming zero initial conditions), we get $38s^2X(s) + 30sX(s) + 63X(s) = 5F(s)$.

How is the transfer function G(s) expressed in terms of the Laplace variable s for this ODE?

Solving for X(s)/F(s), the transfer function is $G(s) = 5 / (38s^2 + 30s + 63)$.

What are the key steps to verify the correctness of the derived transfer function?

Verify by checking the units, ensuring proper algebraic manipulation, and optionally comparing the response with known inputs or using simulation tools to confirm the transfer function's behavior.

What is the significance of the transfer function G(s) in analyzing the system described by the ODE?

The transfer function characterizes the system's input-output relationship in the frequency domain, allowing analysis of stability, transient, and steady-state responses to various inputs.

Additional Resources

Problem 9 (12 Pts.) Determine The Transfer Function For The Following ODE: 38 + 30x + 63x = 5f(t), X(0)

Introduction to Transfer Function Analysis

The process of determining the transfer function of a system described by a differential equation is fundamental in control systems engineering. The transfer function encapsulates the input-output relationship of a system in the Laplace domain, enabling engineers to analyze system stability, design controllers, and predict system behavior effectively. In the case of the given differential equation:

\[
38 + 30x(t) + 63x(t) = 5f(t),
\]

the goal is to derive its transfer function \(\frac{X(s)}{F(s)} \), which relates the Laplace transform of the system's output \(x(t) \) to the Laplace transform of the input \(f(t) \).

Understanding the Differential Equation

Before proceeding to the transfer function derivation, it’s crucial to interpret the given differential equation correctly. The original expression reads:

\[
38 + 30x + 63x = 5f(t),
\]

which appears to have some ambiguity due to the repetition of \(x(t)\). Typically, in such problems, the differential equation involves derivatives of $x(t)$, such as:

$$
a \frac{d^n x(t)}{dt^n} + \dots + b x(t) = c f(t),
$$

or a similar form. Given the problem statement, and to maintain consistency with standard transfer function derivation, we will interpret the equation as:

$$
38 + 30 \frac{d x(t)}{dt} + 63 x(t) = 5 f(t),
$$

where the derivatives are involved, as the original form likely intended to include a differential term. Assuming this, the equation becomes a first-order linear differential equation with constant coefficients, which is common in control systems.

Note: If the original problem differs in the actual derivatives involved, the methodology remains similar, with adjustments for order.

Step-by-Step Derivation of the Transfer Function

1. Express the Differential Equation in Standard Form

Assuming the differential equation is:

$$
38 + 30 \frac{d x(t)}{dt} + 63 x(t) = 5 f(t).
$$

Rearranged, it becomes:

$$
30 \frac{d x(t)}{dt} + 63 x(t) = 5 f(t) - 38.
$$

However, in control systems, constant terms in the differential equation typically represent initial conditions or system offsets. For the transfer function derivation, the standard form neglects such constants unless they influence the system dynamics directly. Since transfer functions relate to zero-input response and the system's inherent properties, the constant term 38 can be interpreted as an offset or a bias.

In the Laplace domain, initial conditions are represented as $X(0)$. If initial conditions are zero, the derivation simplifies.

For clarity, and in line with typical transfer function derivations, we focus on the homogeneous differential equation:

$$
30 \frac{d x(t)}{dt} + 63 x(t) = 5 f(t).
$$

---

2. Take Laplace Transform of Both Sides

Applying Laplace transforms, assuming zero initial conditions:

$$
30 \left[s X(s) - x(0)\right] + 63 X(s) = 5 F(s).
$$

If initial conditions are zero ($ x(0) = 0 $), then:

$$
30 s X(s) + 63 X(s) = 5 F(s).
$$

---

3. Solve for $X(s)/F(s)$

Factor $X(s)$:

$$
X(s) (30 s + 63) = 5 F(s),
$$

which leads to:

$$
\boxed{
\frac{X(s)}{F(s)} = \frac{5}{30 s + 63}.
}
$$

Simplify numerator and denominator:

$$
\frac{X(s)}{F(s)} = \frac{5}{3(10 s + 21)}.
$$

Or, equivalently:

$$
\boxed{
\frac{X(s)}{F(s)} = \frac{5/3}{10 s + 21}.
}
$$

This is the transfer function of the system.

Final Transfer Function and Interpretation

The transfer function derived is:

$$
H(s) = \frac{X(s)}{F(s)} = \frac{5/3}{10 s + 21}.
$$

This transfer function reveals several important features:

- It is a first-order system, characterized by a single pole at $s = -\frac{21}{10} = -2.1$.
- The numerator is a constant, indicating that the system's gain is frequency-independent and primarily determined by the ratio $\frac{5/3}{1}$.
- The pole's location indicates system stability; since the pole is in the left-half plane, the system is stable.

Features and Implications of the Transfer Function

Features:
- First-order system: The transfer function's form indicates a simple, first-order dynamic.
- Pole location: The pole at $s = -2.1$ suggests a stable, exponentially decaying response.

- Gain: The static gain is $\frac{5/3}{10 \times 0 + 21} = \frac{5/3}{21} = \frac{5}{3 \times 21} = \frac{5}{63}$.

Implications:
- Time response: The system's response to inputs will be characterized by a single exponential decay.
- Frequency response: The system acts as a low-pass filter, attenuating high-frequency inputs.
- Stability: The negative pole ensures that the system will settle to a steady state without oscillations.

Pros and Cons of the Derived Transfer Function

Pros:
- Simplicity: The first-order form enables straightforward analysis and controller design.
- Predictability: The exponential response can be readily computed.
- Stability assurance: The pole's position guarantees a stable system.

Cons:
- Limited dynamics: First-order systems cannot model complex behaviors like oscillations or overshoot.
- Constant offset: The original constant term (38) was omitted in the transfer function derivation, which could influence steady-state bias if not accounted for properly.
- Assumption of zero initial conditions: Real systems may have initial states that influence the transient response.

Conclusion and Final Remarks

The process of deriving the transfer function for the given differential equation illustrates the fundamental steps common to control systems analysis: interpret the differential equation, transform to the Laplace domain, assume initial conditions, and solve algebraically for the ratio $\frac{X(s)}{F(s)}$. The resulting transfer function,

$$
H(s) = \frac{5/3}{10 s + 21},
$$

provides a comprehensive representation of the system's dynamics, enabling further analysis such as stability assessment, frequency response, and controller design.

Understanding these steps and features offers valuable insights into system behavior and lays the foundation for designing robust control strategies to meet desired performance criteria. The simplicity of the first-order transfer function makes it an excellent example for educational purposes and initial system analysis.

Note: If the original differential equation involved higher derivatives or different constants, the methodology would adapt accordingly, involving higher-order Laplace transforms and more complex algebra.

[Problem 9 12 Pts Determine The Transfer Function For The Following Ode 38 30x 63x 5f T X 0]

Problem 9 12 Pts Determine The Transfer Function For The Following Ode 38 30x 63x 5f T X 0

Related Articles

- PLEASE HELP ME What Is A Major Risk The Trans-Alaska Pipeline Poses To The Environment?A. It Allows Many
- Ou Have A Portfolio That Is Equally Invested In Stock F With A Beta Of .94, Stock G With A Beta Of 1.36,
- Sophie Has The Ability To Work Around Impediments, Rise Above Obstacles, And Adapt To The Most Demanding

Back to Home